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The Werner Arithmetic

Frank H. Hall

Book II

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ARITHMETIC

ORAL AND WRITTEN

BOOK TWO—PARTS I AND II

BY

FRANK H. HALL

AUTHOR OF "ARITHMETIC READERS," AND "ARITHMETIC OF FARM AND WORKSHOP"

CHICAGO NEW YORK

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PREFACE.

A book may be as noteworthy for what it omits as for what it contains. "Arithmetical conundrums," "mathematical monstrosities," "examples," in which the thing exemplified is lost to the pupil in long rows of figures, and problems suited only to maturity, are omitted from this book. The work presented can be thoughtfully performed by students ten to thirteen years of age if they have been properly prepared for it in the preceding years.

As in Book I. of this series, classification is made subordinate to gradation. Every set of problems—every problem, has been selected with careful reference to the supposed thought-power of the pupil, and bears close relation to what precedes and to what follows.

The decimal arrangement of the parts of the book whereby a topic is re-presented on every tenth page, insures at once systematic review and the proper relating of the new to the old.

F. H. H.

WAUKEGAN, ILL., December, 1896.

CONTENTS.

	Pages.
"The Foundation"	5–9
Suggestions to Teachers	10
Simple Numbers	11, 21, 31, 41, 51, 61, etc.
Common Fractions	12, 22, 32, 42, 52, 62, etc.
Decimals	13, 23, 33, 43, 53, 63, etc.
Denominate Numbers	14, 24, 34, 44, 54, 64, etc.
Measurements	15, 25, 35, 45, 55, 65, etc.
Ratio and Proportion	16, 26, 36, 46, 56, 66, etc.
Percentage	17, 27, 37, 47, 57, 67, etc.
Percentage	18, 28, 38, 48, 58, 68, etc.
Review Problems	19, 29, 39, 49, 59, 69, etc.
Miscellaneous	20, 30, 40, 50, 60, 70, etc.
Definitions and Tables	271–288

THE FOUNDATION.

To THE TEACHER.—If the pupil into whose hands this book may fall has been trained along the lines suggested in the First Book of the Werner Arithmetics, "the foundation" upon which to make further arithmetical effort will have been properly laid, and but little attention to pages 5 to 9 of this book will be necessary. Otherwise much time must be devoted to securing a clear conception of the meaning and application of these fundamental problems before the work in the body of this book is attempted. Require the pupil, if necessary, to recite each problem again and again; 1st, reading it and telling its meaning; 2d, solving it; and, 3d, illustrating with a concrete problem. Then make other similar problems to be treated in a similar manner. Put great emphasis upon the meaning of the problem. If the child knows that the expression, $\frac{1}{2} \div \frac{1}{6}$, means, *find how many times* 1 *sixth is contained in* 1 *half*, he solves the problem without difficulty. Do not permit the pupil to "juggle with figures" and then ask him to explain the juggling. Rather, see that he has a clear conception of what is to be done, and allow him the privilege of doing it in a thoughtful manner.

To THE PUPIL.—Read each problem and (a) tell its meaning, (b) solve it, and (c) tell the suggested number story. Do this until you can easily give the meaning of all problems similar to these, solve them, and tell the suggested number stories without reference to the notes that follow.

ADDITION.	SUBTRACTION.	MULTIPLICATION.	DIVISION.	DIVISION.
$6\not{\phi} + 2\not{\phi} = *$	$6\not{\phi} - 2\not{\phi} = *$	$6\not{\phi} \times 2 = *$	$6\not{\phi} \div 2\not{\phi} = *$	$6\not{\phi} \div 2 = *$
$\frac{2}{3} + \frac{1}{4} = (1)$	$\frac{3}{4} - \frac{1}{3} = (2)$	$\frac{3}{4}$ bu. $\times 7 = (3)$	$4 \div \frac{1}{3} = (4)$	$\frac{1}{3}$ bu. $\div 4 = (5)$
$6\frac{3}{4} + 5\frac{1}{8} = (6)$	$6\frac{3}{4} - 5\frac{1}{8} = (7)$	$12\not{\phi} \times \frac{3}{4} = (8)$	$\frac{2}{3} \div \frac{1}{6} = (9)$	$6\frac{3}{4}$ yd. $\div 2 = (10)$
2 bu. 3 pk.	5 bu. 1 pk.	$\$\frac{3}{8} \times \frac{1}{4} = (12)$	$7\frac{1}{2} \div 2\frac{1}{2} = (14)$	$5\frac{3}{4}$ tons $\div 2 = (15)$
2 bu. 2 pk.	1 bu. 3 pk.	$12\not{\phi} \times 2\frac{3}{4} = (16)$	2 ft.)46 ft. (17)	2)46 ft.
(11)	(12)	$12\frac{1}{2} \times 3 = (19)$	$2\frac{1}{2}$ ft.)15 ft. (20)	(18)
		$12\frac{1}{2} \times 2\frac{1}{2} = (22)$	$2\frac{1}{4}$ bu.)$13\frac{1}{2}$ bu.	3)$27\frac{1}{2}$ ft.
			(23)	(21)

* See First Book of the Werner Arithmetics, pp. 89 and 155.

(1) $\frac{2}{3}$ plus $\frac{1}{4}$, means, $\frac{2}{3}$ *and* $\frac{1}{4}$. I can change thirds and fourths to twelfths; $\frac{2}{3} =$ ——— twelfths; $\frac{1}{4} =$ ——— twelfths; ——— twelfths and ——— twelfths = ——— twelfths. *Story—Harry played ball* $\frac{2}{3}$ *of an hour, and "hide and seek"* $\frac{1}{4}$ *of an hour; in all he played* ——— *twelfths of an hour.* (See First Book, pages 125, 135, 145, 147, 149.)

(2) $\frac{3}{4}$ minus $\frac{1}{3}$, means, $\frac{3}{4}$ *less* $\frac{1}{3}$. I can change fourths and thirds to twelfths. $\frac{3}{4} =$ ——— twelfths; $\frac{1}{3} =$ ——— twelfths; ——— twelfths

less ——— twelfths = ——— twelfths. *Story—William had a piece of wire* $\frac{3}{4}$ *of a foot long; he cut from it a piece* $\frac{1}{8}$ *of a foot long; what remained was ——— twelfths of a foot long.*

(3) $\frac{3}{4}$ of a bushel multiplied by 7, means, take 7 times three fourth-bushels; 7 times 3 fourth-bushels = ——— fourth-bushels, or ——— and ——— ——— bushels. *Story—A farmer fed to his horses* $\frac{3}{4}$ *bu. of oats each day for a week; in all he fed ——— and ——— ——— bushels.*

(4) 4 divided by $\frac{1}{3}$, means, find how many times 1 third is contained in 4. I can change 4 to thirds. 4 is 12 thirds. 1 third is contained in 12 thirds ——— times. *Story—The teacher divided 4 oranges among some boys, giving to each* $\frac{1}{3}$ *of an orange; there were ——— boys.*

(5) $\frac{1}{2}$ of a bushel divided by 4, means, find 1 fourth of $\frac{1}{2}$ of a bushel. 1 fourth of $\frac{1}{2}$ of a bushel = ——— ——— of a bushel. *Story—A boy divided* $\frac{1}{2}$ *of a bushel of oats equally among 4 ponies; each pony received ——— ——— of a bushel.*

(6) $6\frac{3}{4}$ plus $5\frac{1}{8}$, means, $6\frac{3}{4}$ *and* $5\frac{1}{8}$. $6\frac{3}{4}$ and $5\frac{1}{8}$ = ——— and ———. *Story—In one jar there were* $6\frac{3}{4}$ *lb. of butter; in another there were* $5\frac{1}{8}$ *lb.; in both there were ——— and ——— ——— pounds.*

(7) $6\frac{3}{4}$ minus $5\frac{1}{8}$, means, $6\frac{3}{4}$ *less* $5\frac{1}{8}$. $6\frac{3}{4}$ less $5\frac{1}{8}$ = ——— and ———. *Story—A grocer had* $6\frac{3}{4}$ *lb. cheese, from which he sold* $5\frac{1}{8}$ *lb.; there remained ——— and ——— ——— pounds.*

(8) 12¢ multipled by $\frac{3}{4}$, means, take $\frac{3}{4}$ of 12¢. 3 fourths of 12¢ = ——— cents. *Story—At 12¢ a pound,* $\frac{3}{4}$ *of a pound of meat costs ——— cents.*

(9) $\frac{2}{3}$ divided by $\frac{1}{6}$, means, find how many times 1 sixth is contained in 2 thirds. I can change thirds to sixths; 2 thirds = ——— sixths; 1 sixth is contained in 4 sixths ——— times. *Story—Mrs. Smith divided* $\frac{2}{3}$ *of a pie among some boys, giving to each* $\frac{1}{6}$ *of a pie; there were ——— boys.* (See First Book of the Werner Arithmetics, page 150.)

(10) $6\frac{3}{4}$ yd. divided by 2, means, find 1 half of $6\frac{3}{4}$ yards. 1 half of $6\frac{3}{4}$ yards = ——— and ——— ——— yards. *Story—A salesman cut* $6\frac{3}{4}$ *yd. of ribbon into 2 equal pieces; there were ——— and ——— ——— yards in each piece.*

(11) 2 bu. 3 pk. plus 2 bu. 2 pk., means, 2 bu. 3 pk. *and* 2 bu. 2 pk. 2 bu. 3 pk. and 2 bu. 2 pk. = ——— bu. ——— pk. *Story—A dealer put 2 bu. 3 pk. of oats into one bag and 2 bu. 2 pk. into another bag; in both there were ——— bushels ——— peck.*

(12) 5 bu. 1 pk. minus 1 bu. 3 pk., means, 5 bu. 1 pk. *less* 1 bu. 3 pk. 5 bu. 1 pk. less 1 bu. 3 pk. = ——— bu. ——— pk. *Story—Mr. Bean had 5 bu. 1 pk. of oats, from which he fed 1 bu. 3 pk; there remained ——— bushels ——— pecks.*

(13) $3\frac{3}{8}$ multiplied by $\frac{1}{2}$, means, take 1 half of 3 fifths of a dollar. 1 half of 3 fifths of a dollar = ——— ——— of a dollar. *Story—At* $\frac{3}{8}$ *of a dollar a yard, 1 half of a yard of ribbon is worth ——— ——— of a dollar.* (See First Book, page 114.)

(14) $7\frac{1}{2}$ divided by $2\frac{1}{2}$, means, find how many times $2\frac{1}{2}$ is contained in $7\frac{1}{2}$. I can change $2\frac{1}{2}$ and $7\frac{1}{2}$ to halves; $2\frac{1}{2}$ = ——— halves; $7\frac{1}{2}$ = ——— halves. ——— halves are contained in ——— halves ——— times. *Story—A farmer put* $7\frac{1}{2}$ *bushels of oats into bags, putting* $2\frac{1}{2}$ *bushels in each bag; there were* ——— *bags.*

(15) $5\frac{3}{4}$ tons divided by 2, means, find 1 half of $5\frac{3}{4}$ tons. 1 half of $5\frac{3}{4}$ tons = ——— and ——— ——— tons. *Story—*$5\frac{3}{4}$ *tons of coal were divided equally between two families; each family received* ——— *and* ——— ——— *tons.*

(16) 12¢ multiplied by $2\frac{3}{4}$, means, take 2 times 12¢ plus 3 fourths of 12¢ ($2\frac{3}{4}$ times 12¢). 2 times 12¢ plus 3 fourths of 12¢ = ——— cents. *Story—At* 12¢ *a pound* $2\frac{3}{4}$ *lb. of cheese cost* ——— *cents.*

(17) 46 ft. divided by 2 ft., means, find how many times 2 ft. are contained in 46 ft. 2 ft. are contained in 46 ft. ——— times. *Story—A mechanic cut* 46 *ft. of moulding into pieces, each piece being* 2 *ft. long; there were* ——— *pieces.*

(18) 46 ft. divided by 2, means, find 1 half of 46 ft. 1 half of 46 ft. = ——— ft. *Story—Henry divided* 46 *ft. of wire into* 2 *equal parts; each part was* ———*feet long.*

(19) $12\frac{1}{2}$ multiplied by 3, means, take 3 times $12\frac{1}{2}$ (3 times $\frac{1}{2}$ plus 3 times 12). 3 times $12\frac{1}{2}$ = ———. *Story—One side of a triangle having equal sides is* $12\frac{1}{2}$ *ft.; the perimeter of the triangle is* ——— *and* ——— ———*feet.* (See First Book, pages 53 and 63.)

(20) 15 feet divided by $2\frac{1}{2}$ feet, means, find how many times $2\frac{1}{2}$ feet are contained in 15 feet. I can change 15 ft. and $2\frac{1}{2}$ ft. to half-feet; 15 ft. = ——— half-feet; $2\frac{1}{2}$ feet = ——— half-feet; ——— half-feet are contained in ——— half-feet ——— times. *Story—A mechanic had* 15 *feet of moulding which he cut into pieces, each piece being* $2\frac{1}{2}$ *ft. long; there were* ——— *pieces.*

(21) $27\frac{1}{2}$ feet divided by 3, means, find 1 third of $27\frac{1}{2}$ feet. 1 third of $27\frac{1}{2}$ feet = ——— and ——— ——— feet. *Story—The perimeter of a triangle having equal sides is* $27\frac{1}{2}$ *feet; each side is* ——— *and* ——— ———*feet.*

(22) $12\frac{1}{2}$ multiplied by $2\frac{1}{2}$, means, take 2 times $12\frac{1}{2}$ plus $\frac{1}{2}$ of $12\frac{1}{2}$. 2 times $12\frac{1}{2}$ plus $\frac{1}{2}$ of $12\frac{1}{2}$ = ——— and ——— ———. *Story—At* \$$12\frac{1}{2}$ *per ton,* $2\frac{1}{2}$ *tons of hay cost* ——— *and* ——— ——— *dollars.*

(23) $13\frac{1}{2}$ bu. divided by $2\frac{1}{4}$ bu., means, find how many times $2\frac{1}{4}$ bu. are contained in $13\frac{1}{2}$ bu. I can change $2\frac{1}{4}$ and $13\frac{1}{2}$ to fourths. $2\frac{1}{4}$ = ——— fourths. $13\frac{1}{2}$ = ——— fourths. ——— fourths are contained in ——— fourths ——— times. *Story—A farmer had* $13\frac{1}{2}$ *bu. of oats which he put into bags, putting* $2\frac{1}{4}$ *bu. in each bag. There were* ——— *bags.*

ADDITION.	SUBTRACTION.	MULTIPLICATION.	DIVISION.	DIVISION.
.6 + .2 = *	.6 — .2 = *	.6 × 2 = *	.6 ÷ .2 = *	.6 ÷ 2 = *
.24 + .05 = (1)	.64 — .05 = (2)	.12 × 9 = (3)	.08 ÷ .02 = (4)	$.08 ÷ 2 = (5)
		20 × .2 = (8)	8 ÷ .4 = (9)	$6.36 ÷ 3 = (10)
.24 + .5 = (6)	.64 — .5 = (7)	20 × 3.2 = (13)	4 ÷ .05 = (14)	5.6 tons ÷ 4 = (15)
176.4	276.4	.1 × .1 = (16)	2.5 ÷ .05 = (17)	5)$24.5 (18)
148.75	148.23	.3 × .2 = (19)	$.5)$24.5 (20)	5)$8.2 (21)
(11)	(12)	64		
		2.3 (22)	$.05)$8.2 (23)	5)$24. (24)

* See First Book of the Werner Arithmetics, pages 155 and 157.

(1) .24 plus .05, means, 24 hundredths *and* 5 hundredths. .24 and .05 = ——— hundredths. *Story—William had* $.24; *he earned* $.05; *he then had* ———.

(2) .64 minus .05, means, 64 hundredths *less* 5 hundredths. .64 less .05 = ———hundredths. *Story—Martha had* $.64; *she spent* $.05; *she then had* ———.

(3) .12 multiplied by 9, means, take 9 times 12 hundredths. 9 times .12 = ——— hundredths, or ——— and ——— hundredths.

(4) .08 divided by .02, means, find how many times 2 hundredths are contained in 8 hundredths. .02 are contained in .08 ——— times. *Story—I paid* $.08 *for oranges at* $.02 *each; I bought* ——— oranges.

(5) $.08 divided by 2, means, find 1 half of 8 hundredths of a dollar. One half of $.08 = ———. *Story—I paid* $.08 *for* 2 *lemons; one lemon cost* ———.

(6) .24 plus .5, means, 24 hundredths *and* 5 tenths. 5 tenths = 50 hundredths. .24 and .50 = ——— hundredths. *Story—John had* $.24; *Alfred had* $.5; *together they had* ———.

(7) .64 minus .5, means, 64 hundredths *less* 5 tenths. 5 tenths = 50 hundredths. .64 less .50 = ——— hundredths. *Story—Sarah had* $.64; *Mary had* $.5; *Sarah had* $——— *more than Mary.*

(8) 20 multiplied by .2, means, find 2 tenths of 20. One tenth of 20 = ———; 2 tenths of 20 = ———. *Story—At* $20 *an acre,* .2 *of an acre of land would cost* ——— *dollars.*

(9) 8 divided by .4, means, find how many times 4 tenths are contained in 8. 8 = 80 tenths. 4 tenths are contained in 80 tenths ——— times. *Story—I paid* $8 *for potatoes at* $.4 (4 dimes) *a bushel; I bought* ——— *bushels.*

(10) $6.36 divided by 3, means, finds 1 third of $6.36. One third of $6.36 = ———. *Story—I paid* $6.36 *for* 3 *barrels of apples;* 1 *barrel cost* ———.

(11) The sum of 176.4 and 148.75 is ———.

(12) The difference of 276.4 and 148.23 is ———.

(12) 20 multiplied by 3.2, means, take 3 times 20 plus 2 tenths of 20. 3 times 20 = ———. 2 tenths of 20 = ———. 3 times 20 plus 2 tenths of 20 = ———. *Story—At* $20 *an acre,* 3.2 *acres of land are worth* ——— *dollars.*

(14) 4 divided by .05, means, find how many times 5 hundredths are contained in 4. 4 = 400 hundredths. 5 hundredths are contained in 400 hundredths ——— times. *Story—I paid* $4 *for tablets at* $.05 (5¢) *each; I bought* ——— *tablets.*

(15) 5.6 tons divided by 4, means, find 1 fourth of 5.6 tons. One fourth of 5.6 = ——— and ——— ——— tons. *Story—A farmer sold* 4 *loads of hay; the entire weight of the hay was* 5.6 *tons; the loads averaged* ——— *and* ——— ——— *tons.*

(16) .1 multiplied by .1, means, find 1 tenth of 1 tenth. One tenth of 1 tenth = ——— ———.

(17) 2.5 divided by .05, means, find how many times 5 hundredths are contained in 2.5. 2.5 = 250 hundredths. 5 hundredths are contained in 2.5 (250 hundredths) ——— times. *Story—I paid* $2.5 *for pencils at* $.05 (5¢) *each; I bought* ——— *pencils.*

(18) $24.5 divided by 5, means, find 1 fifth of $24.5. One fifth of $24.5 = ———. *Story—I paid* $24.5 *for* 5 *tons of coal;* 1 *ton cost* ———.

(19) .3 multiplied by .2, means, find 2 tenths of 3 tenths. One tenth of 1 tenth = ———. One tenth of 3 tenths = ———. Two tenths of 3 tenths = ———.

(20) $24.5 divided by $.5, means, find how many times 5 tenths of a dollar are contained in $24.5 (245 tenths dollars). 5 tenths are contained in 24.5 (245 tenths) ——— times. *Story—I paid* $24.5 *for apples at* $.5 (5 dimes) *a bushel; I bought* ——— *bushels.*

(21) $8.2 divided by 5, means, find 1 fifth of $8.2. One fifth of $8.2 ($8.20) = ———. *Story—I paid* $8.2 *for* 5 *yards of cloth;* 1 *yard cost* ———.

(22) 64 multiplied by 2.3, means, find 2 times 64 plus 3 tenths of 64. One tenth of 64 = ———; 3 tenths of 64 = ———; 2 times 64 = ———; 2 times 64 plus 3 tenths of 64 = ———. *Story—At* $64 *an acre,* 1 *tenth of an acre costs* ———; 3 *tenths of an acre cost* ———; 2 *acres cost* ———; 2.3 *acres cost* ———.

(23) $8.2 divided by $.05, means, find how many times 5 hundredths of a dollar are contained in $8.2. 8.2 = 820 hundredths. 5 hundredths are contained in 8.2 (820 hundredths) ——— times. *Story—I paid* $8.2 *for lemons at* $.05 (5¢) *each; I bought* ——— *lemons.*

(24) $24 divided by 5, means, find 1 fifth of $24. One fifth of $24 = ———. *Story—I paid* $24 *for* 5 *loads of straw;* 1 *load cost* ———.

SUGGESTIONS TO TEACHERS.

1.

Order of Procedure.

STEP 1.—Prepare the pupil by means of oral instruction for the work of a given page. This preparation may be, in part, the slow reading to the pupil of the *figure problems** upon the page, the teacher hesitating at each blank for the pupil to supply the word. In this preparatory work, no book should be in the hands of the pupil. New words should be first presented through the voice of the teacher in the expression of thought. They may then be written upon the blackboard by the teacher, erased, and the pupil exercised in both oral and written reproduction.

STEP 2.—The book should be put into the hands of the pupil and he should read, first silently, then orally, the *figure problems* upon any page for which proper preparation has been made.

STEP 3.—The pupil may attempt the *letter problems** at his desk without further assistance. If he is unable to solve these, review the *figure problems* and give others similar to them. In some instances it may be well to have the *figure problems* solved upon the blackboard as preparation for the *letter problems*.

2.

Wherever the word "*story*" follows a problem, as on pp. 12 and 13, require the pupil to make a statement showing how the problem might originate in business or other experience; thus, the story for problem (f), page 12, might be—*A man divided* $4\frac{3}{4}$ *acres of land into lots each containing* $\frac{1}{10}$ *of an acre. There were* 95 *lots.*

3.

The separatrix, as an aid in "pointing off" in multiplication and division of decimals, is mentioned in a note on page 133. Its use is illustrated on pp. 143, 153, 253. Its occasional use in blackboard work earlier in the course may be helpful; but a too early reliance upon formal rules is to be avoided if the purpose of the work is the development of the thought-power of the pupil.

* The *figure problems* are those designated by figures; the *letter problems*, those designated by letters.

THE WERNER ARITHMETIC

BOOK TWO—PART ONE.

Simple Numbers.

Teach orally the meaning and use of the expressions, **exact divisor, exactly divisible, odd number, even number.**

1. Three is an exact divisor of 6, of 9, of ——, of ——, of ——, and of ——. Four is an exact divisor of ——.

2. Eighteen is exactly divisible by 2, by ——, by —— and by ——. 28 is exactly divisible by ——.

3. Two is an exact divisor of 4, 6, ——, ——, ——, etc.

4. A number that is exactly divisible by 2 is called an even number; 8, 10, ——, ——, ——, are even numbers.

5. Two is not an exact divisor of 1, 3, 5, ——, ——, etc.

6. A number that is not exactly divisible by 2 is called an odd number; 9, 11, —— ——, ——, ——, are odd numbers.

7. 37 is an —— number. 24 is an —— number.

8. Tell which of the following are odd numbers and which are even numbers: 375, 256, 320, 197, 281, 378, 584, 252, 323, 569.

(a) Copy the numbers given in problem 8 and find their sum.*

(b) Find the sum of all the odd numbers from 1 to 15 inclusive.

(c) Find the sum of all the even numbers from 2 to 16 inclusive.

* Problems *designated by letters* are for the slate.

Common Fractions.

Teach orally the meaning and use of the expression, **common denominator.** Pupils must be prepared for this page by work similar to that found in the Werner Arithmetic, Book I., pp. 135 and 145.

1. When two or more fractions have denominators that are alike, they are said to have a *common denominator.* $\frac{1}{12}$, $\frac{7}{12}$, $\frac{5}{12}$, and $\frac{9}{12}$ have a ——— ———. $\frac{2}{3}$ and $\frac{5}{7}$ do not have a ——— ———.

Reduce to equivalent fractions having a common denominator.

2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. (Change to 12ths.) $\frac{1}{2}$ = $\frac{1}{3}$ = $\frac{1}{4}$ =

3. $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$. (Change to ———.) $\frac{1}{5}$ = $\frac{1}{4}$ = $\frac{1}{2}$ =

4. $\frac{1}{6}$, $\frac{1}{7}$. (Change to ———.) $\frac{1}{6}$ = $\frac{1}{7}$ =

5. $\frac{1}{4}$, $\frac{1}{7}$. (Change to ———.) $\frac{1}{4}$ = $\frac{1}{7}$ =

(a) $\frac{3}{4}$ and $\frac{2}{7}$. (b) $\frac{3}{8}$ and $\frac{5}{7}$. (c) $\frac{3}{8}$ and $\frac{2}{5}$.

6. Add $\frac{5}{8}$ and $\frac{1}{3}$. I can change eighths and thirds to ———ths. $\frac{5}{8} = \frac{\quad}{24}$. $\frac{1}{3} = \frac{\quad}{24}$. ——— twenty-fourths and ——— twenty-fourths = ——— twenty-fourths.

(d) Find the sum of $576\frac{5}{8}$ and $239\frac{1}{3}$.

7. From $\frac{5}{7}$ subtract $\frac{1}{3}$. I can change sevenths and thirds to ———sts. $\frac{5}{7} = \frac{\quad}{21}$. $\frac{1}{3} = \frac{\quad}{21}$. ——— twenty-firsts less ——— twenty-firsts = ——— twenty-firsts.

(e) Find the difference of $572\frac{5}{7}$ and $368\frac{1}{3}$.

8. Divide $\frac{3}{4}$ by $\frac{1}{20}$. This means, find how many times $\frac{1}{20}$ is contained in $\frac{3}{4}$. I can change $\frac{3}{4}$ to twentieths. $\frac{3}{4} = \frac{\quad}{20}$. 1 twentieth is contained in ——— twentieths ——— times. *Story.**

(f) Find the quotient of $4\frac{3}{4}$ divided by $\frac{1}{20}$. (Change $4\frac{3}{4}$ to twentieths.) *Story.*†

(g) $147\frac{2}{7} + 38\frac{2}{3}$. (h) $374\frac{5}{8} - 181\frac{1}{3}$. (i) $27\frac{1}{2}$ bu. + $2\frac{1}{2}$ bu.‡

* See Note (9) page 6. † See paragraph 2, page 10.

‡ Tell the meaning. Change $27\frac{1}{2}$ and $2\frac{1}{2}$ to halves. *Story.* See note (14) page 7.

Decimals—Thousandths.°

Teach one pupil to write thousandths decimally; as, seven thousandths (.007), thirty-seven thousandths (.037), two hundred thirty-seven thousandths (.237), four thousand two hundred thirty-seven thousandths (4.237).

Teach the pupil to read such expressions as 2.345 in two ways; thus, (a) *two and three hundred forty-five thousandths;* (b) *two thousand three hundred forty-five thousandths.* Also lead the pupil to perceive that the expression, 2.345, stands for a number that is made up of 2 wholes, 3 tenths, 4 hundredths, and 5 thousandths; that 3 tenths = 300 thousandths; that 4 hundredths = 40 thousandths. All this must be made clear to the pupil by means of oral instruction *before the work that follows is attempted.*

The problems designated by figures should be solved twice; first without a pencil, then with a pencil. They thus become the stepping-stones to the problems designated by letters, which are for the slate.

1. .125 and .006 are ——— thousandths.
2. .125 and .06 are ——— thousandths.
3. .125 and .6 are ——— thousandths.
4. .275 less .006 are ——— thousandths.
5. .275 less .06 are ——— thousandths.
6. Add 2.4 and 3.104. (a) 27.006 + 14.8 + 25.06 =
7. From 36.025 subtract 1.008. (b) 8.175 − 3.023 =
8. Multiply 2.005 by 3. (c) 26.008 × 4 =
9. Divide .035 by .007. (This means ——— 4* ———.)

(d) Find the quotient of .375 divided by .005. *Story.*†

10. Divide .035 by 7. (This means ——— 5* ———.)

(e) Find the quotient of 35.049 acres divided by 7. *Story.*

(f) Add.	(g) Subtract.	(h) Multiply.	(i) Divide.	(j) Divide.
46.27	54.75	7.054‡	$.005)$.385§	5)$.385§
25.308	8.326	5		

° See the Werner Arithmetic, Book I., pp. 155 and 195.

* These figures refer to notes on page 8.

† See page 10, paragraph 2.

‡ Write the decimal point in the product *immediately after writing the tenths' figure of the product.*

§ Lead the pupil to "point off" in division of decimals by thinking what the problems mean. Problem (i) means, *find how many times* 5 *thousandths are contained in* 385 *thousandths.* 5 thousandths are contained in 385 thousandths 77 times. Problem (j) means, *find* 1 *fifth of* 385 *thousandths.* ⅕ of 385 thousandths is 77 thousandths, or .077.

Denominate Numbers.

2000 lb. = 1 ton.

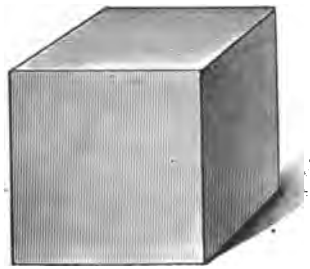
1. 500 lb. are ——— ——— of a ton.
2. 1500 lb. are ——— ——— of a ton.
3. 1000 lb. are ——— ——— of a ton.
4. 200 lb. are ——— ——— of a ton.
5. I weigh ——— pounds.
6. My father's horse weighs ——— pounds.
7. A keg of nails weighs 100 pounds.
8. One ton of nails is ——— kegs.
9. A load of hay weighs from ——— to ——— pounds.*
10. A load of coal weighs from ——— to ——— pounds.*
11. When hay is $12 a ton, 1000 lb. cost ———.
12. When coal is $8 a ton, 1500 lb. cost ———.
13. When meal is $16 a ton, 500 lb. cost ———.
14. When hay is $6 a ton, 200 lb. cost ———.
15. When hay is $7 a ton, 200 lb. cost ———.
16. When hay is $5 a ton, 200 lb. cost ———.
17. When hay is $8 a ton, 200 lb. cost ———.
18. When hay is $6 a ton, 2200 lb. cost ———.
19. When hay is $8 a ton, 2200 lb. cost ———.
20. When hay is $5 a ton, 2200 lb. cost ———.
21. 2500 lb. are ——— and ——— ——— tons.
22. 3000 lb. are ——— and ——— ——— tons.
23. 4500 lb. are ——— and ——— ——— tons.
24. 5000 lb. are ——— and ——— ——— tons.
25. When hay is $12 a ton, 2500 lb. cost ———.
26. When hay is $10 a ton, 3000 lb. cost ———.
27. When hay is $8 a ton, 4500 lb. cost ———.
28. When hay is $6 a ton, 5000 lb. cost ———.
29. When hay is $12 a ton, 3500 lb. cost ———.
30. When hay is $9 a ton, 3000 lb. cost ———.

* When you see a load of hay or a load of coal, courteously ask the man in charge of it to tell you how much it weighs.

Measurements.

By means of oral instruction, make the pupil familiar with the expressions, cubic inch, 1-inch cube, 2-inch cube, 2-foot cube, solid content, etc.

At least, 27 1-inch cubes should be provided. With these the pupils should build, first a 2-inch cube then a 3-inch cube.



1-INCH CUBE.

Tell the pupil that a cubic inch is a 1-inch cube *or its equivalent;* a cubic foot, a 1-foot cube *or its equivalent.*

1. A 2-inch square contains — — square inches.*
2. A 2-inch cube contains —— cubic inches.
3. A 3-inch square contains —— square inches.
4. A 3-inch cube contains —— cubic inches.

5. A 2-foot square contains —— square feet.
6. A 2-foot cube contains —— cubic feet.
7. A 3-foot square contains —— square feet.
8. A 3-foot cube contains —— cubic feet.

9. The area of a 2-inch square is —— —— inches.
10. The solid content of a 2-inch cube is ————.
11. The area of a 3-inch square is —— —— inches.
12. The solid content of a 3-inch cube is ————.

13. The area of a 2-foot square is —— —— feet.
14. The solid content of a 2-foot cube is ————.
15. Find the area of—

    (1) A 4-inch square.
    (2) A 5-inch square.
    (3) A 6-inch square.
    (4) A 7-inch square.
    (5) A 12-inch square.

    Find the solid content of—†

    (a) A 4-inch cube.
    (b) A 5-inch cube.
    (c) A 6-inch cube.
    (d) A 7-inch cube.
    (e) A 12-inch cube.

* If necessary, review the measurement work in Werner Arithmetic, Book 1, found on pages 53, 63, 73, 83, 93, etc.

† Problems designated by letters are for the slate.

Ratio and Proportion.

1. One fourth of 20 is ——. 20 is $\frac{1}{4}$ of ——.
2. One half of 11 is ——. 11 is $\frac{1}{2}$ of ——.
3. One third of 10 is ——. 10 is $\frac{1}{3}$ of ——.
4. One fifth of 45 is ——. 45 is $\frac{1}{5}$ of ——.

5. Two thirds of 60 are ——. 60 is $\frac{2}{3}$ of ——.*
6. Two thirds of 36 are ——. 36 is $\frac{2}{3}$ of ——.*
7. Two thirds of 15 are ——. 15 is $\frac{2}{3}$ of ——.
8. Two thirds of 21 are ——. 21 is $\frac{2}{3}$ of ——.

9. 10 is —— of 15. 15 is ——† of 10, or — times 10.
10. 15 is —— of 20. 20 is —— of 15, or — times 15.
11. 20 is —— of 25. 25 is —— of 20, or — times 20.
12. 25 is —— of 30. 30 is —— of 25, or — times 25.

13. Three fourths of 16 are 2 thirds of ————.‡
14. Two thirds of 15 are 1 half of ————.
15. Three fourths of 24 are 2 thirds of ————.
16. Two thirds of 12 are 1 third of ————.

17. Eight is —— —— of 12. A man can earn —— —— as much in 8 days as he can earn in 12 days. If he can earn $21 in 12 days, in 8 days he can earn —— dollars.

18. Fifteen is —— and —— —— times 6. A man can earn —— times as much in 15 days as he can earn in 6 days. If he can earn $10 in 6 days, in 15 days he can earn —— dollars.

(a) If a man can earn $104 in 6 weeks, how much can he earn in 15 weeks?

* It may be well after the pupil has solved "mentally" such problems as Nos. 5 and 6, to *require him* to solve them with a pencil—this, as a preparation for similar problems in which larger numbers are used.

† Allow the pupil to say, "15 is 3 halves of 10," but remind him that the expression "3 halves of 10," means, 3 *times* 1 *half of* 10

‡ If the pupil finds difficulty with this problem, require him to pause after saying "16," until he thinks how many 3 fourths of 16 are.

Percentage.*

Tell the pupil that *per cent means hundredth or hundredths.* Teach the meaning and use of the sign %. Note that there are *three cases* in percentage.

50 per cent = .50 = $\frac{1}{2}$.
25 per cent = .25 = $\frac{1}{4}$.
$33\frac{1}{3}$ per cent = .$33\frac{1}{3}$ = $\frac{1}{3}$.
20 per cent = .20 = $\frac{1}{5}$.

(1)†	(2)†
1. 50 per cent of 10 =	10 is 50 % of ——.
2. 25 per cent of 12 =	12 is 25 % of ——.
3. $33\frac{1}{3}$ per cent of 12 =	12 is $33\frac{1}{3}$ % of ——.
4. 20 per cent of 10 =	10 is 20 % of ——.
5. 50 per cent of 8 =	8 is 50 % of ——.
6. 25 per cent of 8 =	8 is 25 % of ——.
7. 20 per cent of 15 =	15 is 20 % of ——.
8. $33\frac{1}{3}$ per cent of 15 =	15 is $33\frac{1}{3}$ % of ——.

(3)†	(3)†
9. 5 is —— per cent of 10.	5 is —— % of 20.
10. 5 is —— per cent of 15.	5 is —— % of 25
11. 4 is —— per cent of 8.	4 is —— % of 12.
12. 4 is —— per cent of 16.	4 is —— % of 20.
13. 3 is —— per cent of 6.	3 is —— % of 15.
14. 3 is —— per cent of 9.	3 is —— % of 12.

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15. Henry had 25 cents; he spent 20 per cent of his money; he spent —— cents.

16. Peter spent 10 cents; this was 25 per cent of all he earned; he earned —— cents.

17. Roscoe earned 60 cents and spent 30 cents; he spent —— per cent of what he earned.

* To the Teacher.—Frequently review this page and *give many similar oral problems* as a preparation for page 27.

† These figures designate the *percentage cases* to which the problems below them belong.

Percentage.

(*1*)

1. 50 % of 18 is ——. 50 % of 19 is ——.

(a) Find 50 per cent of 724;* (b) of 725.

2. 25 % of 16 is ——. 25 % of 17 is ——.

(c) Find 25 per cent of 896; (d) of 897.

3. 33⅓ % of 18 is ——. 33⅓ % of 19 is ——.

(e) Find 33⅓ per cent of 726; (f) of 727.

4. 20 % of 25 is ——. 20 % of 26 is ——.

(g) Find 20 per cent of 875; (h) of 877.

(*2*)

5. 7 is 50 % of ——. 7½ is 50 % of ——.

(i) 376 is 50 % of what?† (j) 376½ is 50 % of what?

6. 9 is 25 % of ——. 9½ is 25 % of ——.

(k) 524 is 25 % of what? (l) 524½ is 25 % of what?

7. 8 is 33⅓ % of ——. 8½ is 33⅓ % of ——.

(m) 652 is 33⅓ % of what? (n) 652½ is 33⅓ % of what?

8. 7 is 20 % of ——. 7½ is 20 % of ——.

(o) 478 is 20 % of what? (p) 478½ is 20 % of what?

(*3*)

9. 11 is —— % of 22.‡ 11 is —— % of 33.

10. 11 is —— % of 44. 11 is —— % of 55.

11. 15 is —— % of 30. 30 is —— % of 60.

12. 30 is —— % of 120. 30 is —— % of 150.

(*1*)

13. One per cent (1 %) of a number is 1 hundredth of the number; 2 % is 2 hundredths; 3 % is 3 hundredths.

14. One per cent of 500 is ——. 2 % of 500 is ——.

15. One per cent of 600 is ——. 3 % of 600 is ——.

* Problems designated *by letters* are for the slate. Find 50 % of 724 by finding ½ of 724.

† Lead the pupil to see that he can solve this problem by finding the number of which 376 is 1 half.

‡ 11 is (½) one half of 22; therefore it is 50 % of 22.

Review.

1. Tell which of the following are odd numbers and which are even numbers: 14, 17, 19, 20, 24, 27.

2. The fractions $\frac{1}{15}$, $\frac{7}{15}$, and $\frac{11}{15}$ have a —— denominator.

3. Change $\frac{2}{3}$ and $\frac{4}{5}$ to equivalent fractions having a common denominator. I can change thirds and fifths to ——. $\frac{2}{3}$ = $\frac{4}{5}$ =

4. Add $\frac{3}{7}$ and $\frac{1}{3}$. I can change 7ths and 3rds to ——. $\frac{3}{7}$ = $\frac{9}{21}$. $\frac{1}{3}$ = $\frac{7}{21}$. —— twenty-firsts and —— twenty-firsts = —— twenty-firsts.

5. Divide $\frac{3}{5}$ by $\frac{1}{20}$. This means, find how many times $\frac{1}{20}$ is contained in $\frac{3}{5}$. I can change 5ths to ——. $\frac{3}{5}$ = $\frac{12}{20}$. One twentieth is contained in —— twentieths —— times.*

6. Multiply 6.05 by 3. Three times ————.

(a) Find the product of 34.02 multiplied by 4.

7. Divide $6.28 by 2. (This means, —— 10† ——.)

(b) Find the quotient of $475.65 divided by 5.

8. When hay is $12 a ton, 1500 lb. cost —— dollars.

9. When coal is $7 a ton, 3000 lb. cost —— dollars.

10. A 2-inch square is how many times as large as a 1-inch square?

11. A 2-inch cube is how many times as large as a 1-inch cube?

(c) How many square inches in an oblong 9 inches by 23 inches?

12. Three fourths of 20 are 1 half of ——.

(d) Three fourths of 84 are 1 half of what number?

13. Two thirds of 15 are 1 third of ——.

(e) Two thirds of 69 are 1 third of what number?

* Tell a suggested number story. See note (9), page 6.

† See note (10), page 8.

Miscellaneous.

1. Two thirds of 18 are ——. $\frac{2}{3}$ of 21 =

(a) Find $\frac{2}{3}$ of 48; (b) of 348.

2. Mr. A. owned $15\frac{1}{2}$ acres of land; he sold $6\frac{1}{4}$ acres; he then had —— acres.

(c) Mr. B. owned $546\frac{1}{2}$ acres of land; he sold $228\frac{1}{4}$ acres. How many acres had he remaining?

3. A bushel of oats weighs 32 lb.; 2 bushels of oats weigh —— pounds; 3 bushels weigh —— pounds.

(d) Sixty bushels of oats weigh how many pounds less than one ton?

4. Two pounds of coffee @ $.23 a pound cost ——.

(e) Find the cost of 84 pounds of coffee @ $.23 a pound.*

5. From a piece of cloth containing 12 yards, there were sold $5\frac{1}{2}$ yards and $2\frac{1}{2}$ yards; there were left —— yards.

(f) From a piece of cloth containing 55 yards, there were sold $24\frac{5}{8}$ yd. and $17\frac{1}{4}$ yd. How many yards were left?

6. A boarding school uses one gallon of milk each day; at 5¢ a quart the milk for one week costs ——.

(g) A boarding school uses 4 gal. of milk each day. At 5¢ a quart, how much will the milk for the month of October cost?

7. John bought 20 oranges at 3¢ each and sold them at 5¢ each; he gained —— cents.

(h) John's father bought 25 bushels of potatoes at 32¢ a bushel and sold them at 45¢ a bushel. How much did he gain?

* In multiplication of decimal fractions, require the pupil to locate the decimal point in the product *immediately after* he has written the figure to the right of the decimal point in any whole or partial product. Thus, in the multiplication of $.23 by 84 he writes (and thus locates) the decimal point of the product immediately after writing the figure 9 in the first partial product. This will be quite clear to the pupil if he understands that he first finds 4 times .23; then 80 times .23; then the sum of these two partial products.

$$\begin{array}{r} \$.23 \\ 84 \\ \hline .92 \\ 18.4 \\ \hline 19.32 \end{array}$$

Simple Numbers.

Teach orally the meaning and use of the expressions, **integral number, fractional number, mixed number.**

1. Seventeen is an *integral number.*
2. Five eighths is a *fractional number.*
3. Seventeen and five eighths is a *mixed number.*

4. 25 is an ——— number. $5\frac{1}{2}$ is a ——— number.
5. $\frac{3}{4}$ is a ——— number. 3.5 is a ——— number.
6. .8 is a ——— number. 18 is an ——— number.

7. *A number is exactly divisible by* 2 *if the right-hand figure is* 0, 2, 4, 6, *or* 8. Tell which of the following are exactly divisible by 2: 241, 136, 274, 393, 247, 826.

8. *A number is exactly divisible by* 5 *if its right-hand figure is* 0 *or* 5. Tell which of the following are exactly divisible by 5: 184, 275, 320, 145.

9. The following are exactly divisible by 2: 16, 38, 54, 68, ———, ———, ———, ———.

10. The following are exactly divisible by 5: 85, 140, 175, 180, ———, ———, ———, ———.

11. The following are exactly divisible by 10: 30, 40, 80, 120, ———, ———, ———, ———.

12. There are twice as many 5's as there are 10's in a number. 90 is ——— tens. 90 is ——— fives.

13. Ten is contained in 150 ——— times; 5 is contained in 150 ——— times.

(a) Divide $2150 by 50.* (b) Divide $215.00 by 50.†

(c) Divide $2295 by 51. (d) Divide $290.70 by 51.‡

(e) Divide $2058 by 49. (f) Divide $156.80 by 49.

(g) Divide $3276 by 52. (h) Divide $322.40 by 52.

(i) Divide $2496 by 48. (j) Divide $254.40 by 48.

* This means, find 1 fiftieth of $2150. 1 fiftieth of $2150 is —— dollars. *Story*—If 50 acres of land cost $2150, 1 acre cost ——.

† This means, find 1 fiftieth of $215.00. 1 fiftieth of $215.00 is ——. *Story*—If 50 tons of coal cost $215, 1 ton cost ———.

‡ In solving such problems as (b), (d), etc., require the pupil to write the decimal point in the quotient, *immediately after writing the units' figure* of the quotient. If the pupil thinks what the problem means, he will easily "point off" correctly.

Common Fractions.*

Tell the pupil that the **terms of a fraction** are its numerator and its denominator. Teach the meaning and use of the expression, *Reduce to its lowest terms.*

Reduce to their lowest terms:

1. $\frac{8}{12}$ = $\frac{15}{20}$ = $\frac{16}{20}$ = $\frac{80}{30}$ = $\frac{15}{30}$ = $\frac{15}{35}$ = $\frac{10}{40}$ =
2. $\frac{14}{20}$ = $\frac{20}{25}$ = $\frac{18}{30}$ = $\frac{18}{30}$ = $\frac{18}{30}$ = $\frac{18}{20}$ = $\frac{20}{50}$ =

Reduce to equivalent fractions having a common denominator:

3. $\frac{1}{4}$ and $\frac{1}{3}$. (Change to ———.) $\frac{1}{4}$ = $\frac{1}{3}$ =

(a) $\frac{2}{5}$ and $\frac{3}{5}$. (b) $\frac{3}{4}$ and $\frac{5}{6}$. (c) $\frac{3}{5}$ and $\frac{7}{3}$.

4. Add $\frac{3}{5}$ and $\frac{1}{3}$. (Change to ———ths.)†

(d) Find the sum of $248\frac{3}{5}$ and $467\frac{1}{3}$.

5. From $\frac{3}{4}$ subtract $\frac{2}{5}$. (Change to ———ths.)‡

(e) Find the difference of $837\frac{3}{4}$ and $284\frac{2}{5}$.

6. Divide 6 by $\frac{2}{3}$. This means, find how many times $\frac{2}{3}$ are contained in 6. I can change 6 to thirds. 6 = $\frac{18}{3}$. 2 thirds are contained in ——— thirds ——— times. *Story.*§

(f) Find the quotient of 48 divided by $\frac{2}{3}$. *Story.*‖

7. Divide $8\frac{1}{2}$ feet by 2. This means, find 1 half of $8\frac{1}{2}$ feet. 1 half of $8\frac{1}{2}$ feet = *Story.*

(g) Find the quotient of $86\frac{1}{2}$ miles divided by 2. *Story.***

(h) $3\frac{1}{2}$ miles)154 miles. This means, *find how many times* $3\frac{1}{2}$ *miles are contained in* 154 *miles.* (Change $3\frac{1}{2}$ and 154 to halves.) *Story*—A canal boat moved at the rate of $3\frac{1}{2}$ miles an hour; to move 154 miles would require ——— hours.

(i) 3)$165\frac{1}{2}$ miles. This means, *find* 1 *third of* $165\frac{1}{2}$ *miles.* *Story*—A train moved $165\frac{1}{2}$ miles in 3 hours; it moved at the rate of ——— miles an hour.

* Review page 12. † See page 12, problem 6.
‡ See page 12, problem 7. § See note (4), page 6.
‖ 48 feet of ribbon was cut into pieces $\frac{2}{3}$ of a foot long; there were —— pieces.
** A train moved $86\frac{1}{2}$ miles in 2 hours; this was at the rate of —— miles an hour

Decimal Fractions.

Teach the pupil to read in the several ways indicated and to write such expressions as the following: $.245 = 245 thousandths of a dollar; = 2 dimes, 4 cents, and 5 mills; = 2 tenths, 4 hundredths, and 5 thousandths of a dollar. The pupil should also be led to perceive that $.245 = 245 mills; that $3.245 = 3245 mills, etc.

1. One thousandth of a dollar is ——— mill.
2. Four thousandths of a dollar are 4 ———.
3. Four hundredths of a dollar are 4 ———.
4. Four tenths of a dollar are 4 ———.
5. $.375 = 3 ———, 7 ———, 5 ———, or 375 ———.
6. $2.746 = ——— mills. $2.50 = ——— mills.

7.	.1 of $60 =	.1 of $4 =	.1 of $64 =*
8.	.1 of $.1 =	.1 of $.5 =	.1 of $.7 =
9.	.1 of $6 =	.1 of $8.5 =	1 of $5.3 =
10.	.1 of $.01 =	.1 of $.03 =	.1 of $.07 =
11.	.1 of $6.42 =	.1 of $7.56 =	.1 of $8.42 =
12.	.1 of $875 =	.1 of $74.2 =	.1 of $5.35 =*

(a) Multiply $374 by .3. This means, *find* 3 *tenths of* $374.

Operation.	*Explanation.*
$374 †	One tenth of $374 =
.3	Three tenths of $374 =
$112.2	

Number Story.

If one acre of land is worth $374,
1 tenth of an acre of land is worth ———.
3 tenths of an acre of land are worth ———.

(b) Multiply.	(c) Multiply.	(d) Multiply.	(e) Divide.	(f) Divide.
346	23.5	2.73	$.05)$.85 ‡	5)$.85 §
.3	.3	3		

* By means of problems 7 to 12 and other similar exercises, lead the pupil to see that he can obtain 1 tenth of an integral number by "pointing off" one figure on the right, and of a mixed decimal by removing the point one place to the left.

† The pupil should understand tnat he multiplies $37.4 (not $374) by 3, and should be taught to write the decimal point in the product *immediately after writing the tenths' figure of the product.* See page 10, paragraph 3.

‡ This means, *find how many times* 5 *hundredths are contained in* 85 *hundredths.*

§ This means, *find* 1 *fifth of* 85 hundredths.

Denominate Numbers.

A bushel of oats weighs 32 lb.
A bushel of wheat weighs 60 lb.
A bu. of corn (shelled) weighs 56 lb.
A bu. of corn (not shelled) weighs 70 lb.*

1. Two bushels of oats weigh ——— lb.; 5 bu. weigh ———.

(a) How much more than 1 ton do 75 bu. of oats weigh?

2. Two bu. of wheat weigh ———; 3 bu. weigh ———.

(b) Seventy-five bushels of wheat weigh how much more than two tons? (c) How much less than three tons?

3. Two bushels of shelled corn weigh ———; 3 bu. weigh ———.

(d) Forty-five bushels of shelled corn weigh how much more than 1¼ tons? (e) How much less than 1½ tons?

4. Two bushels of corn (not shelled) weigh ———; 3 bushels weigh ———; 4 bushels weigh ———.

(f) How much more than three fourths of a ton do 25 bushels of corn (unshelled) weigh? (g) How much less than one ton?

5. One hundred pounds of oats are ——— bushels and ——— pounds, or ——— and ——— 32nds bushels.

(h) Five hundred pounds of oats are how many bushels?

6. Two hundred pounds of wheat are ——— bushels and ——— pounds, or ——— and ——— ——— bushels.

(i) How many bushels in one and one half tons of wheat?

7. One hundred pounds of corn (not shelled) are ——— bushel and ——— pounds, or ——— and ——— ——— bushels.

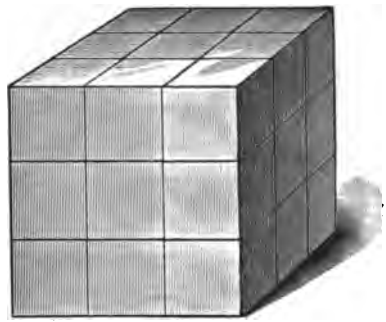
(j) How many bushels in two tons of unshelled corn?

(k)	(l)	(m)	(n)
32 lb.)768 lb.†	32 lb.)769 lb.	32 lb.)770 lb.	32 lb.)772 lb.

* Explain to the pupil that the expression "a bushel of corn (not shelled)," means, *the amount of unshelled corn required to make 1 bushel of shelled corn.*

† 32 lb. are contained in 768 lb. 24 times. *Story—In 768 lb. of oats there are 24 bushels.*

Measurements.



1. A 3-foot square (or its equivaient) is called a square yard. It contains —— square feet.

2. A 3-foot cube (or its equivalent) is called a cubic yard. It contains —— cubic feet.

3. The area of a 4-foot square is —— square feet, or —— and —— —— square yards.

4. The solid content of a 4-foot cube is —— cubic feet.

(a) How many cubic yards in a 4-foot cube?

5. The area of a 5-foot square is —— square feet, or —— and —— —— square yards.

6. The solid content of a 5-foot cube is —— cubic feet.

(b) How many cubic yards in a 5-foot cube?

7. The area of a 6-foot square is —— square feet, or —— square yards.

8. The solid content of a 6-foot cube is —— cubic feet.

(c) How many cubic yards in a 6-foot cube?

9. Find the area of—	Find the solid content of—
(1) A 7-foot square.	(d) A 7-foot cube.
(2) An 8-foot square.	(e) An 8-foot cube.
(3) A 9-foot square.	(f) A 9-foot cube.
(4) A 10-foot square.	(g) A 10-foot cube.
(5) An 11-foot square.	(h) An 11-foot cube.
(6) A 12-foot square.	(i) A 12-foot cube.

Ratio and Proportion.*

1. One fifth of 25 is ——. 25 is $\frac{1}{5}$ of ——.
2. One fourth of 21 is ——. 21 is $\frac{1}{4}$ of ——.
3. One third of 22 is ——. 22 is $\frac{1}{3}$ of ——.
4. One half of 45 is ——. 45 is $\frac{1}{2}$ of ——.

5. Three fourths of 24 are ——. 24 is $\frac{3}{4}$ of ——.
6. Three fourths of 36 are ——. 36 is $\frac{3}{4}$ of ——.

(a) Find $\frac{3}{4}$ of 96. (b) 96 is $\frac{3}{4}$ of what number?
(c) Find $\frac{2}{3}$ of 96. (d) 96 is $\frac{2}{3}$ of what number?
(e) Find $\frac{1}{2}$ of 96. (f) 96 is $\frac{1}{2}$ of what number?

7. 12 is —— —— of 18. 18 is —— —— of 12.
8. 18 is —— —— of 24. 24 is —— —— of 18.
9. 24 is —— —— of 30. 30 is —— —— of 24.
10. 30 is —— —— of 36. 36 is —— —— of 30.

11. Two thirds of 18 are 1 half of ——.
12. Three fourths of 24 are 2 thirds of ——.
13. Two thirds of 30 are 1 half of ——.
14. Three fourths of 20 are 2 thirds of ——.

(g) One third of 132 is 1 half of what number?
(h) Two thirds of 252 are 1 half of what number?
(i) Three fourths of 96 are 2 thirds of what number?
(j) Two thirds of 96 are 3 fourths of what number?

15. Nine is —— —— of 12. A man can earn —— —— as much in 9 days as he can earn in 12 days. If he can earn $40 in 12 days, in 9 days he can earn —— dollars.

(k) If a man can earn $896 in 12 months, how many dollars can he earn in 9 months?

(l) If Mr. Conrad's horses consume 726 bushels of oats in a year, how many bushels will be required to feed them 8 months?

* See foot-notes, page 16. The thoughtful teacher will not fail to see the bearing of this work upon percentage problems.

Percentage.*

$16\frac{2}{3}$ per cent = .16$\frac{2}{3}$ = $\frac{1}{6}$.
$14\frac{2}{7}$ per cent = .14$\frac{2}{7}$ = $\frac{1}{7}$.
$12\frac{1}{2}$ per cent = .12$\frac{1}{2}$ = $\frac{1}{8}$.

(1)	(2)
1. $16\frac{2}{3}$ per cent of 12 =	12 is $16\frac{2}{3}$ % of ——.
2. $14\frac{2}{7}$ per cent of 21 =	21 is $14\frac{2}{7}$ % of ——.
3. $12\frac{1}{2}$ per cent of 16 =	16 is $12\frac{1}{2}$ % of ——.
4. $16\frac{2}{3}$ per cent of 18 =	18 is $16\frac{2}{3}$ % of ——.
5. $14\frac{2}{7}$ per cent of 28 =	28 is $14\frac{2}{7}$ % of ——.
6. $12\frac{1}{2}$ per cent of 24 =	24 is $12\frac{1}{2}$ % of ——.
7. 50 per cent of 12 =	12 is 50 % of ——.
8. 25 per cent of 16 =	16 is 25 % of ——.
9. 20 per cent of 25 =	25 is 20 % of ——.
10. $33\frac{1}{3}$ per cent of 18 =	18 is $33\frac{1}{3}$ % of ——.

(3)	(3)
11. 2 is —— per cent of 6.	2 is ——% of 4.
12. 2 is —— per cent of 8.	2 is ——% of 14.
13. 2 is —— per cent of 12.	2 is ——% of 10.
14. 2 is —— per cent of 16.	1 is ——% of 2.

15. There were 40 pears on a tree; 25 % of them fell off; —— pears remained on the tree.

16. William sold 7 melons; these were $12\frac{1}{2}$ % of all the melons he raised; he raised —— melons.

17. Mary had 30 little chickens; a hawk killed five of them; the hawk killed —— per cent of her chickens.

(a) Find $12\frac{1}{2}$ per cent of $992.

(b) Find $16\frac{2}{3}$ per cent of $852.

* Review page 17. Be sure that the pupil clearly understands that *per cent means hundredth or hundredths;* that 25 % means 25 hundredths = 1 fourth; that $33\frac{1}{3}$ % means $33\frac{1}{3}$ hundredths = 1 third, etc. See foot-notes, page 17. Frequently review this page, and give many similar oral problems as a preparation for page 37.

Percentage.

(*1*)

1. $16\frac{2}{3}$ % of 24 = $16\frac{2}{3}$ % of 25 =

(a) Find $16\frac{2}{3}$ per cent of 342; (b) of 343.

2. $12\frac{1}{2}$ % of 24 = $12\frac{1}{2}$ % of 25 =

(c) Find $12\frac{1}{2}$ per cent of 976; (d) of 978.

3. $14\frac{2}{7}$ % of 28 = $14\frac{2}{7}$ % of 30 =

(e) Find $14\frac{2}{7}$ per cent of 994; (f) of 996.

(*2*)

4. 5 is $12\frac{1}{2}$ % of ——. $5\frac{1}{2}$ is $12\frac{1}{2}$ % of ——.

(g) 246 is $12\frac{1}{2}$ % of what? (h) $246\frac{1}{2}$ is $12\frac{1}{2}$ % of what?

5. 5 is $14\frac{2}{7}$ % of ——. $5\frac{1}{2}$ is $14\frac{2}{7}$ % of ——.

(i) 351 is $14\frac{2}{7}$ % of what? (j) $356\frac{1}{4}$ is $14\frac{2}{7}$ % of what?

6. 5 is $16\frac{2}{3}$ % of ——. $5\frac{1}{2}$ is $16\frac{2}{3}$ % of ——.

(k) 239 is $16\frac{2}{3}$ % of what? (l) $241\frac{1}{8}$ is $16\frac{2}{3}$ % of what?

(*3*)

7. 12 is ——% of 24. 12 is ——% of 36.
8. 12 is ——% of 48. 12 is ——% of 60.
9. 12 is ——% of 72. 12 is ——% of 84.
10. 12 is ——% of 96. 15 is ——% of 45.

(*1*)*

11. One per cent of \$300 is ——. 2 % of \$300 =
12. One per cent of \$320 is ——. 2 % of \$320 =
13. One per cent of \$325 is ——. 2 % of \$325 =
14. One per cent of \$342 is ——. 2 % of \$342 =

(m) Find 3 % of \$342. (n) Find 4 % of \$342.
(o) Find 5 % of \$342. (p) Find 6 % of \$342.
(q) Find 3 % of \$536. (r) Find 4 % of \$536.

* Review problems 13, 14, 15, on page 18.

Review.

1. Two is an exact divisor of 28, 274, ——, ——.
2. Five is an exact divisor of 75, 230, ——, ——.
3. Ten is an exact divisor of 80, ——, ——, ——.
4. Twenty is an integral number. $\frac{3}{4}$ and .5 are —— numbers. $5\frac{1}{2}$ and 3.2 are —— numbers.*

5. The fractions $\frac{3}{10}$, $\frac{9}{10}$, ——, ——, have a common denominator. $\frac{3}{7}$ and —— do not have a common denominator.

6. The fractions $\frac{3}{7}$, $\frac{3}{5}$, ——, ——, are expressed in their lowest terms.

7. The sum of .135 and .6 is ——. .135 − .06 =

8. Divide .045 by .009. (This means —— 4 † ——.)

(a) Find the quotient of .875 divided by .005.

9. Divide .045 by 9. (This means —— 5 † ——.)

(b) Find the quotient of 54.063 divided by 9.

10. When hay is $8 a ton, 2500 lb. cost ——. ‡

11. Five bushels of wheat weigh —— pounds. §

(c) Sixty-five bushels of wheat weigh how much more than 65 bushels of oats? §

12. The area of a 10-inch square is —— square inches.

(d) How many square feet in a 15-foot square?

(e) How many square yards in a 15-foot square?

13. The solid content of a 10-inch cube is —— cu. in.

(f) How many cubic feet in a 15-foot cube?

(g) How many cubic yards in a 15-foot cube?

14. Three fourths of 60 are ——. 60 is $\frac{3}{4}$ of ——.

(h) Find $\frac{3}{4}$ of 132. (i) 132 is $\frac{3}{4}$ of what number?

15. Two thirds of 27 are 1 half of ——.

(j) Two thirds of 132 are 1 half of what number?

* See page 21. † These figures refer to notes on page 8.
‡ See page 14. § See page 24.

Miscellaneous Problems.

1. Byron bought 6 melons for 54¢; he sold them at 15¢ each; on each melon he gained —— cents.

(a) A merchant bought 6 barrels of apples for $13.50, and sold them at $2.75 a barrel. How much did he gain on each barrel?

2. James carried 4 dozen eggs to market; he sold them for 12¢ a dozen; the dealer paid James for the eggs with sugar at 5¢ a lb.; James should receive —— and —— —— pounds of sugar.*

(b) A farmer took four cords of wood to market; he sold it for $6.75 a cord and took its value in coal at $6 a ton. How many tons of coal should he receive?

3. At 5¢ a quart, 1 gallon of milk is worth —— cents. $2\frac{1}{2}$ gallons are worth —— cents.

(c) At 5¢ a quart, how much are $45\frac{1}{2}$ gallons of milk worth?

4. Two and 1 half feet are —— inches. 4 weeks are —— days.

(d) Forty-six and 1 half feet are how many inches?

(e) Fifty-two weeks are how many days? †

(f) Seventy-five gallons are how many quarts?

(g) Two hundred eighty-four quarts are how many pints?

5. William had 30¢; he spent $\frac{3}{5}$ of his money for ink and pencils and the remainder for paper; the paper cost —— cents.

(h) William's father had $585; he spent $\frac{2}{9}$ of his money for a carriage and harness and the remainder for horses. How much did the horses cost?

6. If 2 lb. of raisins cost 18¢, 3 lb. will cost —— cents.

(i) If 2 acres of land cost $376.50, at the same rate, how much will 3 acres cost?

* If the pupil, after solving problem 2, has difficulty with problem (b), ask him to solve problem 2 upon the blackboard.

† Lead the pupil to see that in the solution of this problem he may take fifty-two 7's or seven 52's. If he adopts the latter method his thought may be—*One* day in each week would make 52 days, and *seven* days in each week would make 7 times 52 days; or he may think that fifty-two 7's are equal to seven 52's, and that, as a matter of convenience, he finds seven 52's.

Simple Numbers.*

Teach orally the meaning and use of the expressions prime number and composite number.

1. A number that has no exact integral divisors except itself and 1, is called a *prime number*. 1, 2, 3, 5, 7, 11, ——, ——, ——, ——, ——, are prime numbers.

2. Integral numbers that are not prime are said to be *composite*. 4, 6, 8, 9, ——, ——, ——, ——, are composite numbers.

3. 24 is a —— number. 23 is a —— number.
29 is a —— number. 26 is a —— number.
28 is a —— number. 27 is a —— number.
31 is a —— number. 37 is a —— number.
25 is a —— number. 40 is a —— number.

4. If an integral number is expressed by two or more figures, and the right-hand figure is 5, the number is ——.

5. Every even number, except 2, is ——.

6. Some odd numbers are ——, and some are ——. 9 is ——. 11 is ——. 13 is ——. 21 is ——.

(a) Write all the prime numbers from 1 to 53 inclusive, and find their sum.

(b) Add.	(c) Subtract.	(d) Multiply.	(e) Divide.	(f) Divide.
346	5423	$754\frac{1}{2}$	\$52)\$7436 †	52)\$7436 ‡
275	$1896\frac{5}{9}$	26		
142				
879	(g)	(h)	(i)	(j)
27	$7462\frac{4}{8}$	846	53 bu.)2438 bu. §	53)2438 bu. §
624	1827	$35\frac{1}{2}$		

* Review pages 11 and 21.

† This means, find how many times \$52 are contained in \$7436. \$52 are contained in \$7436, 143 times. *Story—A man paid \$7436 for land at \$52 an acre; there were 143 acres.*

‡ This means, find 1 fifty-second of \$7436. 1 fifty-second of \$7436 is \$143. *Story—A man paid \$7436 for 52 acres of land; one acre cost \$143.*

§ Tell the meaning. Tell a suggested number story.

Common Fractions.*

Teach the meaning and use of the expression, *Reduce to a whole or mixed number.*

Reduce to whole or mixed numbers:

1. $\frac{15}{4}$ = $\frac{40}{7}$ = $\frac{33}{8}$ = $\frac{63}{9}$ = $\frac{46}{9}$ = $\frac{46}{6}$ = $\frac{37}{6}$ =

(a) $17\frac{5}{6}$ = (b) $24\frac{6}{4}$ = (c) $53\frac{4}{5}$ =

Reduce to the their lowest terms:

2. $\frac{18}{40}$ = $\frac{12}{16}$ = $\frac{5}{35}$ = $\frac{8}{30}$ = $\frac{8}{36}$ = $\frac{10}{42}$ =

(d) $\frac{75}{150}$ = (e) $\frac{24}{120}$ = (f) $\frac{36}{150}$ =

Reduce to equivalent fractions having a common denominator:

3. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$. (Change to ———.) † $\frac{1}{2}$ = $\frac{1}{3}$ = $\frac{1}{8}$ =

(g) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$. (h) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$. (i) $\frac{7}{8}$, $\frac{1}{3}$, $\frac{3}{4}$.

4. Add $\frac{1}{3}$, $\frac{1}{8}$, and $\frac{1}{4}$. (Change to ———ths.)

(j) Find the sum of $75\frac{3}{8}$, $86\frac{2}{3}$, and $47\frac{1}{4}$.

5. From $\frac{7}{8}$ subtract $\frac{1}{3}$. (Change to ———ths.)

(k) Find the difference of $946\frac{5}{8}$ and $388\frac{1}{3}$.

6. Divide 7 by $\frac{3}{4}$. (This means ————.) *Story.* ‡

(l) Find the quotient of 36 divided by $\frac{3}{4}$. *Story.* §

(m) $2\frac{1}{4}$ gallons)126 gallons. This means, *find how many times* $2\frac{1}{4}$ *gallons are contained in* 126 *gal.* (Change $2\frac{1}{4}$ and 126 to fourths.) *Story.*¶

(n) 4)$276\frac{1}{2}$ gallons. This means, *find* 1 *fourth of* $276\frac{1}{2}$ *gallons. Story*—In 4 days Mr. Smith sold $276\frac{1}{2}$ gallons of milk; this was at the rate of ——— gallons per day.

* Review pages 12 and 22.

† The pupil is expected to find by trial that he can change halves, thirds, and eighths, to twenty-fourths.

‡ See note (4) page 6, and problem 6, page 22.

§ Mr. Brown put 36 bushels of peaches into $\frac{3}{4}$-bushel baskets; there were ——— baskets.

¶ Put 126 gal. of milk into $2\frac{1}{4}$ gal. cans.

Decimal Fractions.

1. One tenth of $245 is ———. .1 of $24.5 =

2. One tenth of $2.45 is ———. .2 of $2.45 =

(a) Multiply $2.45 by 2.3. This means, *find* 2 *times* $2.45 *plus* 3 *tenths of* $2.45.

Operation.	*Explanation.*
$2.45	One tenth of $2.45 is ———.
2.3	Three tenths of $2.45 are———.
$.735*	Two times $2.45 are ———.
$4.90	$.735 + $4.90 = $5.635.
$5.635	

Number Story.

If 1 ton of coal is worth $2.45,
1 tenth of a ton of coal is worth ———.
3 tenths of a ton of coal are worth ———.
2 tons of coal are worth ———.
2.3 tons of coal are worth ———.

(b) Multiply $3.65 by 2.4.* (.1 of $3.65 is $.365.)

(c) Multiply $52.8 by 3.2. (.1 of $52.8 is $5.28.)

3. If $\frac{1}{2}$ a dozen eggs cost 5 cents, $2\frac{1}{2}$ dozen will cost ——— cents.

(d) If $\frac{1}{2}$ of a ton of coal costs $2.15, how much will 3.6 tons cost?

4. I bought $7\frac{2}{3}$ yards of print at 6¢ a yd. and gave the salesman half a dollar; I should receive in change ———.

(e) I bought 5.3 tons of coal at $4.20 a ton and gave the salesman 3 10-dollar bills. How much change should I receive?

(f) Add.	(g) Subtract.	(h) Multiply.	(i) Divide.	(j) Divide.
$64\frac{2}{3}$	78	3.75	$.5)$47.5 †	5)$47.5 ‡
28.37	24.42	2.6		

* The pupil should be required to write the decimal point in each partial product *immediately after making the figure that represents the tenths in that product.*

† This means, *find how many times* 5 *tenths are contained in* 475 *tenths.*

‡ This means, *find* 1 *fifth of* $47.5.

Denominate Numbers.

1. One ton is —— pounds.
2. One tenth of a ton is —— pounds.
3. Three tenths of a ton are —— pounds.
4. Nine tenths of a ton are —— pounds.

5. One hundredth of a ton is —— pounds.
6. Three hundredths of a ton are —— pounds.
7. Seven hundredths of a ton are —— pounds.

8. One thousandth of a ton is —— pounds.
9. Two thousandths of a ton are —— pounds.
10. Six thousandths of a ton are —— pounds.

11. 2400 lb. = 1 and —— tenths (1.2) tons.*
12. 2800 lb. = 1 and —— tenths ( ) tons.
13. 3200 lb. = 1 and —— tenths ( ) tons.
14. 2460 lb. = 1 and —————— tons.
15. 2468 lb. = 1 and —————— tons.*

(a) Change 4870 lb. to tons. (b) 4980 lb.
(c) Change 5260 lb. to tons. (d) 5750 lb.
(e) Change 6480 lb. to tons. (f) 7260 lb.

16. 2 tons are —— lb. .3 of a ton are —— lb.
17. 2.3 tons are —— lb. 2.4 tons are —— lb.
18. .03 of a ton are —— lb. .04 of a ton are —— lb.
19. 2.04 tons are —— lb. 2.03 tons are —— lb.
20. 2.34 tons are —— lb. 3.23 tons are —— lb.

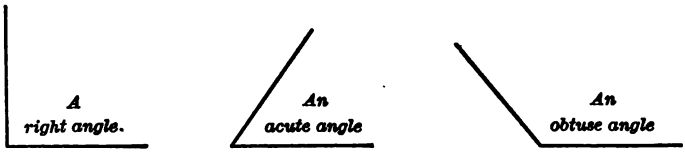
(g) Change 3.52 tons to lb. (h) 4.37 tons.
(i) Change 5.18 tons to lb. (j) 3.72 tons.
(k) Change 1.48 tons to lb. (l) 2.324 tons.
(m) Change 3.7 tons to lb. (n) 4.9 tons.

(o) At ½ a cent a pound, find the cost of 3.45 tons of scrap iron.

* By means of problems 11 to 15 and other similar exercises, lead the pupil to see that he can change pounds to tons by "pointing off" three figures from the right and dividing by 2.

Measurements.

Teach the meaning and use of the terms, **angle, right angle, acute angle, obtuse angle.** Review the terms, *square, oblong, perimeter.*



1. When two lines meet at a point, they are said to form an angle.

2. When a jackknife is half way open, the handle and blade form a right angle. If it is less than half way open, they form an —— angle. If it is more than half way open, but not fully open, they form an —— angle.

3. A square □ has 4 right angles.

4. An oblong ▭ has —— right angles.

5. **Rectangular** means right angled.

6. A rectangular figure, or **rectangle**, may be a *square*, or it may be an *oblong*.

7. A rectangle 6 inches by 6 inches is a ——.

8. A rectangle 4 in. by 6 in. is an ——.

9. A rectangular surface 4 feet by 6 feet is an ——; its area is —— square feet and its perimeter * is —— feet.

10. A rectangular surface 8 feet by 8 feet is a ——; its area is —— square feet; its perimeter is —— ft.

Find the area and the perimeter of each of the following rectangular surfaces:

(a) 23 feet by 6 feet. (b) 25 inches by 7 inches.
(c) 15 yards by 8 yards. (d) 32 feet by 23 feet.
(e) 30 inches by 20 inches. (f) 21 yards by 24 yards.

* See Werner Arithmetic, Book I., page 53.

Ratio and Proportion.

1. One sixth of 12 is ——. 12 is $\frac{1}{6}$ of ——.
2. One fifth of 12 is ——. 12 is $\frac{1}{5}$ of ——.
3. One fourth of 11 is ——. 11 is $\frac{1}{4}$ of ——.

(a) Find 1 third of 275. (b) 275 is 1 third of what?

(c) Find 1 half of 377. (d) 377 is 1 half of what?

4. Two fifths of 30 are ——. 30 is $\frac{2}{5}$ of ——.
5. Three fifths of 30 are ——. 30 is $\frac{3}{5}$ of ——.

(e) Find $\frac{2}{3}$ of 90. (f) 90 is $\frac{2}{3}$ of what number?

(g) Find $\frac{3}{5}$ of 90. (h) 90 is $\frac{3}{5}$ of what number?

6. 14 is —— —— of 21. 21 is —— —— of 14.*
7. 21 is —— —— of 28. 28 is —— —— of 21.
8. 28 is —— —— of 35. 35 is —— —— of 28.
9. 35 is —— —— of 42. 42 is —— —— of 35.

10. Two fifths of 30 are 1 half of ——.
11. Three fifths of 30 are 2 thirds of ——.

(i) Two fifths of 90 are 1 half of what number?

(j) Three fifths of 90 are 2 thirds of what number?

12. Twelve is —— —— of 8, or —— and —— —— times 8. A man can earn —— and —— —— times as much in 12 days as he can earn in 8 days. If he can earn $20 in 8 days, in 12 days he can earn ——.

(k) If a man can earn $73, in 8 months, how many dollars can he earn in 12 months?

13. If 8 lb. of sugar are worth 50¢, 12 lb. are worth —— cents.

(l) If 8 tons of coal are worth $34.20, how much are 12 tons worth?

(m) If 12 barrels of apples are worth $27.60, how much are 8 barrels worth?†

* See note (†), page 16.

† Compare problems 15, (k), and (l), page 26.

Percentage.

$11\frac{1}{9}$ per cent = .$11\frac{1}{9}$ = $\frac{1}{9}$.
10 per cent = .10 = $\frac{1}{10}$.

(1)	(2)
1. $11\frac{1}{9}$ per cent of 27 =	27 is $11\frac{1}{9}$ % of ——.
2. 10 per cent of 20 =	20 is 10 % of ——.
3. $11\frac{1}{9}$ per cent of 18 =	18 is $11\frac{1}{9}$ % of ——.
4. 10 per cent of 30 =	30 is 10 % of ——.
5. 50 per cent of 3 =	3 is 50 % of ——.
6. 25 per cent of 9 =	9 is 25 % of ——.
7. $33\frac{1}{3}$ per cent of 7 =	7 is $33\frac{1}{3}$ % of ——.
8. 20 per cent of 11 =	11 is 20 % of ——.
9. $16\frac{2}{3}$ per cent of 19 =	19 is $16\frac{2}{3}$ % of ——.
10. $14\frac{2}{7}$ per cent of 15 =	15 is $14\frac{2}{7}$ % of ——.
11. $12\frac{1}{2}$ per cent of 25 =	25 is $12\frac{1}{2}$ % of ——.

(3)	(3)
12. 6 is —— per cent of 24.	6 is —— % of 18.
13. 6 is —— per cent of 12.	6 is —— % of 30.
14. 3 is —— per cent of 24.	3 is —— % of 21.
15. 3 is —— per cent of 18.	3 is —— % of 27.
16. 3 is —— per cent of 30.	$1\frac{1}{2}$ is —— % of 3.

17. Helen bought a piece of flannel that was 40 inches long; by washing, it shrank 10 % in length; after washing, it was —— inches long.

18. A farmer sold 8 barrels of apples; these were 20 % of all he harvested; he harvested —— barrels and had —— barrels left.

19. Before washing, a piece of flannel was 40 inches in length; after washing it was 35 inches long; it shrank by washing —— per cent.*

20. A dealer had 25 bu. of apples; he lost 20 % of them by decay; there remained —— bushels.

(a) A dealer had 2375 bu. of apples; he lost 20 % of them by decay. How many bushels remained?

* It shrank *what part* of its original length?

Percentage.

(1)

1. $11\frac{1}{9}$ % of 36 = $11\frac{1}{9}$ % of 38 =

(a) Find $11\frac{1}{9}$ per cent of 1044; (b) of 1047.

2. 10 % of 50 = 10 % of 51 = 10 % of 52 =

(c) Find 10 % of 870; (d) of 874.

(2)

3. 7 is $11\frac{1}{9}$ % of ——. $7\frac{1}{2}$ is $11\frac{1}{9}$ % of ——.

(e) 371 is $11\frac{1}{9}$ % of what? (f) $371\frac{1}{3}$ is $11\frac{1}{9}$ % of what?

4. 8 is 10 % of ——. $8\frac{1}{3}$ is 10 % of ——.

(g) 89 is 10 % of what? (h) 89.2 is 10 % of what?

(3)

5. 12 is —— % of 24. $12\frac{1}{2}$ is ——% of 50.

6. 6 is —— % of 60. 6 is —— % of 54.

7. $3\frac{1}{2}$ is —— % of 7. $3\frac{1}{3}$ is —— % of 10.

8. 36 is —— % of 72. 72 is —— % of 144.

(1)

9. One per cent of \$700 is ——. 2 % of \$700 =

10. One per cent of \$730 is ——. 2 % of \$730 =

11. One per cent of \$732 is ——. 2 % of \$732 =

(i) Find 3 % of \$732. (j) Find 4 % of \$732.

(k) Find 5 % of \$732. (l) Find 6 % of \$732.

(m) Find 7 % of \$320. (n) Find 7 % of \$326.

(2)

12. Two dollars are 1 hundredth of \$200.

13. Two dollars are 1 % of ——. \$4 are 2 % of ——.

14. Six dollars are 3 % of ——. \$8 are 4 % of ——.

15. Five dollars are 1 % of ——. \$10 are 2 % of ——.

16. Twelve dollars are 2 % of ——.* \$14 are 2 % of ——.

* The foregoing problems should lead the pupil to think that if \$12 are 2 hundredths of some number, \$6 are 1 hundredth of the same number, and therefore the number is 100 times \$6, or \$600. He may also be led to discover that 2 % = $\frac{1}{50}$, and that 50 times \$12 = \$600.

Review.

1. All integral numbers are either prime or composite. 17 is ——. 31 is ——. 95 is ——. 242 is ——. 370 is ——.

2. Name at least one exact divisor of each of the following numbers: 250, 175, 792, 430, 855.

3. Reduce each of the following improper fractions to a whole number or to a mixed number: $\frac{11}{4}$, $\frac{35}{5}$, $\frac{37}{9}$, $\frac{18}{7}$, $\frac{25}{6}$.

(a) $\frac{324}{5}$. (b) $\frac{185}{5}$. (c) $\frac{256}{4}$. (d) $\frac{231}{7}$.

4. Reduce each of the following to its lowest terms:

$\frac{8}{20}$ $\frac{9}{27}$ $\frac{12}{24}$ $\frac{15}{35}$ $\frac{11}{33}$ $\frac{12}{48}$ $\frac{10}{100}$

(e) $\frac{65}{195}$. (f) $\frac{539}{1870}$. (g) $\frac{130}{170}$. (h) $\frac{12}{180}$.

5. Reduce the following to equivalent fractions having a common denominator:

$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$. (Change to ——.) $\frac{1}{3}$ = $\frac{1}{6}$ = $\frac{1}{8}$ =

(i) $\frac{3}{5}$, $\frac{5}{6}$, $\frac{1}{2}$. (j) $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{2}$. (k) $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{2}$.

6. If $\frac{1}{2}$ of a bushel of potatoes costs 20¢, $2\frac{1}{2}$ bushels will cost ——.

(l) If $\frac{1}{2}$ of a ton of straw costs $2.25, how much will 3.2 tons cost?

7. Change the following to tons: 4200 lb.

(m) 5750 lb. (n) 7320 lb. (o) 3150 lb.

8. Find the area and the perimeter of each of the following rectangular surfaces: 7 feet by 5 feet.

(p) 27 feet by 12 feet. (q) 15 inches by 15 inches.

9. If 2 gallons of molasses are worth 60¢, 3 gallons are worth —— cents.

(r) If 2 loads of brick are worth $12.60, how much are 3 loads worth?

Miscellaneous Problems.

1. In a school there are 35 pupils; $\frac{4}{7}$ of the pupils are boys; ——— ——— of the pupils are girls; there are ——— boys and ——— girls.

(a) In a school there are 392 pupils; $\frac{4}{7}$ of the pupils are boys. How many girls are in the school?

2. If 3 melons cost 36¢, at the same rate 5 melons will cost ——— cents.

(b) If 3 acres of land cost $525, how much will 5 acres cost at the same rate?

3. Harry exchanged 5 lb. of butter at 20¢ a pound for coffee at 25¢ a pound; he should receive ——— pounds of coffee.

(c) Harry's father exchanged 6 cords of wood at $5.20 a cord for cedar posts at 20¢ each. How many posts should he receive?

4. 3 + 2 + 6 + 4 + 5 + 8 + 1 + 7 + 9 + 4 + 7 + 3 + 2 + 8 =

(d) 275 + 361 + 554 + 732 + 598 + 236 + 347 + 256 =

5. If Mark saves $4 a month, in 1 year he will save ——— dollars.

(e) If Mark's father saves $21.50 a month, how much will he save in 1 year?

6. A common brick is 8 inches long, 4 in. wide, and 2 in. thick; it has two faces each of which is 4 in. by 8 in., two faces each of which is ——— in. by ——— in., and two faces each of which is ——— in. by ——— in.

(f) Find the sum of the areas of all the faces of a common brick.

7. Sixty-seven inches are ——— feet and ——— inches.

(g) Change 674 inches to feet and inches.

(h) Change 975 days to weeks and days.

(i) Change 874 ounces to pounds and ounces.

Simple Numbers.

Teach orally the meaning and use of the expressions, **factor, prime factor.**

1. Any exact integral divisor of a number (except the number itself and 1) is called a *factor* of the number. The factors of 6 are —— and ——. The factors of 10 are —— and ——.

2. A factor that is itself a prime number is called a *prime factor.* A factor that is itself a composite number is called a *composite factor.*

2 and 3 are —— factors of 24.
4 and 6 are —— factors of 24.

3. Every composite number may be resolved into prime factors.

The prime factors of 12 are 2, 2, and 3.
The prime factors of 18 are 2, 3, and 3.
The prime factors of 15 are —— and ——.
The prime factors of 14 are —— and ——.
The prime factors of 30 are ——, ——, and ——.

4. From the above it will be seen that a number is equal to the product of its prime factors. 3 and 7 are the prime factors of ——; 2, 2, and 7, of ——.

$$\begin{array}{r|l} 2 & 30 \\ 3 & 15 \\ \hline & 5 \end{array}$$

The prime factors of 30 are 2, 3, and 5.
$2 \times 3 \times 5 = 30.$

$$\begin{array}{r|l} 2 & 50 \\ 5 & 25 \\ \hline & 5 \end{array}$$

The prime factors of 50 are 2, 5, and 5.
$2 \times 5 \times 5 = 50.$

(a) What are the prime factors of 40? (b) of 60?
(c) of 65? (d) of 72? (e) of 86? (f) of 85?

Multiply.	Divide.	Divide.
(g) 724 by 28.*	(h) 748 lb. by 32 lb.*	(i) 1254 lb. by 9.*
(j) 846 by 23.	(k) 834 lb. by 32 lb.	(l) 1046 lb. by 9.
(m) 637 by 25.	(n) 928 lb. by 32 lb.	(o) 1134 lb. by 9.
(p) 926 by 27.	(q) 796 lb. by 32 lb.	(r) 1341 lb. by 9.

* Tell the meaning. Tell a number story.

Common Fractions.*

Teach the meaning and use of the expression, *Reduce to an improper fraction.*

Reduce to improper fractions:

1. $7\frac{1}{2}$ = ——. $8\frac{3}{4}$ = ——. $9\frac{2}{3}$ = ——. $7\frac{5}{8}$ = ——.

(a) $27\frac{1}{3}$ = (b) $48\frac{3}{4}$ = (c) $75\frac{1}{8}$ =

Reduce to whole or mixed numbers:

2. $\frac{17}{6}$ = $\frac{35}{8}$ = $\frac{29}{4}$ = $\frac{28}{3}$ = $\frac{45}{9}$ = $\frac{39}{6}$ =

(d) $\frac{286}{5}$ = (e) $\frac{514}{7}$ = (f) $\frac{281}{6}$ =

Reduce to equivalent fractions having a common denominator:

3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{7}$. (Change to ——). $\frac{1}{2}$ = $\frac{1}{4}$ = $\frac{1}{7}$ =

(g) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{7}$. (h) $\frac{5}{6}$, $\frac{2}{3}$, $\frac{3}{8}$. (i) $\frac{2}{5}$, $\frac{3}{4}$, $\frac{1}{2}$.

4. Add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{2}$. (Change to ——.)

(j) Find the sum of $86\frac{2}{3}$, $95\frac{3}{4}$, and $87\frac{1}{2}$.

5. From $\frac{5}{7}$ subtract $\frac{1}{4}$. (Change to ——.)

(k) Find the difference of $873\frac{5}{7}$ and $249\frac{3}{4}$.

6. Divide $\frac{3}{4}$ by $\frac{1}{3}$. This means, *find how many times* $\frac{1}{3}$ *is contained in* $\frac{3}{4}$. I can change fourths and thirds to ——ths. $\frac{3}{4}$ = ——. $\frac{1}{3}$ = ——. —— twelfths are contained in —— twelfths —— times. *Story*—James can husk a row of corn in $\frac{1}{3}$ of an hour; in $\frac{3}{4}$ of an hour he can husk —— and —— —— rows.

(l) $\frac{4}{5} + \frac{1}{4}$.† (m) $\frac{2}{3} + \frac{1}{4}$. (n) $\frac{1}{5} + \frac{1}{3}$.

(o) $5\frac{1}{3}$ acres)288 acres. This means, *find how many times* $5\frac{1}{3}$ *acres are contained in* 288 *acres.*‡

(p) 5)$625\frac{1}{2}$ acres. This means, *find* 1 *fifth of* $625\frac{1}{2}$ *acres. Story.*§

* Review pages 12, 22, and 32. † Change to twentieths.

‡ Divide a farm of 288 acres into $5\frac{1}{3}$-acre lots.

§ Divide a farm of $625\frac{1}{2}$ acres equally among 5 children.

Decimal Fractions.

1. One hundredth of $500 is ———. .02 of $500 =
2. One hundredth of $540 is ———. .02 of $540 =
3. One hundredth of $542 is ———. .02 of $542 =

4. .01 of $600 =	.01 of $60 =	.01 of $660 = *
5. .01 of $.1 =	.01 of $.5 =	.01 of $.7 =
6. .01 of $6.4 =	.01 of $7.5 =	.01 of $3.2 =
7. .01 of $24.2 =	.01 of $37.1 =	.01 of $53.1 =
8. .01 of $721 =	.01 of $841 =	.01 of $924 = *

(a) Multiply $374 by .03. This means, *find* 3 *hundredths of* $374.

Operation.

$374 †
.03
$11.22

Explanation.

One hundredth of $374 is ———.
Three hundredths of $374 are ———.

Number Story.

If one acre of land is worth $374,
1 hundredth of an acre of land is worth ———.
3 hundredths of an acre of land are worth ———.

(b) Multiply $347 by .03. (c) $537 × .04.

(d) Multiply $24.6 by .03. (e) $39.4 × .04.

(f) At $875 an acre, how much will .04 of an acre of land cost?

(g) Add.	(h) Subtract.	(i) Multiply.	(j) Divide.	(k) Divide.
286.3	$146\frac{2}{5}$	356	$.04)$5.76 ‡	4)$5.76 §
$184\frac{4}{5}$	78.2	.05		

* By means of problems 4 to 8 and other similar exercises, lead the pupil to see that he can obtain 1 hundredth of an integral number by "pointing off" two figures on the right, and of a mixed decimal by removing the point two places to the left.

† The pupil should understand that he multiplies $3.74 (not $374) by 3, and should be taught to write the decimal point in the product *immediately after* writing the tenths' figure of the product.

‡ This means, *find how many times* 4 *hundredths are contained in* 576 *hundredths.*

§ This means, *find* 1 *fourth of* $5.76.

Denominate Numbers.*

1. 4000 lb. are ——— tons.
2. 4200 lb. are ——— tons.
3. 4400 lb. are ——— tons.
4. 4600 lb. are ——— tons.

(a) At $5.20 a ton, how much will 4600 lb. of coal cost?

Operation.	*Explanation.*
4600 lb. = 2.3 tons.	One ton costs $5.20.
$5.20	1 tenth of a ton costs $.52.
2.3	3 tenths of a ton cost ———.
$1.560 †	2 tons cost ———.
$10.40	2.3 tons cost ———.
$11.960	

Find the cost:

(b) 4800 lb. @ $5.40 per ton. (c) 4200 lb. @ $5.60.
(d) 6200 lb. @ $5.35 per ton. (e) 6400 lb. @ $5.25.
(f) 5600 lb. @ $5.70 per ton. (g) 5400 lb. @ $5.80.

Review of Fourth Grade Work.

5. Three yd. are — feet. 3 yd. are — inches.
6. Three hours are — minutes. 3 yr. are — months.
7. Three minutes are — seconds. 3 wk. are — days.
8. Three bu. are — quarts. 3 gal. are — quarts.
9. Three lb. are — ounces 3 pk. are — pints.

(h) Change 148 yd. to feet. (i) Change 15 yd. to inches.
(j) Change 24 h. to minutes. (k) Change 24 h. to seconds.
(l) Change 35 yr. to months. (m) Change 52 wk. to days.
(n) Change 37 lb. to ounces. (o) Change 49 gal. to quarts.
(p) Change 43 bu. to quarts. (q) Change 56 pk. to pints.

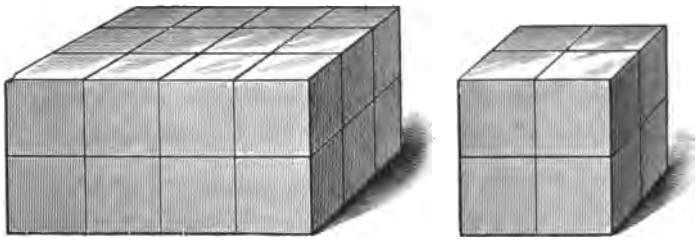
(r) How many square inches in a 5-foot square?

(s) How many square feet in a rectangular floor that is 5 yd. by 7 yd.?

* Review page 34.

† Write the decimal point in each partial product *immediately after* writing the tenths' figure of that product.

Measurements—Rectangular Solids.



1. A rectangular solid has —— faces. Each face is a rectangle.

2. Some or all of the faces of a rectangular solid may be squares.

3. If each face of a rectangular solid is a square, the solid is called a ——.

4. If some of the faces of a rectangular solid are oblongs, the solid is not a cube.

5. The area of a rectangular surface 3 inches by 4 inches is —— square inches. *

6. The solid content of a rectangular solid 3 in. by 4 in. by 2 in. is —— cubic inches.*

7. The area of a rectangular surface 3 inches by 5 inches is —— square inches.

8. The solid content of a rectangular solid 3 in. by 5 in. by 2 in. is —— cubic inches.

9. The area of a rectangular surface 2 inches by 5 inches is —— square inches.

10. The solid content of a rectangular solid 2 in. by 5 in. by 3 in. is —— cubic inches.

* By means of problems 5 to 10 and the pictures at the top of the page (or, better, by use of one-inch cubes as suggested on page 15), lead the pupil to perceive that the number of square inches in one face of a rectangular solid indicates the number of cubic inches in one "layer" of the solid. If there are 6 *square units* in 1 face of a rectangular solid, there are 6 *cubic units* in 1 "layer" of the solid.

Ratio and Proportion.

1. One seventh of 28 is ——. 28 is $\frac{1}{7}$ of ——.*
2. One fifth of 28 is ——. 28 is $\frac{1}{5}$ of ——.
3. One third of 28 is ——. 28 is $\frac{1}{3}$ of ——.

(a) Find 1 fourth of 387. (b) 387 is 1 fourth of what?
(c) Find 1 fifth of 724. (d) 724 is 1 fifth of what?

4. Five sixths of 60 are ——. 60 is $\frac{5}{6}$ of ——.
5. Four fifths of 40 are ——. 40 is $\frac{4}{5}$ of ——.

(e) Find $\frac{5}{6}$ of 420. (f) 420 is $\frac{5}{6}$ of what number?
(g) Find $\frac{4}{5}$ of 920. (h) 920 is $\frac{4}{5}$ of what number?

6. 16 is —— —— of 24. 24 is —— —— of 16.
7. 24 is —— —— of 32. 32 is —— —— of 24.
8. 32 is —— —— of 40. 40 is —— —— of 32.
9. 40 is —— —— of 48. 48 is —— —— of 40.

10. Five sixths of 30 are 1 third of ——.
11. One sixth of 48 is 2 thirds of ——.

(i) Five sixths of 366 are 1 half of what number?
(j) One sixth of 852 is 2 thirds of what number?

12. Twelve is —— —— of 9, or —— and —— —— times 9. A man can earn —— and —— —— times as much in 12 days as he can earn in 9 days. If he can earn $24 in 9 days, in 12 days he can earn —— dollars.

(k) If a man can earn $840 in 9 months, how many dollars can he earn in 12 months?

13. If 9 lb. of nails are worth 33 cents, 12 lb. are worth —— cents.

(l) If 9 cords of wood are worth $42.75, how much are 12 cords worth?

(m) If 12 barrels of apples are worth $31.20, how much are 9 barrels worth? †

* Use a pencil if necessary.

† See page 26, problems 15, (j), and (k).

Percentage.

$66\frac{2}{3}$ per cent = .66⅔ = ⅔.
75 per cent = .75 = ¾.

(1)	(2)
1. $66\frac{2}{3}$ per cent of 12 =	12 is $66\frac{2}{3}$ % of ——.
2. 75 per cent of 12 =	12 is 75 % of ——.
3. $66\frac{2}{3}$ per cent of 24 =	24 is $66\frac{2}{3}$ % of ——.
4. 75 per cent of 24 =	24 is 75 % of ——.
5. 50 per cent of 5 =	5 is 50 % of ——.
6. 25 per cent of 13 =	13 is 25 % of ——.
7. $33\frac{1}{3}$ per cent of 10 =	10 is $33\frac{1}{3}$ % of ——.
8. 20 per cent of 16 =	16 is 20 % of ——.
9. $16\frac{2}{3}$ per cent of 25 =	25 is $16\frac{2}{3}$ % of ——.
10. $14\frac{2}{7}$ per cent of 22 =	22 is $14\frac{2}{7}$ % of ——.
11. $12\frac{1}{2}$ per cent of 33 =	33 is $12\frac{1}{2}$ % of ——.
12. $11\frac{1}{9}$ per cent of 19 =	19 is $11\frac{1}{9}$ % of ——.
13. 10 per cent of 21 =	21 is 10 % of ——.

(3)	(3)
14. 4 is —— per cent of 24.*	7 is ——% of 35.
15. 4 is —— per cent of 28.	7 is ——% of 28.
16. 4 is —— per cent of 32.	7 is ——% of 21.
17. 4 is —— per cent of 36.	7 is ——% of 14.
18. 4 is —— per cent of 40.	8 is ——% of 12.
19. 9 is —— per cent of 12.	12 is ——% of 18

20. Twenty-five per cent of 80 sheep are —— sheep.
21. Twenty-one sheep are 25 % of —— sheep.
22. Twenty-five sheep are —— per cent of 75 sheep.

* In solving Case 3 percentage problems similar to the above, lead the pupil to think somewhat as suggested by the following: Teacher. *Four is what % of* 24? Pupil. *Four is* ($\frac{4}{24}$) 1 *sixth of* 24; 1 *sixth* = $16\frac{2}{3}$ *per cent.* The pupil must understand that to tell what per cent one number is of another he must think what part of it it is, and give his answer in hundredths.

Percentage.

(*1*)

1. 75 per cent of 36 = * $66\frac{2}{3}$ per cent of 36 = †

(a) Find 75 % of 796. (b) Find $66\frac{2}{3}$ % of 822.

(c) Find 3 % of $375. This means, find 3 hundredths of $375.

Operation.	*Explanation.*
$375	One per cent of $375 = $3.75.
.03	Three per cent of $375 = $11.25.
$11.25	

Number Story.

Mr. A collected money for Mr. B. It was agreed that Mr. A should keep 3 % of all he might collect to pay him for his trouble. He collected $375; he should keep $11.25 and "pay over" the remainder to Mr. B.

(d) How much should Mr. A "pay over" to Mr. B?

(e) Find 7 % of $465. (f) Find 9 % of $324.

(g) Find 3 % of $422. (h) Find 7 % of $538.

(*2*)

2. 36 is 75 % of ———.‡ 36 is $66\frac{2}{3}$ % of ———.§

(i) 453 is 75 % of what? (j) 562 is $66\frac{2}{3}$ % of what?

(k) Forty-eight dollars are 3 % of what? This means, $48 are 3 hundredths of how many dollars?

Operation.	*Explanation.*
$48 ÷ 3 = $16.	One hundredth of the unknown number is $16.
$16 × 100 = $1600.	100 hundredths (the whole) are ———.

Number Story.

A lawyer collected some money for 3 % of the amount collected; his share (commission) was $48; the amount collected was $1600.

(l) How much should the lawyer pay over to the man for whom he collected the money?

(m) $98 are 7 % of what? (n) $198 are 9 % of what?

* Find $\frac{1}{3}$ of 36. † Find $\frac{2}{3}$ of 36.
‡ 36 is $\frac{1}{4}$ of what number? § 36 is $\frac{1}{3}$ of what number?

Review.

1. The prime factors of 45 are ——, ——, and ——.*

(a) What are the prime factors of 100? (b) of 125?

2. Two, 3, and 5 are the prime factors of ——.*

(c) Of what number are 3, 3, 2, and 7 the prime factors?

3. Reduce $\frac{12}{18}$ to its lowest terms. $\frac{12}{18}$ =

(d) Reduce $\frac{126}{140}$ and $\frac{48}{144}$ to their lowest terms.

4. Reduce $8\frac{2}{3}$ to an improper fraction. $8\frac{2}{3}$ =

(e) Reduce $57\frac{3}{8}$ and $72\frac{3}{4}$ to improper fractions.

5. Reduce $\frac{38}{7}$ to a mixed number. $\frac{38}{7}$ =

(f) Reduce $\frac{324}{7}$ and $\frac{513}{7}$ to mixed numbers.

6. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to equivalent fractions having a common denominator. $\frac{2}{3}$ = $\frac{3}{4}$ =

(g) Reduce $\frac{7}{12}$ and $\frac{5}{9}$ to equivalent fractions having a common denominator.

7. Multiply 60 by .7. This means, *find* 7 *tenths of* 60. One tenth of 60 = ——; 7 tenths of 60 = ——.

(h) Multiply \$537 by .07. This means, *find* 7 *hundredths of* \$537. (See page 43.)

8. At \$1 per ton, 4240 lb. of coal cost ——.

(i) Find the cost of 4240 lb. of coal at \$7 per ton.

9. The volume of a rectangular solid 5 inches by 4 inches by 2 inches is —— cubic inches.

(j) Find the volume of a rectangular solid 9 inches by 7 inches by 7 inches.

10. If 9 lb. of tea are worth \$6, 12 lb. are worth ——.

(k) If 9 acres of land are worth \$346.50, how much are 12 acres worth at the same rate? (12 is $1\frac{1}{3}$ times 9.)

* See page 41.

Miscellaneous Problems.

1. From June, 1881 to June, 1899, it is —— years.

(a) How many years from Aug., 1492 to Aug., 1897?

2. A man sold a horse for one hundred twenty dollars; this was seventeen dollars and twenty cents more than the horse cost him; the horse cost him ——.

(b) A man sold a farm for fourteen thousand seven hundred and fifty dollars; this was eight hundred seventy-five dollars more than he paid for the farm. How much did the farm cost him?

3. From 5460 lb. of coal there were sold 2 tons. —— pounds remained.

(c) From 18940 lb. of coal there were sold $8\frac{1}{4}$ tons. How many pounds were left?

4. From Mendota to Galesburg it is 80 miles; a train going 30 miles an hour, that leaves Mendota at 8:30, should arrive at Galesburg at ——.

(d) From Chicago to Denver it is about 1000 miles. If a train leaves Chicago for Denver at 9 o'clock Monday morning and goes at the rate of 30 miles an hour, when will it arrive at Denver?

5. If butter is 25¢ a pound and coffee is 30¢ a pound, 3 lb. butter pays for —— lb. coffee.

(e) Fifteen and one half pounds of butter at 24¢ a pound, will pay for how many pounds of coffee at 32¢ a pound?

6. If $\frac{3}{4}$ of a yard of lace is worth 15¢, $2\frac{1}{4}$ yards are worth —— cents.

(f) If $\frac{3}{4}$ of a yard of cloth costs $1.05, how much will $27\frac{1}{2}$ yards cost?

(g) If $\frac{3}{5}$ of an acre of land is worth $37.40, how much are $2\frac{1}{2}$ acres worth at the same rate?

Simple Numbers.

Teach orally the meaning and use of the terms, **multiple, common multiple,** and **least common multiple.***

1. Four, 6, 8, 10, 12, etc., are *multiples* of 2.

2. Six, 9, 12, 15, 18, etc., are multiples of 3.

3. Twelve is a multiple of 2. 12 is also a multiple of 3, and of 4, and of 6.

4. Twelve is a *common multiple* of 2, 3, ——, and ——.

5. Fifteen is a common multiple of —— and ——.

6. Common multiples of 4 and 6 are, 12, 24, ——, etc. The *least common multiple* or 4 and 6 is 12.

7. Common multiples of 6 and 8 are, 24, 48, ——, etc. The least common multiple of 6 and 8 is ——.

8. Common multiples of 8 and 12 are, ——, ——, etc. The least common multiple of 8 and 12 is ——.

9. Common multiples of 6 and 9 are, ——, ——, etc. The least common multiple of 6 and 9 is ——.

10. The prime factors of 18 are ——, ——, and ——.

(a) What are the prime factors of 140. (b) of 160?
(c) of 135? (d) of 175? (e) of 250? (f) of 225?

Multiply.	Divide.	Divide.
(g) 635 by 53.	(h) 944 lb. by 56 lb.†	(i) 1536 lb. by 12.†
(j) 728 by 54.	(k) 846 lb. by 56 lb.	(l) 1445 lb. by 12.
(m) 834 by 52.	(n) 739 lb. by 56 lb.	(o) 1374 lb. by 12.
(p) 947 by 51.	(q) 873 lb. by 56 lb.	(r) 1653 lb. by 12.
(s) 836 by 55.	(t) 965 lb. by 56 lb.	(u) 1738 lb. by 12.

* Do much oral work in preparation for this page. *By using these terms*, make the pupil as familiar with *multiple, common multiple,* and *least common multiple* as he is with *house, schoolhouse, and stone schoolhouse.* He must not only know these *words*, but he must know the "*things*" for which the words are symbols.

† Tell the meaning. Tell a number story.

Common Fractions.

Teach the abbreviation l. c. m. for least common multiple.

1. Add $\frac{5}{8}$ and $1\frac{1}{12}$.
   L. c. m. of 8 and 12 is ——.
   $\frac{5}{8} = \frac{\quad}{24}$. $1\frac{1}{12} = \frac{\quad}{24}$.
   $\frac{15}{24} + \frac{\quad}{24} = \frac{\quad}{24} = 1\frac{\quad}{24}$.

2. From $\frac{8}{9}$ subtract $\frac{1}{6}$.
   L. c. m. of 9 and 6 is ——.
   $\frac{8}{9} = \frac{\quad}{18}$. $\frac{1}{6} = \frac{\quad}{18}$.
   $\frac{16}{18} - \frac{\quad}{18} = \frac{\quad}{18}$.

3. Divide $1\frac{1}{12}$ by $\frac{1}{8}$.*
   L. c. m. of 12 and 8 is ——.
   $1\frac{1}{12} = \frac{\quad}{24}$. $\frac{1}{8} = \frac{\quad}{24}$.
   $\frac{\quad}{24} \div \frac{\quad}{24} =$ ————. †

4. Multiply 12 by $2\frac{3}{4}$.
   This means ————. ‡
   $2\frac{3}{4}$ times 12 = ——. *Story.* ‡

5. Multiply 12 by $\frac{3}{4}$.
   This means ————. §
   $\frac{3}{4}$ of 12 = ——. *Story.* §

(a) $\frac{5}{6} + \frac{3}{6}$.
(b) $\frac{3}{4} + \frac{5}{6}$.
(c) $\frac{5}{6} + \frac{3}{8}$.
(d) $\frac{3}{5} + \frac{3}{7}$.

(e) $\frac{5}{6} - \frac{1}{4}$.
(f) $\frac{1}{6} - \frac{1}{8}$.
(g) $1\frac{7}{8} - \frac{3}{8}$.
(h) $\frac{5}{7} - \frac{2}{5}$.

(i) $\frac{7}{8} \div \frac{1}{3}$.
(j) $\frac{3}{5} \div \frac{1}{6}$.
(k) $\frac{5}{8} \div \frac{1}{6}$.
(l) $\frac{5}{7} \div \frac{1}{5}$.

(m) $48 \times 2\frac{3}{4}$.
(n) $252 \times 3\frac{3}{4}$.
(o) $175 \times 2\frac{1}{5}$.

(p) $96 \times \frac{3}{4}$.
(q) $84 \times \frac{2}{3}$.
(r) $95 \times \frac{3}{5}$.

(s) $6\frac{1}{4}$ dollars)375 dollars. This means, *find how many times* $6\frac{1}{4}$ *are contained in* \$375. Change $6\frac{1}{4}$ and 375 to fourths.) *Story.* ‖

(t) 6)756 dollars. This means, *find* 1 *sixth of* \$756. *Story.* ¶

* This means, *find how many times* $\frac{1}{8}$ *is contained in* $1\frac{1}{12}$.
† 3 twenty-fourths are contained in 28 twenty-fourths $7\frac{1}{3}$ times.
‡ See note (16), page 7. § See note (8), page 6.
‖ At $6\frac{1}{4}$ a ton, for \$375 I can buy —— tons of coal.
¶ A man paid \$756 for 6 acres of land; 1 acre cost ——.

Decimal Fractions.

1. One tenth of $432 is ———. .2 of $432 =

2. One hundredth of $432 is ———. .02 of $432 =

(a) Multiply $432 by .25. This means, *find* 2 *tenths of* $432, *plus* 5 *hundredths of* $432.

Operation.	*Explanation.*
$432	One hundredth of $432 is ———.
.25	5 hundredths of $432 are ———.
$21.60 *	One tenth of $432 is ———.
$86.4 †	2 tenths of $432 are ———.
$108.00	$21.60 + $86.4 =

Number Story.

If 1 acre of land is worth $432,
1 hundredth of an acre is worth ———.
5 hundredths of an acre are worth ———
1 tenth of an acre is worth ———.
2 tenths of an acre are worth ———.
25 hundredths of an acre are worth ———.

(b) Multiply $325 by .23. (c) $482 × .32.

(d) Multiply $278 by .43. (e) $356 × .36.

(f) Multiply $536 by .07. (g) $351 × .7.

(h) Multiply $249 by 2.6. (i) $426 × .45.

(j) At $756 an acre, how much will .34 of an acre of land cost?

(k) Add.	(l) Subtract.	(m) Multiply.	(n) Divide.	(o) Divide.
8.75	56½	675	$.05)$6. ‡	5)$6. §
7.324	12.9	.36		

* The pupil should understand that he multiplies $4.32 (not $432) by 5, and should be taught to write the decimal point in the partial product *immediately after writing the tenths' figure*, 6, of the partial product.

† The pupil should understand that he multiplies $43.2 (not $432) by 2, and should be taught to write the decimal point in the partial product *immediately after writing the tenths' figure*, 4, of the partial product.

‡ This means, *find how many times* 5 *hundredths are contained in* 600 hundredths (6.00).

§ This means. *find* 1 *fifth of* $6.

Denominate Numbers.

1. *Sixteen and one half feet are* 1 *rod.*
2. *Three hundred twenty rods are* 1 *mile.*
3. One rod is —— and —— —— yards.*

4. Two rods are —— feet. 4 rods are —— feet.†
5. Six rods are —— feet. 10 rods are —— feet.
6. One mile is —— rods. $\frac{1}{2}$ mile is —— rods.
7. $\frac{1}{4}$ mile is —— rods. $\frac{1}{8}$ mile is —— rods.†

8. The telegraph poles along the line of a railroad are usually ten rods apart; they are —— feet apart. From the first telegraph pole to the third it is —— rods; from the first to the fifth it is —— rods.

(a) How far is it from the first telegraph pole to the thirty-third?

9. The roads in the country are usually 4 rods wide. A 4-rod road is —— feet wide.
10. A 100-foot street is —— rods and —— foot wide.
11. From the schoolhouse to ——, it is —— —— of a mile, or —— rods.

(b) Change 5 miles to rods. (c) Change 40 rods to feet.
(d) Change 28 rods to yards. (e) Change 7 miles to rods.

Review.

12. When hay is $12 a ton, 3000 lb. will cost ——
13. When oats are 30¢ a bushel, 96 lb. cost ——.
14. When wheat is 80¢ a bushel, 180 lb. cost ——.

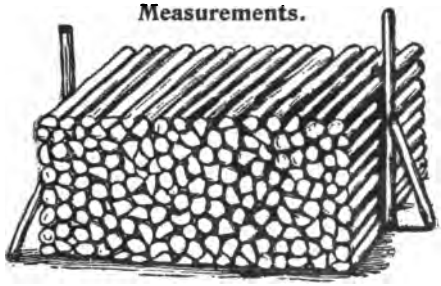
(f) When corn (not shelled) is 45¢ a bushel, how much will 3640 lb. cost?

(g) When coal is $5.40 a ton, how much will 2600 lb. cost?

* Lead the pupil to discover by actual measurement the number of yards in a rod.

† The pupil may profitably memorize the eight answers to problems 4 to 7.

Measurements.



1. *A pile of wood* 8 *feet long*, 4 *feet wide, and* 4 *feet high (or its equivalent) is called a cord.*

(a) How many cubic feet in a rectangular solid 8 feet by 4 feet by 4 feet? *

2. A pile of wood 16 feet long, 4 feet wide, and 4 feet high contains ——— cords.

(b) How many cubic feet in 2 cords?

3. A pile of wood 8 feet long, 4 feet wide, and 6 feet high contains ——— cords.

(c) How many cubic feet in $1\frac{1}{2}$ cords? (d) In $\frac{3}{4}$ of a cord?

4. A pile of wood 28 feet long, 4 feet wide, and 4 feet high contains ——— cords.

(e) How many cubic feet in $5\frac{1}{4}$ cords? (f) In $6\frac{3}{4}$ cords?

Review.

5. A 1-inch square = ——— ——— of a 2-inch square.

6. A 1-inch cube = ——— ——— of a 2-inch cube.

7. A 2-inch square = ——— ——— of a 3-inch square.

8. A 2-inch cube = ——— ——— of a 3-inch cube.

(g) Find the area of a surface 12 ft. by 24 ft. †

(h) Find the volume of a solid 12 ft. by 8 ft. by 4 ft. ‡

* Require the pupil to memorize the answer to this question. Lead the pupil to see that a pile of wood 4 ft. wide and 4 ft. high contains a cord for every 8 ft. in the length of the pile; one eighth of a cord for every foot in length, etc.

† Take care that the pupil understands that he does not (cannot) multiply 24 ft. by 12 ft., but 24 square feet by 12, or 12 square feet by 24.

‡ In this problem the pupil multiplies 12 cubic feet by 8, and the product thus obtained by 4.

Ratio and Proportion.

1. One eighth of 24 is ——. 24 is $\frac{1}{8}$ of ——.
2. One seventh of 24 is ——. 24 is $\frac{1}{7}$ of ——.
3. One fifth of 24 is ——. 24 is $\frac{1}{5}$ of ——.

(a) Find 1 sixth of 49.26. (b) 49.26 is 1 sixth of what?
(c) Find 1 seventh of $9.31. (d) $9.31 is 1 seventh of what?

4. Four sevenths of 56 are ——. 56 is $\frac{4}{7}$ of ——.
5. Three sevenths of 42 are ——. 42 is $\frac{3}{7}$ of ——.

(e) Find $\frac{4}{7}$ of 875. (f) 876 is $\frac{4}{7}$ of what number?
(g) Find $\frac{3}{7}$ of 924. (h) 927 is $\frac{3}{7}$ of what number?

6. 18 is —— —— of 27. 27 is —— —— of 18.
7. 27 is —— —— of 36. 36 is —— —— of 27.
8. 45 is —— —— of 54. 54 is —— —— of 45.
9. 63 is —— —— of 72. 72 is —— —— of 63.

10. Three sevenths of 28 are 2 thirds of ——.
11. Six sevenths of 28 are 3 fourths of ——.

(i) Three sevenths of 364 are 1 half of what number?
(j) One seventh of 434 is 2 thirds of what number?

12. Twenty is —— —— of 25. If 25 bags of salt are worth $15, 20 bags of salt are worth —— dollars.

(k) If 25 acres of land are worth $640, how much are 20 acres worth at the same rate?

13. Twenty-five is —— —— of 20, or —— and —— —— times 20. If 20 bushels of apples are worth $12, 25 bushels of apples are worth —— dollars.

(l) If 20 barrels of salt are worth $22.40, how much are 25 barrels of salt worth at the same rate?

14. If 12 qt. of nuts are worth 40¢, 9 qt. of nuts are worth —— cents.

Percentage.

40 per cent = .40 = $\frac{2}{5}$.
60 per cent = .60 = $\frac{3}{5}$.

	(1)	(2)
1.	40 per cent of 50 =	50 is 40 % of ——.
2.	60 per cent of 15 =	15 is 60 % of ——.
3.	40 per cent of 45 =	16 is 40 % of ——.
4.	60 per cent of 45 =	18 is 60 % of ——.
5.	75 per cent of 28 =	18 is 75 % of ——.
6.	$66\frac{2}{3}$ per cent of 21 =	18 is $66\frac{2}{3}$ % of ——.
7.	40 per cent of 60 =	20 is 40 % of ——.
8.	60 per cent of 60 =	24 is 60 % of ——.
9.	75 per cent of 32 =	27 is 75 % of ——.
10.	$66\frac{2}{3}$ per cent of 36 =	22 is $66\frac{2}{3}$ % of ——.

	(3)	(3)
11.	8 is —— per cent of 32.	8 is —— % of 24.
12.	8 is —— per cent of 40.	8 is —— % of 16.
13.	8 is —— per cent of 64.	8 is —— % of 56.
14.	8 is —— per cent of 48.	8 is —— % of 72.
15.	8 is —— per cent of 80.	8 is —— % of 20.*
16.	9 is —— per cent of 15.	18 is —— % of 27.
17.	18 is —— per cent of 24.	$4\frac{1}{3}$ is —— % of 13.

18. Russel earned 60¢; he spent 10 % of his money for a tablet and 20 % of it for a book; the tablet cost —— cents and the book cost —— cents.

19. Ten per cent of the sheep in a certain flock were black; there were 8 black sheep; in all there were —— sheep.

20. Two quarts are —— per cent of 1 peck.

21. Two quarts are —— per cent of 1 gallon.

* Eight is ($\frac{8}{10}$) $\frac{4}{5}$ of 20; $\frac{4}{5}$ = —— per cent.

Percentage.*

(1)

1. 40 per cent of 55 = 60 per cent of 55 =

(a) Find 40 % of 575. (b) Find 60 % of 365.

(c) Find 75 % of 676. (d) Find 66⅔ % of 591.

(e) Find 3 % of $254. † (f) Find 7 % of $254.

(g) A lawyer's commission for collecting money was 7 %; he collected $635. How much of the money should he keep and how much should he "pay over" to the man for whom he collected the money?

(2)

2. 18 is 40 % of ——. 18 is 60 % of ——.

(h) 346 is 40 % of what? (i) 345 is 60 % of what?

(j) 534 is 75 % of what? (k) 534 is 66⅔ % of what?

(l) $36 is 3 % of what? ‡ (m) $84 is 7 % of what?

(n) A lawyer's commission for collecting money was 7 %; § his commission amounted to $63. How much did he collect and how much should he "pay over" to the man for whom he collected the money?

(3)

3. 18 is —— per cent of 45. 33 is ——% of 55.

4. 33 is —— per cent of 44. 18 is ——% of 27.

(o) Eighteen dollars are what per cent of $600? This means, *find how many hundredths of* $600, $18 *are.*

Operation.	*Explanation.*
$600 ÷ 100 = $6.	One per cent of $600 is ——.
$18 ÷ $6 = 3 times.	$18 are as many per cent of $600
$18 are 3 % of $600.	as $6 are contained times in $18.

5. $84 are —— % of $1200. $72 are —— % of $800.

* If any part of this page seems too difficult, review thoroughly pages 17, 18; 27, 28; 37, 38; 47, 48, and 57.

† This means, find 3 hundredths of $354. See page 48, problem (c).

‡ This means, $36 is 3 hundredths of what? See page 48, problem (k).

§ 7 % of what? Ans. 7 % of the amount collected. Tell the pupil that commission for collecting money is reckoned on the amount collected. See page 158, but *do not* "smite the pupil with a definition" to be committed to memory.

Review.

1. The prime factors of 63 are ——, ——, and ——.

(a) What are the prime factors of 215? (b) of 470?

2. Three, 3, 3, and 2 are the prime factors of ——.

(c) Of what number are 3, 5, 2, and 23 the prime factors?

3. Add $\frac{1}{8}$ and $\frac{3}{8}$. (The l. c. m. of 8 and 3 is ——.)

(d) Add $\frac{7}{9}$, $\frac{5}{12}$, and $\frac{2}{3}$. (The l. c. m. of 9, 12, and 3 is ——.)

(e) Divide $\frac{3}{8}$ by $\frac{1}{12}$. (The l. c. m. of 9 and 12 is ——.)

4. Multiply $324 by $\frac{1}{4}$. (This means ——.*)

(f) Multiply $324 by .25. [See page 53, problem (a).]†

5. One half of a rod is —— and —— —— feet.

(g) Twenty-six and one half rods are how many feet?

6. Three eighths of a mile are —— rods.

(h) How many rods in $7\frac{3}{8}$ miles? (i) in $4\frac{3}{4}$ miles?

7. Two rods are —— yards. 3 rd. are —— yards.

(j) How many yards in 42 rods? (k) in 64 rods?

8. When wheat is 70¢ a bushel, 240 lb. cost ——.

(l) When wheat is 90¢ a bushel, how much will one ton of wheat cost?

9. A pile of wood 32 feet long, 4 feet wide, and 4 feet high contains —— cords.

(m) How many cu. ft. in a pile of wood 32 ft. by 4 ft. by 2 ft.?

10. If 12 lb. of sugar are worth 45¢, at the same rate 8 lb. are worth —— cents.

(n) If 12 boxes of soap are worth $22.50, how much are 8 boxes worth at the same rate?

* See page 6, note 8. † Compare the answer to problem 4 with the answer to problem (f). Why are they alike?

Miscellaneous Problems.

1. The perimeter of a rectangular surface 7 feet by 9 feet is —— feet.

(a) How many rods of fence are required to enclose a rectangular field 38 rods by 54 rods?

2. At 14¢ a dozen, 30 eggs would cost —— cents.

(b) At 16¢ a dozen, how much would 414 eggs cost?

3. If the sun rises at 7 o'clock and sets at 4:30, from sunrise to sunset it is —— hours and —— minutes.

(c) If the sun rises at 7:14 and sets at 4:28, how long is it from sunrise to sunset?

4. A man bought 16 dozen eggs at 10¢ a dozen; he lost 25 per cent of them by decay; he sold the remainder at 12¢ a dozen; he lost —— cents.

(d) A man bought 360 dozen eggs at 9¢ a dozen; he lost 25 % of them by decay; he sold the remainder at 11¢ a dozen. How much money did he lose?

5. From midnight Monday to midnight Wednesday it is —— hours.

(e) How many hours in a week? (f) In the month of January?

6. At 45¢ each, 3 copies of "Robinson Crusoe" will cost —— dollar and —— cents; for $1.80, I can buy —— copies.

(g) How much will 7 copies of "Robinson Crusoe" cost?

(h) How many copies of "Robinson Crusoe" can I buy for $10.35? (i) For $4.95?

7. Sixteen hundredths and seven tenths are ——.

(j) Add four thousand three hundred twenty-four and twenty-five thousandths, and seven hundred forty-six and thirty-four hundredths, and seventy-five and eight tenths.

(k) From forty-five hundred subtract forty-five hundredths.

Simple Numbers.

1. Ten times 60 are ——. 10 times 40 are ——.
2. Ten times 46 are ——. 10 times 25 are ——.
3. Ten times 300 are ——. 10 times 325 are ——.

4. *Ten times* 2 *times a number are* —— *times the number.*

(a) Multiply 347 by 20.

Operation.	*Explanation.*
347	Two times 347 are ——.
20	Ten times 694 are ——.
6940	Ten times 2 times (20 times) 347 are —— —.

(b) Multiply 564 by 30. (c) Multiply 468 by 40.
(d) Multiply 735 by 50. (e) Multiply 642 by 60.

5. *One hundred times* 2 *times a number are* —— *times the number.*

(f) Multiply 86 by 200. (g) Multiply 94 by 300.
(h) Multiply 75 by 400. (i) Multiply 48 by 700.

Review.

6. Common multiples of 12 and 10 are ——, ——, etc. The least common multiple of 12 and 10 is ——.

7. The prime factors of 48 are ——, ——, ——, ——, and ——.

(j) What are the prime factors of 290? * (k) of 430?

8. 2, 3, and 5 are the prime factors of ——. Thirty is exactly divisible by 2; by 3; by 5; by (2 times 5) 10; by (3 times 5) 15; by (2 times 3) 6.

Multiply.	Divide.	Divide.
(l) 347 by 70.	(m) 8540 lb. by 70 lb.†	(n) 1480 lb. by 20.‡
(o) 575 by 70.	(p) 9470 lb. by 70 lb.	(q) 1350 lb. by 20.§

* See page 41, problem 4.

† This means, find how many times 70 lb. are contained in 8540 lb. 70 lb. are contained in 8540 lb. —— times. *Story—In* 8540 *lb. of ear corn there are* —— *bushels.*

‡ This means, find 1 twentieth of 1480 lb. 1 twentieth of 1480 lb. is —— lb. *Story—I wish to put* 1480 *lb. of wheat into* 20 *grain sacks; I must put* —— *lb. in each sack.*

§ Lead the pupil to see that 1 twentieth, is 1 half of 1 tenth.

Common Fractions.

TO THE TEACHER.—Spend much time upon this page and upon work similar to the problems here given. The figures in parentheses refer to notes on pp. 6 and 7. By means of these notes the pupil should be able to tell the meaning of each problem and to tell a suggested number story, *i.e*, to show its possible application in experience. Before attempting to solve problems (a) to (n), the pupil should be required to *tell the meaning of each problem from* 1 *to* 14 *inclusive, to solve it,* and *to tell a number story for it.*

1. Add $\frac{1}{10}$ and $\frac{1}{12}$.*	(a) $375\frac{3}{10} + 256\frac{5}{12}$.	
2. From $7\frac{1}{8}$ subtract $2\frac{1}{2}$.†	(b) $3467\frac{1}{8} - 1282\frac{1}{2}$.	
3. Divide $\frac{3}{5}$ by $\frac{1}{10}$.	(c) $3\frac{3}{5} \div \frac{1}{10}$.	(9)
4. Divide 8 by $\frac{2}{3}$.	(d) $28 \div \frac{2}{3}$.	(4)
5. Divide $3\frac{3}{4}$ by $1\frac{1}{4}$.	(e) $47\frac{1}{4} \div 1\frac{3}{4}$.	(14)
6. Divide $21\frac{1}{4}$ ft. by 3.	(f) $87\frac{1}{4}$ ft. $\div$ 3.	(21)
7. Divide $1\frac{3}{4}$ ft. by 3.	(g) $5\frac{3}{4}$ ft. $\div$ 3.	(15)
8. Divide $13\frac{3}{4}$ ft. by 3.	(h) $745\frac{3}{4}$ ft. $\div$ 3.	(31)
9. Multiply 20¢ by $2\frac{3}{4}$. ‡	(i) $96 \times 2\frac{3}{4}$.	(16)
10. Multiply 20¢ by $\frac{3}{4}$.	(j) $124 \times \frac{3}{4}$.	(8)
11. Multiply $\$3\frac{1}{5}$ by 6.	(k) $64\frac{1}{5} \times 6$.	(19)
12. Multiply $\$\frac{1}{2}$ by $\frac{1}{2}$.	(l) $\$\frac{1}{2} \times 26\frac{1}{2}$.	(13)
13. Multiply $\$1\frac{1}{2}$ by $\frac{1}{2}$.	(m) $\$27\frac{1}{2} \times \frac{1}{2}$.	(13)
14. Multiply $\$6\frac{1}{2}$ by $2\frac{1}{2}$. §	(n) $\$86\frac{1}{2} \times 2\frac{1}{2}$.	(29)

* The pupil is expected to find by trial that the l. c. m. of 10 and 12 is ———.

† Lead the pupil to take 1 from the 7, change it to fifths and add it to the $\frac{1}{5}$.

‡ In problems 9 to 13 inclusive, think of the number to be multiplied as the price per yard; this will suggest the following:

Problem 9, $2\frac{3}{4}$ yd. of ribbon at 20¢ a yard.
Problem 10, $\frac{3}{4}$ yd. of ribbon at 20¢ a yard.
Problem 11, 6 yd. of ribbon at $\$\frac{3}{5}$ a yard.
Problem 12, $\frac{1}{2}$ yd. of ribbon at $\$\frac{1}{2}$ a yard.
Problem 13, $\frac{1}{2}$ yd. of ribbon at $\$1\frac{1}{2}$ a yard.

§ This means, 2 times $\$6\frac{1}{2}$ plus $\frac{1}{2}$ of $\$6\frac{1}{2}$. 2 times $\$6\frac{1}{2}$ are —— dollars. $\frac{1}{2}$ of $\$6\frac{1}{2}$ is —— dollars. $\$13 + \$3\frac{1}{4} =$ —— dollars. *Story*—At $\$6\frac{1}{2}$ a ton, $2\frac{1}{2}$ tons of coal will cost ———.

Decimal Fractions.

(a) Multiply $546 by 3.24. This means, *find* 3 *times* $546, plus 2 tenths of $546, plus 4 hundredths of $546.

Operation.	*Explanation.*
$546 *	One hundredth of $546 is ———.
3.24	4 hundredths of $546 are ———.
$21.84	One tenth of $546 is ———.
$109.2	2 tenths of $546 are ———.
$1638	3 times $546 are ———.
$1769.04	$21.84 + $109.2 + $1638 =

Number Story.

If one acre of land is worth $546,
1 hundredth of an acre is worth ———.
4 hundredths of an acre are worth ———.
1 tenth of an acre is worth ———.
2 tenths of an acre are worth ———.
3 acres are worth ———.
3.24 acres are worth ———.

(b) Multiply $437 by 2.36; (c) $375 by 5.27.

(d) Multiply $352 by 6.21; (e) $284 by 7.32.

(f) Multiply four hundred seventy-three dollars by four and thirty-five hundredths.

(g) When oil meal is $28 a ton, how much will 4680 lb. cost? (4680 lb. = ——— tons.)

(h) When hay is $12 a ton, how much will 860 lb. cost? (860 lb. = ——— hundredths of a ton.)

(i) Add.	(j) Subtract.	(k) Multiply.	(l) Divide.	(m) Divide.
.886	8.744	8.2 †	$.5)$31 ‡	5)$31. §
.075	.956	.43		

* See foot-notes, page 53.

† One hundredth of 8.2 is .082. See that the pupil writes the decimal point in the first partial product *immediately after writing the tenths' figure* of the partial product.

‡ This means, *find how many times* 5 *tenths are contained in* 310 ($31.0) tenths.

§ This means, find 1 fifth of $31.

Denominate Numbers.

Teach the meaning and use of the expressions, **gross weight, net weight,** and **tare.**

CITY SCALE.

No. 186

Load of Hay, *Dec. 31, 1896.*

From Clyde H. Hall *To* Fay D. Winslow.

Gross Weight, 4760 *lb.*

Tare, 1960 *lb.*

Net Weight, 2800 *lb.*

Clarence Marshall, *Weigher.*

(a) Find the value of the load of hay, ticket No. 186, at $8.25 a ton.

Find the value of the following:

	Commodity.	Gross Weight.	Tare.	Price per Ton
(b)	Hay.	5610	1870	$12.50
(c)	Coal.	6380	2140	$6.50
(d)	Bran.	3340	1560	$13.50

1. One hundred feet are —— rods and —— foot.
2. Two hundred feet are —— rods and —— feet.
3. Three hundred feet are —— rods and —— feet.
4. Four hundred feet are —— rods and —— feet.
5. Five hundred feet are —— rods and —— feet.

(e) Change 8 miles to rods. (f) Change 30 rods to feet.

(g) Change 36 rods to yards. (h) Change 2160 lb. oats to bu.

(i) Change 5240 lb. to tons. (j) Change 274 ft. to inches.

Measurements.

Make the pupil familiar with that for which the term square rod stands. "Stake off" a piece of land one rod square on the schoolground or elsewhere; also a piece four rods square.

1. *One hundred sixty square rods are one acre.* A piece of land 1 rod wide and 160 rods long is one ——.

2. A piece of land $\frac{1}{2}$ of a mile long and 1 rod wide is —— ——.

3. Land 2 rd. wide and —— rd. long is one acre.

4. Land 4 rd. wide and —— rd. long is one acre.

5. Land 8 rd. wide and —— rd. long is one acre.

6. Land 10 rd. wide and —— rd. long is one acre.

7. Land 5 rd. wide and —— rd. long is one acre.

8. Land 4 rods by 4 rods is —— ——th of an acre.

9. Land 4 rods by 10 rods is —— —— of an acre.

10. Land 8 rods by 10 rods is —— —— of an acre.

11. Land 1 rod wide and 1 mile long is —— acres.

12. Land 2 rods wide and 1 mile long is —— acres.

13. Land 66 ft. wide and 1 mile long is —— acres.

14. Land 5 rods by 16 rods is —— square rods.

(a) How many square rods in a rectangular piece of land 28 rd. by 36 rd. ?

Review.

(b) Find the area of a rectangular surface 18 ft. by 26 ft.*

(c) Find the volume of a rectangular solid 15 ft. by 8 ft. by 23 ft. †

15. How many cords in a pile of wood 4 ft. wide, 4 ft. high, and 36 ft. long? ‡

(d) How many cords in a pile of wood 4 ft. high, 4 ft. wide, and 276 ft. long?

(e) How many cords in a pile of wood 4 ft. wide, 4 ft. high, and as long as your schoolroom?

* See foot-note (†), page 55. † See foot-note (‡), page 55.
‡ See foot-note (*), page 55.

Ratio and Proportion.

1. One ninth of 36 is ——. 36 is $\frac{1}{9}$ of ——.
2. One eighth of 36 is ——. 36 is $\frac{1}{8}$ of ——.
3. One seventh of 36 is ——. 36 is $\frac{1}{7}$ of ——.

(a) Find 1 eighth of 353.6 (b) 353.6 is $\frac{1}{8}$ of what?
(c) Find 1 fifth of 1685. (d) 1685 is $\frac{1}{5}$ of what?

4. Five eighths of 80 are ——. 80 is $\frac{5}{8}$ of ——.
5. Three fifths of 60 are ——. 60 is $\frac{3}{5}$ of ——.

(e) Find $\frac{5}{8}$ of 680. (f) 680 is $\frac{5}{8}$ of what number?
(g) Find $\frac{3}{5}$ of 435. (h) 435 is $\frac{3}{5}$ of what number?

6. 22 is —— —— of 33. 33 is —— —— of 22.
7. 33 is —— —— of 55. 55 is —— —— of 33.
8. 55 is —— —— of 99. 99 is —— —— of 55.
9. 44 is —— —— of 77. 77 is —— —— of 44.

10. Three eighths of 32 are 2 thirds of ——.
11. Five eighths of 32 are 2 thirds of ——.

(i) Three eighths of 528 are 2 thirds of what number?
(j) Five eighths of 528 are 2 thirds of what number?

12. Twenty is —— —— of 16, or —— and —— —— times 16. A man can earn —— and —— —— times as much in 20 days as he can earn in 16 days. If he can earn $44 in 16 days, in 20 days he can earn —— dollars.

(k) If a man can earn $740 in 16 weeks, how much can he earn in 20 weeks?

13. If 20 bushels of apples are worth $12, 15 bushels are worth —— dollars.

(l) If 20 gal. of milk are worth $3.24, how much are 15 gal. worth?

(m) If 15 gal. of cream are worth $7.80, how much are 20 gal. of cream worth?

Percentage.

80 per cent = .80 = $\frac{4}{5}$.
$83\frac{1}{3}$ per cent = .$83\frac{1}{3}$ = $\frac{5}{6}$.

(1)	(2)
1. 80 per cent of 60 =	60 is 80 % of ——.
2. $83\frac{1}{3}$ per cent of 60 =	30 is $83\frac{1}{3}$ % of ——.
3. 80 per cent of 40 =	40 is 80 % of ——.
4. $83\frac{1}{3}$ per cent of 36 =	35 is $83\frac{1}{3}$ % of ——.
5. 75 per cent of 60 =	60 is 75 % of ——.
6. $66\frac{2}{3}$ per cent of 18 =	32 is $66\frac{2}{3}$ % of ——.
7. 40 per cent of 35 =	18 is 40 % of ——.
8. 60 per cent of 35 =	27 is 60 % of ——.

(3)	(3)
9. 9 is —— per cent of 54.	9 is —— % of 45.
10. 9 is —— per cent of 63.	9 is —— % of 36.
11. 9 is —— per cent of 27.	9 is —— % of 72.
12. 9 is —— per cent of 18.	9 is —— % of 90.
13. 9 is —— per cent of 81.	14 is —— % of 35.*
14. 21 is —— per cent of 35.	14 is —— % of 21.
15. 21 is —— per cent of 28.	35 is —— % of 42.
16. 28 is —— per cent of 35.	$12\frac{1}{2}$ is —— % of 25.

17. Mr. Dow had 80 bushels of apples; he lost 25 % of them by decay; he lost —— bushels and had —— bushels left.

18. Fourth-of-July night Willie had 10¢; this was $12\frac{1}{2}$ per cent of what his father gave him to spend; his father gave him —— cents.

19. Sarah had 45 chicks; a hawk killed 18 of them; the hawk killed —— per cent of her chickens.

* Fourteen is ($\frac{4}{10}$) $\frac{4}{10}$ of 35; $\frac{4}{10}$ = —— per cent.

Percentage.

(1)

1. 80 per cent of 55 = $83\frac{1}{3}$ per cent of 48 =

(a) Find 80 % of 435. (b) Find $83\frac{1}{3}$ % of 492.

(c) Find 40 % of 435. (d) Find 75 % of 492.

(e) Find 7 % of $435.* (f) Find 9 % of $492.

(g) A lawyer's commission for collecting money was 9 %;† he collected $834. How much of this money should he keep and how much should he "pay over" to the man for whom he collected the money?

(2)

2. 20 is 80 % of ——. 20 is $83\frac{1}{3}$ % of ——.

(h) 280 is 80 % of what? (i) 280 is $83\frac{1}{3}$ % of what?

(j) 276 is 60 % of what? (k) 276 is 75 % of what?

(l) 224 is 7 % of what? (m) 531 is 9 % of what?

(n) A lawyer's commission for collecting money was 9 %;† his commission amounted to $54. How much did he collect and how much should he "pay over" to the man for whom he collected the money?

(3)

3. 44 is —— per cent of 55. 55 is ——% of 66.

4. 36 is —— per cent of 48. 48 is ——% of 60.

5. Fifty-six dollars are what per cent of $800? This means, $56 *are how many hundredths of* $800? ‡

6. $36 is ——% of $400. $55 is ——% of $500.

7. $2.40 is ——% of $80. $3.50 is ——% of $50.

8. A lawyer collected $900; he retained as his commission $63 of this sum and paid the remainder, $837, to the man for whom he collected the money; the lawyer's commission for collecting was ——%.

* This means, find 7 hundredths of $435. See page 48, problem (c).

† 9 % of what? Ans. 9 % of the amount collected.

‡ One hundredth of $800 is $8. See page 58, problem (o).

Review.

1. Ten times 6 times a number are —— times the number. 10 times 6 times 8 are ——. $8 \times 60 =$

2. One hundred times 5 times a number are —— times the number. 9 multiplied by 500 =

(a) Multiply 78 by 80. (b) Multiply 96 by 700.

3. Divide $12\frac{1}{2}$ by $2\frac{1}{2}$. (Change to halves.) *Story.*

(c) Divide $345\frac{3}{8}$ by $2\frac{3}{8}$. (Change to ——.) *Story.*

(d) Divide 345.6 by 2.4. (This means, ————.) *Story.*

Compare the answers to problems (c) and (d). Why are they alike?

(e) Multiply $338 by 2.4.* (f) Multiply $338 by 2.43. †

(g) When land is $375 an acre, how much will 3.35 acres cost?

Find the value of the following:

Commodity.	Gross Weight.	Tare.	Price per Ton.
(h) Straw.	4360 lb.	1780 lb.	$4.25
(i) Coal.	6240 lb.	1830 lb.	$6.75

4. One acre is —— square rods. $\frac{1}{2}$ acre =

5. One mile is —— rods. $\frac{1}{2}$ mile =

6. A piece of land $\frac{1}{2}$ of a mile long and 2 rods wide is —— acres.

7. A piece of land 160 rods long and as wide as the schoolroom is about —— acres.

(j) How many square rods in a rectangular piece of land 47 rods by 6 rods? Is this more or less than 1 acre? Is it more or less than 2 acres?

8. One cord is —— cubic feet. $\frac{1}{2}$ cord =

(k) How many cubic feet of wood in a pile 7 feet by 5 feet by 6 feet? Is this more or less than 1 cord? Is it more or less than 2 cords?

* Tell the meaning. See page 33, problem (a).

† Tell the meaning. See page 63, problem (a).

Miscellaneous Problems.

1. If a man can save $5 a month, in 3 years he can save ——— dollars.

(a) If a man can save $27 in a month, how much can he save in 9 years?

2. Byron bought 2 doz. oranges for 40¢; he sold them at 3¢ each; he gained ——— cents.

(b) A merchant bought 35 barrels of apples for $85; he sold them at $2.75 a barrel. How much did he gain?

3. Henry had 25 chickens; a hawk caught 20 % of them; he sold the remainder at 22¢ each; he received ——— dollars and ——— cents.

(c) Henry's father had 75 bushels of apples; he lost 20 % of them by decay; he sold the remainder at 85¢ a bushel. How much did he receive for them?

4. If coffee costs $\$\frac{1}{5}$ a pound, for 1 dollar I can buy ——— pounds. $1 \div \frac{1}{5} =$

(d) If tea costs $\$\frac{1}{5}$ a pound, how many pounds can I buy for $57? (Change $57 to fifth-dollars.)

5. The sum of two numbers is 34; one of the numbers is 12; the other number is ———.

(e) The sum of two numbers is 346.2; one of the numbers is 75.36. What is the other number?

6. In a piece of slate 1 foot square and 1 inch thick there are ——— cubic inches.

(f) How many cubic inches in a piece of slate 2 feet square and 2 inches thick?

7. If $\frac{1}{8}$ of a ton of coal is worth 6 dollars, 1 ton is worth ——— dollars.

(g) If $\frac{1}{8}$ of a ton of coal is worth $4.20, how much is 1 ton worth?

Simple Numbers.

1. One tenth of 6 is ——. 1 tenth of 40 is ——.
2. One tenth of 46 is ——. 1 tenth of 25 is ——.
3. One tenth of 300 is ——. 1 tenth of 325 is ——.

4. *One half of* 1 *tenth of a number is one* ——*th of the number.* One third of 1 tenth of a number =

(a) Divide 472 by 20.

Operation.	*Explanation.*
20) 472 23.6	One tenth of 472 is ——. One half of 1 tenth ($\frac{1}{10}$) of 472 is ——.

(b) Divide 741 by 30. (c) Divide 548 by 40.
(d) Divide 735 by 50. (e) Divide 960 by 60.

5. *One third of* 1 *hundredth of a number is one* ——*th of the number.* One fourth of 1 hundredth of a number =

(f) Divide 972 by 300. (g) Divide 972 by 400.
(h) Divide 895 by 500. (i) Divide 976 by 800.

Review.

6. Common multiples of 15 and 6 are ——, ——, etc. The least common multiple of 15 and 6 is ——.

7. The prime factors of 63 are ——, ——, and ——.

(j) What are the prime factors of 124? (k) of 178?

8. 3, 3, and 5 are the prime factors of ——. Forty-five is exactly divisible by 3; by 5; by ——; by ——.

(l)	(m)
128 cu. feet.)1536 cu. ft.*	12)1584 cu. ft.*
(n)	(o)
160 sq. rd.)2400 sq. rd.*	15)2445 sq. rd.*

* Tell the meaning. Tell a number story. The story suggested by problem (l) may be about *cords of wood;* problem (m), *loads of wood;* problem (n), *acres;* problem (o), *lots of land.*

Common Fractions.

1. Add $\frac{3}{16}$ and $\frac{1}{8}$. (The l. c. m. of 15 and 6 is ——.)

(a) Find the sum of $456\frac{7}{16}$, $341\frac{3}{8}$, $245\frac{1}{6}$, and 564.

2. From $9\frac{3}{16}$ subtract $4\frac{5}{8}$. ($1\frac{3}{16} = \frac{19}{16}$. $\frac{5}{8} = \frac{10}{16}$.)

(b) Find the difference of $4275\frac{3}{16}$ and $1328\frac{5}{8}$.

3. Multiply $\frac{7}{8}$ by 9. (This means —— 3 * ——.)

(c) Find the product of $453\frac{7}{8}$ multiplied by 9.

4. Multiply $2\frac{3}{4}$ by $\frac{1}{8}$. (This means —— 13 * ——.)

(d) Find the product of $458\frac{3}{4}$ multiplied by $\frac{1}{8}$.

5. Multiply 17 by $\frac{3}{4}$. (This means —— 8 * ——.) †

(e) Find the product of 741 multiplied by $\frac{3}{4}$.

6. Multiply 17 by $2\frac{3}{4}$. (This means —— 16 * ——.)

(f) Find the product of 741 multiplied by $2\frac{3}{4}$.

7. Multiply $16\frac{1}{2}$ by $2\frac{1}{4}$. (This means —— 22 * ——).

(g) Find the product of $732\frac{1}{2}$ multiplied by $2\frac{1}{4}$.

8. Divide 8 by $\frac{3}{4}$. (4)* (Change 8 to ——ths.)

(h) Find the quotient of 97 divided by $\frac{3}{4}$. *Story.*

9. Divide $1\frac{1}{8}$ by $\frac{1}{8}$. (9) * (Change to ——ths.)

(i) Find the quotient of $3\frac{1}{16}$ divided by $\frac{1}{8}$. *Story.*

10. Divide $7\frac{1}{2}$ by $2\frac{1}{4}$. (14) * (Change to ——ths.)

(j) Find the quotient of $55\frac{1}{2}$ divided by $2\frac{3}{4}$. *Story.*

11. Divide $\$1\frac{1}{8}$ ($\frac{1}{8}$) by 4. (This means —— 15 * ——.)

(k) Find the quotient of $7\frac{1}{8}$ divided by 8. *Story.*

12. Divide $\$16\frac{3}{4}$ by 3. (This means —— 21 * ——.)

(l) Find the quotient of $532\frac{3}{4}$ divided by 3. *Story.*

(m) Add.	(n) Subtract.	(o) Multiply.	(p) Divide.	(q) Divide.
$375\frac{1}{16}$	$435\frac{1}{4}$	$346\frac{5}{8}$	$3\frac{1}{4}$ ft.)232 ft.	3)$365\frac{3}{4}$ ft.
$246\frac{5}{8}$	$182\frac{11}{16}$	12		

* These figures refer to notes on pages 6 and 7. See also foot-notes, page 62.

† This means, find $\frac{3}{4}$ of 17. The pupil may now be led to see that $\frac{3}{4}$ of 17 = $\frac{1}{4}$ of 3 times 17.

Decimal Fractions.

1. One tenth of $6 is ———. .1 of $6.25 is ———.

2. One hundredth of $6 is ———. .01 of $6.25 is $.0625.*

3. Read each of the following in two ways: $.2436,† $.0532, $.6403, $.0042, $.0002, $.6042, $.8002.

(a) Multiply $6.25 by 4.23. This means, *find* 4 *times* $6.25 + 2 *tenths of* $6.25 + 3 *hundredths of* $6.25.

Operation.	*Explanation.*
$6.25	One hundredth of $6.25 is ———.
4.23	3 hundredths of $6.25 are ———.
$.1875 ‡	One tenth of $6.25 is ———.
$1.250	2 tenths of $6.25 are ———.
$25.00	4 times $6.25 are ———.
$26.4375	$.1875 + $1.250 + $25 =

Number Story.

If one ton of coal is worth $6.25,
1 hundredth of a ton is worth ———.
3 hundredths of a ton are worth ———.
1 tenth of a ton is worth ———.
2 tenths of a ton are worth ———.
4 tons are worth ———.
4.23 tons are worth ———.

(b) Multiply $7.35 by 3.46; (c) $4.45 by 5.24.

(d) When hay is worth $8.75 a ton, how much will 3240 lb. cost? (3240 lb. = ——— tons.)

* The pupil who has been well taught up to this point will, without assistance from the teacher, recognize the fact that the figure 5 in $.0625 stands tor 5 tenths of a mill. He may be led to see that 6¢, 2 m., and 5 tenths of a mill are 625 ten-thousandths of a dollar. *He should now be taught to write decimally, and to read, any number of ten-thousandths.*

† (1) 24¢, 3 m., and 6 tenths of a mill. (2) 2436 ten-thousandths ot a dollar.

‡ To the Teacher.—Read the foot-note on page 133; also, the first part of page 143. If the pupil finds difficulty in "pointing off," teach him to use a *separatrix* in the multiplicand as suggested on page 133. While multiplying 6.25 by 3 hundredths, it may appear on the slate thus: $\overset{v}{0}6.25$ / 4.23 Do not *at this stage of the work* allow the pupil to "point off" by counting the decimal places in the multiplicand and multiplier. Rather, lead him to *think the meaning of the problem.* See foot-notes, page 53.

Denominate Numbers.

1. 5280 *feet are* 1 *mile.* —— rods are 1 mile.

(a) Change 320 rods to feet. (b) Change 160 rods to feet.

2. Six hundred feet are —— rods and —— feet.*

3. Seven hundred feet are —— rods and —— feet.

4. Eight hundred feet are —— rods and —— feet.

5. Two miles are —— rods. $1\frac{1}{2}$ miles are —— rods.

(c) Change 6 miles to rods. (d) Change 30 rods to feet.

(e) Change 2 miles to feet. (f) Change 75 rods to yards.

(g) Change $2\frac{1}{2}$ miles to feet. (h) Change $2\frac{1}{2}$ miles to rods.

Find the value of the following:

Commodity.	Gross Weight.	Tare.	Price.
(i) Corn meal.	4380 lb.	1540 lb.	15.25 per ton.
(j) Oat straw.	3460 lb.	1620 lb.	4.50 per ton.
(k) Old iron.	3240 lb.	1380 lb.	$\frac{1}{2}¢$ per lb.†
(l) Paper-rags.	3120 lb.	1260 lb.	$\frac{3}{4}¢$ per lb.
(m) Ear corn.	3690 lb.	1230 lb.	35¢ per bu.
(n) Oats.	2410 lb.	1250 lb.	28¢ per bu.

6. 120 inches are —— feet. 126 inches are —— feet.

(o) Change 438 in. to feet. (p) Change 580 in. to feet.

7. 72 feet are —— yards. 73 feet are —— yards.

(q) Change 853 ft. to yards. (r) Change 725 ft. to yards.

8. 33 feet are —— rods. 66 feet are —— rods.

(s) Change 627 ft. to rods. ‡ (t) Change 594 feet to rods.

9. 11 yards are —— rods. 22 yards are —— rods.

(u) Change 759 yd. to rods. (v) Change 638 yd. to rods.

* See page 64, problems 1 to 5.

† 1860 lbs. at 1¢ a pound would be worth $18.60; at $\frac{1}{2}$ a cent a pound the value is $\frac{1}{2}$ of $18.60, or ——.

‡ To divide 627 ft. by $16\frac{1}{2}$ ft., change both numbers to halves. See page 7, note (20).

Measurements.

1. A piece of land 10 rods by 16 rods contains —— square rods. It is —— acre.

2. A piece of land 20 rods by 16 rods contains —— square rods. It is —— acres.

(a) Change 640 sq. rd. to acres. (b) Change 1280 sq. rd. to acres. (c) How many acres in a piece of land 40 rods by 64 rods?

3. Eighty square rods are —— —— of an acre.

4. Forty square rods are —— —— of an acre.

5. One hundred twenty square rods are —— —— of an acre.

6. A piece of land 20 rd. by 26 rd. contains —— square rods.

(d) How many acres in a piece of land 20 rods by 26 rods?

Find the number of acres in each of the following:

(e) 20 rods by 28 rods. (f) 20 rods by 30 rods.
(g) 12 rods by 60 rods. (h) 24 rods by 50 rods.
(i) 36 rods by 50 rods. (j) 48 rods by 60 rods.

7. A pile of wood 8 ft. by 4 ft. by 4 ft. contains —— cubic feet. It is —— cord.

8. Sixty-four cubic feet are —— —— of a cord.

9. Thirty-two cubic feet are —— —— of a cord.

10. Ninety-six cubic feet are —— —— of a cord.

(k) How many cubic feet in a pile of wood 12 ft. by 8 ft. by 4 ft.?

(l) How many cords in a pile of wood 12 ft. by 8 ft. by 4 ft.?

Find the number of cords in each of the following:

(m) 6 feet by 4 feet by 8 feet. (n) 6 ft. by 8 ft. by 8 ft.
(o) 6 feet by 12 feet by 4 feet. (p) 6 ft. by 12 ft. by 8 ft.

(q) Find the cost of a pile of wood 12 feet by 8 feet by 6 feet at $4 a cord.

Ratio and Proportion.

1. One fourth of 3.6 is ——.* 3.6 is $\frac{1}{4}$ of ——.
2. One fifth of 3.5 is ——. 3.5 is $\frac{1}{5}$ of ——.
3. One sixth of 2.4 is ——. 2.4 is $\frac{1}{6}$ of ——.

(a) Find 1 fourth of 78.4. (b) 78.4 is $\frac{1}{4}$ of what?
(c) Find 1 fifth of 97.5. (d) 97.5 is $\frac{1}{5}$ of what?

4. Three fourths of 3.6 are ——. † 3.6 is $\frac{3}{4}$ of ——.
5. Two thirds of 2.4 are ——. 2.4 is $\frac{2}{3}$ of ——.

(e) Find $\frac{2}{3}$ of 28.8. (f) 28.8 is $\frac{2}{3}$ of what number?
(g) Find $\frac{3}{5}$ of 37.2. (h) 37.2 is $\frac{3}{5}$ of what number?

6. 1.4 is —— —— of 2.1. ‡ 2.1 is —— —— of 1.4.
7. 2.1 is —— —— of 2.8. 2.8 is —— —— of 2.1.
8. 2.8 is —— —— of 3.5. 3.5 is —— —— of 2.8.
9. 3.5 is —— —— of 4.2. 4.2 is —— —— of 3.5.

10. Two thirds of 2.1 are 1 half of ——.
11. One half of 2.4 is 2 thirds of ——.

(i) Two thirds of 53.7 are 1 half of what number?
(j) One half of 65.6 is 2 thirds of what number?

12. If a man can earn $12 in 2.1 days, in 2.8 days he can earn —— dollars.

(k) If a man can earn $75.45 in 2.1 months, how much can he earn in 2.8 months?

13. If 2.8 tons of coal are worth $12, 2.1 tons are worth —— dollars.

(l) If 2.8 tons of coal are worth $9.60, how much are 2.1 tons worth?

14. If a pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long is worth $5, a pile 4 ft. wide, 4 ft. high, and 32 ft. long is worth —— dollars.

* One fourth of 36 tenths is —— tenths. † Three fourths of 36 tenths are —— tenths, or 2 and —— tenths. ‡ Fourteen tenths are —— —— of 21 tenths.

Percentage.

$37\frac{1}{2}$ per cent = .37½ = $\frac{3}{8}$.
$62\frac{1}{2}$ per cent = .62½ = $\frac{5}{8}$.
$87\frac{1}{2}$ per cent = .87½ = $\frac{7}{8}$.

	(1)	(2)
1.	12½ % of 48 =	48 is 12½ % of ——.
2.	25 % of 48 =	48 is 25 % of ——.
3.	37½ % of 48 =	48 is 37½ % of ——.
4.	50 % of 48 =	48 is 50 % of ——.
5.	62½ % of 40 =	40 is 62½ % of ——.
6.	87½ % of 56 =	56 is 87½ % of ——.

	(3)	(3)
7.	3 is —— % of 24.	9 is —— % of 24.
8.	21 is —— % of 24.	15 is —— % of 24.
9.	4 is —— % of 32.	20 is —— % of 32.
10.	12 is —— % of 32.	28 is —— % of 32.
11.	5 is —— % of 40.	15 is —— % of 40.
12.	35 is —— % of 40.	25 is —— % of 40.

13. Mr. A owed Mr. B $72. Mr. A had suffered many losses and it was hard for him to pay this amount. Mr. B kindly discounted the bill 12½ % and Mr. A paid it. He paid —— dollars.

14. Harry paid 87½ % of his money for books and had $2 left; before he bought the books he had —— dollars; the books cost —— dollars.

15. Mr. Hill set 40 trees in his new orchard; 8 of them died the first year; —— per cent of the trees were dead; —— per cent were alive.

16. Twenty per cent of 50 * is —— per cent of 30.

17. Forty per cent of 50 is —— per cent of 80.

18. Fifty per cent of 12 is —— per cent of 30.

*Require the pupil to pause at this point until he can think how much 20% of 50 is.

Percentage.

(*1*)

1. $37\frac{1}{2}$ per cent of 64 = $62\frac{1}{2}$ per cent of 64 =

(a) Find $37\frac{1}{2}$ % of 192. (b) Find $62\frac{1}{2}$ % of 192.

(c) Find $87\frac{1}{2}$ % of 192. (d) Find 75 % of 192.

(e) Find 5 % of $192.* (f) Find 7 % of $192.

(g) A lawyer's commission for collecting money was 8 %;† he collected $875. How much of this money should he keep and how much should he "pay over" to the man for whom he collected the money?

(*2*)

2. 15 is $37\frac{1}{2}$ per cent of ——. 35 is $62\frac{1}{2}$ % of ——.

(h) 165 is $37\frac{1}{2}$ % of what? (i) 165 is $62\frac{1}{2}$ % of what?

(j) 161 is $87\frac{1}{2}$ % of what? (k) 168 is 75 % of what?

(l) 135 is 3 % of what? (m) 168 is 7 % of what?

(n) A lawyer's commission for collecting money was 8 %;† his commission amounted to $34.80. How much did he collect and how much did he "pay over" to the man for whom he collected the money?‡

(*3*)

3. 18 is —— per cent of 48. 30 is —— per cent of 48.

4. 42 is —— per cent of 48. 36 is —— per cent of 48.

5. Forty-five dollars is what per cent of $500? This means, $45 *are how many hundredths of* $500?§

6. $15 is —— % of $300. $25 is —— % of $500.

7. $2.70 is —— % of $90. $3.50 is —— % of $700.

8. A lawyer collected $500; he retained as his commission $55 of this sum and paid the remainder, $445, to the man for whom he collected the money; the lawyer's commission for collecting was —— %.

* This means, find 5 hundredths of 192. See page 48, problem (c).

† 8 % of what? Ans. 8 % of the amount collected. ‡ $34.80 is 8 hundredths of what?

§ One hundredth of $500 is $5. See page 58, problem (o).

Review.

1. One fourth of 1 tenth of a number is one ——th of the number. 1 fourth of 1 tenth of 120 is ——.

(a) Divide 836 by 40. (b) Divide 75.6 by 40.

2. One fifth of 1 hundredth of a number is one ——th of the number. 1 fifth of 1 hundredth of 3500 is ——.

(c) Divide 950 by 500. (d) Divide 246.5 by 500.

(e) Divide 3660 by 600. (f) Divide 5740 by 700.

3. Multiply 19 by $3\frac{2}{3}$. (This means ——.) *

(g) Multiply 276 by $5\frac{2}{4}$. (h) Multiply 276 by 5.75.

Compare the answers to problems (g) and (h). Why are they alike?

4. The sum of 86 hundredths and 5 tenths is ——.

(i) Add 735 ten-thousandths and 642 thousandths.

5. One mile is —— feet. 1 mile is —— rods.

(j) Change $5\frac{1}{4}$ miles to feet. (k) Change $8\frac{3}{4}$ miles to rods.

6. A piece of land 4 rods by 80 rods is —— acres.

(l) How many acres in a piece of land 18 rods by 60 rods?

7. A pile of wood 4 ft. wide, 4 ft. high, and 40 ft. long contains —— cords.

(m) How many cords in a pile of wood 4 ft. wide, 4 ft. high, and 432 ft. long?

Perform the operations indicated and tell number stories.

(n) 48 cu. ft. × 6 × 8 = (o) 2304 cu. ft. ÷ 128 cu. ft. =†

(p) 96 sq. rods × 25 = (q) 2400 sq. rd. ÷ 160 sq. rd. =‡

(r) 960 rods × 4 = (s) 3840 rods ÷ 320 rods = §

(t) 2350 lb. × 7 = (u) 16450 lb. ÷ 2000 lb. = ‖

* See page 7, note 16.

† Number story for (n) and (o). In a pile of wood 48 ft. long, 6 ft. wide, and 8 ft. high there are —— cords.

‡ Number story for (p) and (q). A rectangular piece of land 96 rods by 25 rods contains —— acres.

§ Think of the perimeter of a square, one side being 960 rods.

‖ Think of 7 loads of coal, each weighing 2350 lb.

Miscellaneous Problems.

1. If I put 5 oz. of candy in each bag, 3 lb. of candy will fill ——— bags, with ——— ounces over.

(a) If I put 2 bushels of oats in each bag, how many bags will one ton of oats fill?

2. Henry was born on January 5, 1885; on Jan. 5, 1896, he was ——— years old.

(b) Queen Victoria was born on May 24, 1819. How old was she on the 24th day of May, 1897?

(c) Columbus discovered America in A.D. 1492. How long ago?

3. A dealer bought 5 gal. of milk at 14¢ a gal.; he sold it at 5¢ a quart, but lost 2 quarts in over-measurement and waste; he gained ——— cents.

(d) A grocer bought 3 bbl. sugar, each containing 340 lb.; he paid 4½¢ a lb. If he sells it at the rate of 20 lb. for a dollar, but loses 40 lb. by over-weights and waste, how much does he gain?

4. Edward bought 2 lb. candy at 18¢ a pound, 2 lb. walnuts at 20¢ a pound, and 1 qt. of pop-corn for 10¢; for all he paid ——— cents.

Find the amount of the following bills:

(e)		(f)	
3 lb. Steak	@ 15¢	5 bu. Apples	@ 35¢
½ bu. Potatoes	@ 40¢ *	½ bu. Beans	@ $1.50
18 lb. Sugar	@ 5¢	7 gals. Milk	@ 15¢
5 cans Corn	@ 8¢	2 cd. Wood	@ $4.50
7 heads Cabbage	@ 8¢	1½ tons Hay	@ $8.50

5. If the divisor is 6 and the quotient is 4½, the dividend is ———. †

(g) If the divisor is 128 and the quotient 16¾, what is the dividend?

* 40¢ a bu. † See Werner Arithmetic, Book I., page 238.

Simple Numbers.

Teach the meaning and use of the term, average.

1. Alfred rode his bicycle for 3 consecutive hours: the first hour he rode 10 miles; the second hour, 8 miles; the third hour, 6 miles; altogether he rode —— miles; his average speed was (24 ÷ 3) —— miles an hour.

2. A farmer bought 20 sheep; for 10 of them he paid $35, and for the other 10, $25; for all he paid —— dollars; the average cost per head was —— dollars.

(a) If 8 cows cost $204, what is the average cost per head?

(b) Five boys were examined in spelling; the marks received were 90, 95, 85, 80, and 75. What was the average standing of the boys?

(c) The temperature at noon of each day for 1 week was as follows: Sunday, 94; Monday, 91; Tuesday, 96; Wednesday, 95; Thursday, 90; Friday, 89; Saturday, 89. What was the average temperature at noon for the week?

(d) Six beef cattle weighed as follows: 1560 lb., 1430 lb., 1640 lb., 1350 lb., 1420 lb., and 1660 lb. What was the average weight per head?

Review.

3. Common multiples of 16 and 6 are ——, ——, etc.

4. The prime factors of 70 are ——, ——, and ——.

(e) What are the prime factors of 195? (f) of 185?

5. 2, 3, and 7 are the prime factors of ——. Forty-two is exactly divisible by 2; by 3; by ——; by ——; by ——, and by ——.

(g) Multiply 864 by 50. (h) Divide 7650 by 50.
(i) Multiply 875 by 700. (j) Divide 8750 by 700.
(k) Multiply 736 by 400. (l) Divide 9384 by 400.
(m) Multiply 834 by 360. (n) Divide 9450 by 450.

Common Fractions.

1. Add $\frac{5}{16}$ and $\frac{1}{8}$. (The l. c. m. of 16 and 6 is ——.)
(a) Find the sum of $376\frac{7}{16}$, $254\frac{1}{2}$, $321\frac{5}{8}$, and 342.

2. From $7\frac{1}{16}$ subtract $3\frac{5}{8}$. ($1\frac{1}{16} = \frac{}{16}$. $\frac{5}{8} = \frac{}{16}$.)
(b) Find the difference of $6274\frac{1}{16}$ and $842\frac{5}{8}$.

3. Multiply $\frac{5}{16}$ by 8. (This means —— 3 * ——).
(c) Find the product of $357\frac{5}{16}$ multiplied by 8.

4. Multiply $1\frac{3}{4}$ by $\frac{1}{4}$. (This means —— 13 * ——.)
(d) Find the product of $573\frac{3}{4}$ multiplied by $\frac{1}{4}$. *Story.*†

5. Multiply 19 by $\frac{3}{8}$. (This means —— 8 * ——.) ‡
(e) Find the product of 643 multiplied by $\frac{3}{8}$.

6. Multiply 19 by $2\frac{3}{8}$. (This means —— 16 * ——.)
(f) Find the product of 643 multiplied by $4\frac{3}{8}$.

7. Multiply $15\frac{1}{2}$ by $2\frac{1}{4}$. (This means —— 22 * ——.)
(g) Find the product of $345\frac{1}{2}$ multiplied by $4\frac{1}{4}$.

8. Divide 8 by $\frac{3}{8}$. (4) * (Change 8 to —— ths.)
(h) Find the quotient of 86 divided by $\frac{3}{8}$. *Story.*

9. Divide $\frac{7}{16}$ by $\frac{1}{8}$. (9) * (Change to —— ths.)
(i) Find the quotient of $3\frac{7}{16}$ divided by $\frac{1}{8}$. *Story.*

10. Divide $8\frac{3}{8}$ by $2\frac{3}{8}$. (14) * (Change to —— ths.)
(j) Find the quotient of $57\frac{3}{8}$ divided by $2\frac{3}{8}$. *Story.*

11. Divide $1\frac{1}{8}$ ($\frac{1}{8}$) by 5. (This means —— 15 * ——.)
(k) Find the quotient of $8\frac{1}{8}$ divided by 9.

12. Divide $21\frac{2}{3}$ yd. by 4. (This means —— 21 * ——.)
(l) Find the quotient of $453\frac{2}{3}$ divided by 4.

(m) Add.	(n) Subtract.	(o) Multiply.	(p) Divide.	(q) Divide.
$436\frac{11}{16}$	$573\frac{1}{8}$	245	$3\frac{3}{8}$ in.)246 in.	3)$246\frac{3}{8}$ in.
$148\frac{3}{8}$	$245\frac{11}{16}$	$24\frac{3}{8}$		

* These figures refer to notes on pages 6 and 7.
† At $573\frac{3}{4}$ an acre, $\frac{1}{4}$ of an acre of land costs ——.
‡ Find $\frac{3}{8}$ of 19.=3 times $\frac{1}{8}$ of 19, or $\frac{1}{8}$ of 3 times 19.

Decimal Fractions.

In solving problems in division of decimals, require the pupil to write the decimal point *immediately after writing the units' figure of the quotient.* Lead him to determine when he has reached this figure by considering *the meaning of the problem.*

DIVISION. CASE I.

1. Divide .45 by .09. (This means ——4*——.)

(a) Find the quotient of 46.75 divided by .25. † *Story.*

2. Divide 9 by .6. (This means ——9*——.)

(b) Find the quotient of 874 divided by .5. ‡ *Story.*

3. Divide 6 by .05. (This means ——14*——.)

(c) Find the quotient of 96 divided by .25. § *Story.*

4. Divide 4.5 by .09. (This means ——23*——.)

(d) Find the quotient of 46.5 divided by .25. ‖ *Story.*

(e) Find the quotient of 57.5 divided by 2.5. ¶ *Story.*

(f) Find the quotient of 68.5 divided by .25. ** *Story.*

(g) Find the quotient of 76 divided by .25. †† *Story.*

DIVISION. CASE II.

5. Divide $.63 by 7. (This means ——5*——.)

(h) Find the quotient of $87.15 divided by 7. *Story.*

(i) Find the quotient of $375.50 divided by 25. *Story.*

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story:

(j) Divide $256 by $8.

(k) Divide $256 by 8.

(l) Divide $24.36 by $.04.

(m) Divide $24.36 by 4.

(n) Divide $53.6 by $.8.

(o) Divide $53.6 by 8.

(p) Divide $46.25 by $.25.

(q) Divide $46.25 by 25.

* These figures refer to notes on pages 8 and 9.
† Find how many times 25 hundredths are contained in 4675 hundredths.
‡ Find how many times 5 tenths are contained in 8740 (874.0) tenths.
§ Find how many times 25 hundredths are contained in 9600 (96.00) hundredths.
‖ Find how many times 25 hundredths are contained in 4650 (46.50) hundredths.
¶ Find how many times 25 tenths are contained in 575 (57.5) tenths.
** Find how many times 25 hundredths are contained in 6850 (68.50) hundredths.
†† Find how many times 25 hundredths are contained in 7600 (76.00) hundredths

Denominate Numbers.

1. 24 *sheets of paper are* 1 *quire.*
2. 20 *quires of paper are* 1 *ream.*

3. One ream of paper is —— sheets.
4. One half of a ream is —— quires, or —— sheets.
5. One fourth of a ream is —— quires, or —— sheets.
6. George bought a ream of paper for $2 and sold it at the rate of 2 sheets for a cent; he gained —— cents.
7. Arthur bought a ream of paper for $2.25 and sold it at 15¢ a quire; he gained —— cents.

Review.

8. Ten thousand feet are nearly —— miles.
9. Fifteen thousand feet are nearly —— miles.

(a) Twenty thousand feet are how many feet less than four miles?

(b) Five miles are how many feet more than 25000 feet?

10. One thousand feet are nearly one ——th of a mile.

(c) Two fifths of a mile are how many feet more than 2000 feet? (d) $\frac{3}{5}$ of a mile, than 3000 ft.?

11. Nine hundred feet are —— rods and —— feet.*
12. One thousand feet are —— rods and —— feet.

13. One hundred feet are —— yards and —— foot.
14. Two hundred feet are —— yards and —— feet.
15. Three hundred feet are —— yards.
16. Four hundred feet are —— yards and —— foot.

(e) Change 3 miles to feet. (f) Change 3 miles to rods.
(g) Change 3 acres to sq. rd. (h) Change 3 cords to cu. ft.
(i) Change $3\frac{1}{2}$ miles to feet. (j) Change $3\frac{1}{2}$ miles to rd.
(k) Change $3\frac{1}{2}$ acres to sq. rd. (l) Change $3\frac{1}{2}$ cords to cu. ft.
(m) Change $3\frac{1}{2}$ rods to feet. (n) Change $3\frac{1}{2}$ yd. to inches.

* See page 64, problems 1 to 5, and page 74, problems 2 to 4.

Measurements.

1. In one cubic yard there are —— cubic feet.*

(a) To dig a cellar 15 feet long, 12 ft. wide, and 6 feet deep would require the removal of how many cubic feet of earth? (b) How many cubic yards?

(c) How many square feet in the floor of the cellar described in problem (a)? (d) How many sq. yards?

2. To make an excavation 2 yards long, 2 yards wide and 2 yards deep would require the removal of —— cubic yards.

(e) How many cubic feet in the excavation described in problem 2?

3. The perimeter of one of the faces of a 2-yard cube is —— yards.

4. The area of one of the faces of a 2-yard cube is —— square yards; of all the faces, —— square yards.

(f) The area of all the faces of a 2-yard cube is how many square feet?

Review.

5. A piece of land 20 rods by 46 rods contains —— square rods.

(g) How many acres in a piece of land 20 rd. by 46 rd.?

6. A shed 16 feet long, 10 feet wide, and 6 feet high contains —— cubic feet.

(h) How many cords of wood will the shed described in problem 6 contain?

Find the number of acres in each of the following:

(i) 80 rods by 24½ rods. (j) 40 rods by 73 rods.

(k) 20 rods by 52 rods. (l) 30 rods by 84 rods.

Find the number of cords in each of the following:

(m) 8 ft. by 12 ft. by 8 ft. (n) 16 ft. by 8 ft. by 8 ft.

(o) 16 ft. by 4 ft. by 8 ft. (p) 20 ft. by 8 ft. by 8 ft.

* Review page 25.

Ratio and Proportion.

1. One fourth of .36 is ——. .36 is $\frac{1}{4}$ of ——.
2. One fifth of .45 is ——. .45 is $\frac{1}{5}$ of ——.
3. One eighth of .32 is ——. .32 is $\frac{1}{8}$ of ——.

(a) Find 1 fourth of 9.86.* (b) 9.78 is $\frac{1}{4}$ of what?

(c) Find 1 fifth of 8.24. (d) 8.24 is $\frac{1}{5}$ of what?

4. Five eighths of .32 are ——. .35 is $\frac{5}{8}$ of ——.
5. Three fifths of .45 are ——. .45 is $\frac{3}{5}$ of ——.

(e) Find $\frac{5}{8}$ of 8.40. (f) 8.40 is $\frac{5}{8}$ of what number?

(g) Find $\frac{3}{5}$ of 9.45. (h) 9.45 is $\frac{3}{5}$ of what number?

6. .32 is —— —— of .40. .40 is —— —— of .32.
7. .24 is —— —— of .40. .40 is —— —— of .24.
8. .24 is —— —— of .56. .56 is —— —— of .24.
9. .40 is —— —— of .64. .64 is —— —— of .40.

10. Three fourths of .32 are 1 half of —— hundredths.
11. One half of .32 is 4 fifths of —— hundredths.

(i) Three fourths of 9.36 are 1 half of what number?

(j) One half of 9.36 is 4 fifths of what number?

12. If .32 of an acre of land is worth $20, .40 of an acre is worth —— dollars.

(k) If .32 of an acre of land is worth $24.56, how much is .40 of an acre worth at the same rate?

13. If .40 of a ton of coal is worth $4.00, .32 of a ton is worth ——.

(l) If .40 of a ton of coal is worth $3.25, how much is .32 of a ton worth?

* Lead the pupil to think as suggested by the following: $\frac{1}{4}$ of 9 is 2 with 1 remainder; 1 = 10 tenths; 10 tenths + 8 tenths are 18 tenths; $\frac{1}{4}$ of 18 tenths is 4 tenths with 2 tenths remainder; 2 tenths = 20 hundredths; 20 hundredths + 6 hundredths = 26 hundredths; $\frac{1}{4}$ of 26 hundredths = 6 hundredths with 2 hundredths remainder; 2 hundredths = 20 thousandths; $\frac{1}{4}$ of 20 thousandths = 5 thousandths.

$$4)\overline{9.86}\quad 2.465$$

Percentage.

30 per cent = .30 = .3 = $\frac{3}{10}$.
70 per cent = .70 = .7 = $\frac{7}{10}$.
90 per cent = .90 = .9 = $\frac{9}{10}$.

(1)	(2)
1. 30 % of 60 =	60 is 30 % of ——.
2. 40 % of 60 =	60 is 40 % of ——.
3. 50 % of 60 =	60 is 50 % of ——.
4. 60 % of 60 =	60 is 60 % of ——.
5. 70 % of 140 =	140 is 70 % of ——.
6. 80 % of 160 =	160 is 80 % of ——.
7. 90 % of 180 =	180 is 90 % of ——.

(3)	(3)
8. 4 is —— % of 40.	12 is —— % of 40.*
9. 28 is —— % of 40.	36 is —— % of 40.
10. 5 is —— % of 50.	45 is —— % of 50.
11. 15 is —— % of 50.	35 is —— % of 50.
12. 6 is —— % of 60.	42 is —— % of 60.
13. 54 is —— % of 60.	18 is —— % of 60.

14. In a certain orchard there were 120 trees; 10% of them were pear trees and 30% of them were cherry trees; there were —— pear trees and —— cherry trees.

15. In another orchard there were 24 pear trees; these were 30% of all the trees in the orchard; there were —— trees in the orchard.

16. There were 60 trees in an orchard; 42 of them were apple trees and the remainder pear trees; there were —— pear trees; —— per cent of the trees in the orchard were apple trees and —— per cent of them were pear trees.

17. Twenty-five % of the trees in an orchard died and were removed; there remained 36 trees;† —— trees died.

* 12 is what part of 40? † 36 trees are what part of all the trees that were in the orchard at first?

Percentage.

(*1*)

1. 30 per cent of 42 = ($\frac{1}{10}$ of 42 is 4.2.)
2. 90 per cent of 51 = 90 per cent of 31 =

(a) Find 30 % of 275. (b) Find 70 % of 362.
(c) Find 90 % of 436. (d) Find 30 % of 475.
(e) Find 3 % of 536. * (f) Find 7 % of 824.

(g) A lawyer's commission for collecting money was 10 %; † he collected $954. How much of the money should he keep and how much should he "pay over" to the man for whom he collected the money?

(*2*)

3. 3.2 is 10 per cent of ——. 4.6 is 10 % of ——.
4. 9.6 is 30 per cent of ——. ‡ 14.7 is 70 % of ——.

(h) 372.3 is 30 % of what? (i) 562.1 is 70 % of what?
(j) 291.6 is 90 % of what? (k) 752.4 is 30 % of what?
(l) 279 is 3 % of what? (m) 539 is 7 % of what?

(n) A lawyer's commission for collecting money was 10 %; † his commission amounted to $76.20. How much did he collect and how much should he "pay over" to the man for whom he collected the money? §

(*3*)

5. 4.2 is —— per cent of 42. 3.25 is —— % of 325.
6. 5.1 is —— per cent of 51. 8.3 is —— % of 83.
7. 84 is —— per cent of 840. 8.40 is —— % of 840.
8. 7.54 is —— per cent of 75.4. 7.54 is —— % of 754.

9. A lawyer collected $754; he retained as his commission $75.40 of this sum and paid the remainder to the man for whom he collected the money; the lawyer's commission for collecting was —— %.

* See page 48, problem (c). † 10% of what?
‡ 9.6 is $\frac{1}{10}$ of what? § $76.20 is $\frac{1}{10}$ of what?

Review.

1. Six sheep cost $33; the average cost per head was ——.

(a) In a certain schoolroom, on Monday there were 35 pupils; Tuesday, 38; Wednesday, 36; Thursday, 37, and Friday, 39. What was the average daily attendance?

2. Divide $\frac{3}{16}$ by $\frac{1}{4}$. (Change to ——ths.) *Story.*

(b) Divide $323\frac{3}{4}$ by $1\frac{1}{4}$. (Change to ——ths.) *Story.*

(c) Divide $323.75 by $1.25. (This means ——.)

Compare the answers to problems (b) and (c). Why are they alike?

3. In 10 quires of paper there are —— sheets.

4. In $\frac{1}{4}$ of a ream of paper there are —— quires.

5. At 3 sheets for 2 cents 1 quire of paper costs ——.

(d) At 3 sheets for 2 cents how much will one ream of paper cost?

6. In 2 yards there are —— feet.

7. In two square yards there are —— square feet.

8. In 2 cubic yards there are —— cubic feet.

(e) In 18 cubic yards there are how many cu. ft.?

(f) In 48 square yards there are how many square feet?

(g) How many square feet of blackboard in this room?*

(h) How many square yards of blackboard in this room?*

(i) How many cubic feet of earth must be removed to make an excavation 6 ft. by 9 ft. by 15 ft.? (j) How many cubic yards?

(k) 326 + 432 + 28 + 175 + 326 + 47 + 63 + 82 + 96 =

(l) 274 + 386 + 27 + 237 + 528 + 35 + 72 + 51 + 46 =

(m) 438 + 275 + 43 + 324 + 136 + 32 + 85 + 37 + 91 =

(n) 243 + 139 + 18 + 914 + 326 + 12 + 72 + 36 + 75 =

* Require an approximate answer only.

Miscellaneous Review.

1. Two boys start from two cities that are 67 miles apart and ride towards each other; one rides 8 miles an hour; the other rides 12 miles an hour; in one hour they ride —— miles; in 3 hours they are —— miles apart.

(a) Two trains start from two cities that are 1000 miles apart and move toward each other; one goes 25 miles an hour; the other goes 30 miles an hour. At the end of 12 hours how far are the trains apart?

2. A man bought land for $80 and sold it for $88; he gained $2 an acre; there were —— acres. It cost him —— dollars an acre and he sold it for —— dollars an acre.

(b) A man bought land for $1850, and sold it for $2220, thereby gaining $5 an acre. How many acres did he buy? (c) How much did he pay per acre for it? (d) At what price per acre did he sell it?

3. A boy earned some money, spent $\frac{5}{8}$ of it, and had $6 left; he spent —— dollars.

(e) A man earned some money, spent $\frac{5}{8}$ of it, and had $25.20 left. How much did he spend?

4. The product of two numbers is 28; one of the numbers is 7; the other number is ——.

(f) The product of two numbers is 240; one of the numbers is 15. What is the other number?

5. Fifty per cent of my money is $12 and John has 3 times as much as I have; John has —— dollars.

(g) Fifty per cent of Mr. A.'s money is $36.45 and Mr. B. has 3 times as much as Mr. A. Mr. B. has ————.

6. I have $24; 50 % of my money equals $33\frac{1}{3}$ per cent of Bernie's money; Bernie has —— dollars.

(h) Mr. C. has $72.30; 50 % of Mr. C.'s money equals $33\frac{1}{3}$ % of Mr. D.'s money. How much money has Mr. D?

Simple Numbers.

NUMBER OF POUNDS OF MILK RECEIVED.

NAME OF PATRON.	M.	T.	W.	T.	F.	S.	S.	Amount.
L. R. Orr,	236	224	259	263	248	227	236	(a) *
Levi Smith,	149	167	134	158	175	136	145	(b)
Henry Judd,	375	384	363	356	325	364	356	(c)
Peter Johnson,	97	84	93	86	92	87	88	(d)
Wm. Jones,	201	196	205	208	199	215	218	(e)
E. C. Ford,	385	397	388	394	396	399	401	(f)
Otto Fisk,	87	146	175	82	185	96	133	(g)
Fay Winslow,	230	231	229	233	228	229	235	(h)
Albert Davis,	155	160	145	170	180	175	194	(i)
Jno. Harris,	435	424	412	456	428	435	457	(j)
Total,	(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r) (s)

The above is a statement of the milk received at a small creamery for one week. (a) to (j) Find the amount delivered by each patron. (k) to (q) Find the amount received at the creamery each day. (r) Find the sum of (k) to (q) inclusive. (s) Find the sum of (a) to (j) inclusive.

(t) Find the average amount received daily at the creamery.

Review.

1. Common multiples of 15 and 10 are ——, ——, etc. The least common multiple of 15 and 10 is ——.

2. The prime factors of 75 are ——, ——, and ——.

(u) What are the prime factors of 2750? (v) of 280?

3. 3, 3, and 7 are the prime factors of ——. Sixty-three is exactly divisible by 3; by ——; by ——, and by ——.

(w) 27 cu. ft.)1296 cu. ft.†

(x) 27)1296 cu. ft.†

* Accuracy in business is so important that great emphasis should be put upon it in the school-room. The pupil must be made to feel that without a high degree of accuracy his work is a failure. A "90-per cent" arithmetic paper is not "good" but "*unsatisfactory*."

† Tell the meaning. Tell a number story.

Common Fractions.

1. Add $\frac{1}{16}$ and $\frac{1}{10}$. (The l. c. m. of 15 and 10 is ——.)

(a) Find the sum of $534\frac{7}{16}$, $257\frac{1}{10}$, $225\frac{1}{8}$, and $324\frac{1}{8}$.

2. From $8\frac{7}{16}$ subtract $5\frac{7}{10}$. ($1\frac{1}{16} = \frac{1}{80}$. $\frac{7}{10} = \frac{56}{80}$.)

(b) Find the difference of $743\frac{3}{16}$ and $181\frac{9}{10}$.

3. Multiply $\frac{7}{16}$ by 8. (This means —— 3 * ——.)

(c) Find the product of $235\frac{7}{16}$ multiplied by 8.

4. Multiply $1\frac{1}{8}$ by $\frac{1}{8}$. (This means —— 13 * ——.)

(d) Find the product of $724\frac{3}{8}$ multiplied by $\frac{1}{8}$. *Story.*

5. Multiply 16 by $\frac{3}{8}$. (This means —— 8 * ——.)†

(e) Find the product of 721 multiplied by $\frac{3}{8}$.

6. Multiply 16 by $2\frac{3}{8}$. (This means —— 16 * ——.)

(f) Find the product of 721 multiplied by $7\frac{3}{8}$.

7. Multiply $18\frac{1}{2}$ by $2\frac{3}{8}$. (This means —— 22 * ——.)

(g) Find the product of $432\frac{1}{2}$ multiplied by $4\frac{1}{8}$.

8. Divide 7 by $\frac{1}{8}$. (4) * (Change 7 to ——ths.)

(h) Find the quotient of 94 divided by $\frac{1}{8}$. *Story.*

9. Divide $\frac{7}{16}$ by $\frac{1}{10}$. (9) * (Change to ——ths.)

(i) Find the quotient of $4\frac{11}{16}$ divided by $\frac{1}{10}$. *Story.*

10. Divide $6\frac{3}{4}$ by $2\frac{1}{2}$. (14) * (Change to ——ths.)

(j) Find the quotient of $74\frac{2}{4}$ divided by $3\frac{1}{2}$.

11. Divide $1\frac{1}{8}$ ($\frac{11}{8}$) by 3. (This means —— 21 * ——.)

(k) Find the quotient of $7\frac{5}{8}$ divided by 3.

12. Divide $21\frac{3}{8}$ by 5. (This means —— 21 * ——.)

(l) Find the quotient of $621\frac{3}{8}$ divided by 5.

(m) Add.	(n) Subtract.	(o) Multiply.	(p) Divide.	(q) Divide.
$375\frac{1}{16}$	$436\frac{1}{8}$	$753\frac{3}{4}$	$3\frac{1}{8}$ yd.)512 yd.	5)$636\frac{3}{8}$ yd.
$286\frac{9}{10}$	$182\frac{3}{10}$	24		

* These figures refer to notes on pages 6 and 7.
† $\frac{3}{8}$ of 16 = 3 times $\frac{1}{8}$ of 16, or $\frac{1}{8}$ of 3 times 16.

Decimal Fractions.

DIVISION. CASE I.

1. Divide .48 by .12. (This means —— 4 * ——.)
(a) Find the quotient of 54.24 divided by .12. *Story.*

2. Divide 6 by .4. (This means —— 9 * ——.)
(b) Find the quotient of 736 divided by .4. *Story.*

3. Divide 3 by .04. (This means —— 14 * ——.)
(c) Find the quotient of 78 divided by .12. *Story.*

4. Divide 4.8 by .12. (This means —— 23 * ——.)
(d) Find the quotient of 40.8 divided by .12. † *Story.*
(e) Find the quotient of 43.2 divided by 1.2. † *Story.*
(f) Find the quotient of 51.6 divided by .12 † *Story.*
(g) Find the quotient of 75 divided by .12. † *Story.*

DIVISION. CASE II.

5. Divide $.56 by 8. (This means —— 5 * ——.)
(h) Find the quotient of $98.40 divided by 8. *Story.*
(i) Find the quotient of $436.80 divided by 12. *Story.*

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story.

(j) Divide $924 by $7.
(k) Divide $924 by 7.
(l) Divide $29.34 by $.06.
(m) Divide $29.34 by 6.
(n) Divide $55.3 by $.7
(o) Divide $55.3 by 7.

(p) Multiply $26.5 by .3. ‡
(q) Multiply $265 by 2.3. §
(r) Multiply $265 by .03. ‖
(s) Multiply $265 by .23. ¶
(t) Multiply $265 by 4.23. **

(u) At $.12 a dozen, how many dozen eggs can I buy for $63.84?
(v) At $356 an acre, how much will 7.2 acres of land cost?

* These figures refer to notes on pages 8 and 9.
† See corresponding questions and notes on page 83.
‡ See page 23. § See page 33. ‖ See page 43. ¶ See page 53. ** See page 63.

Denominate Numbers.

(a) Copy, complete, and receipt the following Bill:

WAUKEGAN, ILL., July 30, 1896.

MR. RICHARD YATES,

Bought of C. F. BROWN & Co.

July	1	15	lb. Sugar,	@	$.05		
"	1	5	lb. Raisins,	@	.07		
"	5	2½	lb. Cheese,	@	.16		
"	8	1	sack Flour,	@		1	25
"	8	½	bu. Potatoes,	@	$.60		
"	19	5	lb. Coffee,	@	.35		
			Received Payment,			(a)	*

Review.

1. 500 sheets of paper are 1 ream and —— sheets.
2. 500 feet are —— rods and —— feet.†
3. 500 rods are 1 mile and —— rods.
4. 500 sq. rd. are 3 acres and —— sq. rd.
5. 500 inches are —— feet and —— inches.
6. 500 feet are —— yards and —— feet.
7. 500 pounds are —— —— of a ton.
8. 500 feet are nearly —— —— of a mile.
9. When hay is $8 a ton, 5000 lb. cost —— dollars.

(b) Load of straw; gross weight, 3380 lb.; tare, 1580 lb. Find the value at $5.50 a ton.

(c) Multiply 40 sq. rods by 24 and divide the product by 160 square rods. *Story.*‡

(d) Multiply 24 cubic feet by 8 and the product thus obtained by 6. Divide the last product by 128 cu. ft. *Story.*

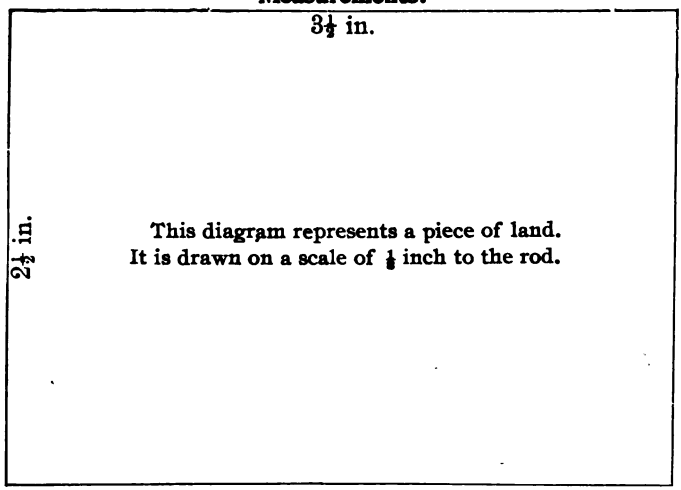
* Receipt it by writing C. F. Brown & Co. at the bottom of the bill, and putting below the firm name your own name or initials following the word per; thus,

C. F. Brown & Co.
per F. H. H.

† See page 64, problems 1 to 5.

‡ I had a piece of land 40 rods long and 24 rods wide; it contained —— acres.

Measurements.



1. The land represented by the above diagram is —— rods long. It is —— rods wide.

(a) Find the perimeter of the piece of land represented by the diagram. (b) Find its area in sq. rods. (c) Find its area in acres. (d) How much is the land worth at $40 an acre?

2. Draw carefully upon your slate or paper, on a scale of ⅛ in. to the rod, a diagram of a rectangular piece of land that is 30 rods by 36 rods.

(e) Find the perimeter of the land represented by the diagram you have drawn. (f) Find its area in sq. rods. (g) Find its area in acres. (h) How much is the land worth at $60 an acre? (i) How much will it cost to build a fence around it at 50¢ a rod?

Find the number of square yards in each of the following:

(j) 24 ft. by 27 ft. (k) 21 ft. by 30 ft. (l) 24 ft. by 28 ft.

Find the number of cubic yards in each of the following:

(m) 36 ft. by 7 ft. by 10 ft. (n) 36 ft. by 12 ft. by 6 ft.

Ratio and Proportion.

1. One fifth of 2 (2.0) is ——.* $\frac{1}{5}$ of 3 is ——.
2. One fourth of 6 (6.0) is ——. $\frac{1}{4}$ of 4 is ——.
3. One eighth of 4 (4.0) is ——. $\frac{1}{8}$ of 2 is ——.

(a) Find 1 fourth of 70.† (b) Find $\frac{1}{4}$ of 79.

(c) Find 1 fifth of 83. (d) Find $\frac{1}{8}$ of 66.

4. 1 is — — of 27. 2 is — — of 27.
5. 3 is — — of 27, or —.‡ 4 is — — of 27.
6. 5 is — — of 27. 6 is — — of 27, or —.
7. 7 is — — of 27. 8 is — — of 27.
8. 9 is — — of 27, or —. 10 is — — of 27.
9. 11 is — — of 27. 12 is — — of 27, or —
10. 13 is — — of 27. 14 is — — of 27.
11. 15 is — — of 27, or —. 16 is — — of 27.
12. 17 is — — of 27. 18 is — — of 27, or —.

13. 1.8 is —— —— of 2.7. 2.7 is —— —— of 1.8.
14. .18 is —— —— of .27. .27 is —— —— of .18.
15. .19 is —— —— of .27. .27 is —— —— of .19.
16. 1.9 is —— —— of 2.7. 2.7 is —— —— of 1.9.

(e) 120 is what part of 140?§ (f) 140 of 120?
(g) 175 is what part of 185? (h) 185 of 175?
(i) 120 is what part of 128? (j) 128 of 120?
(k) 120 is what part of 160? (l) 160 of 120?
(m) 24 is what part of 144? (n) 48 of 144?
(o) 96 is what part of 144? (p) 100 of 144?

* $\frac{1}{5}$ of 20 tenths. † $\frac{1}{4}$ of 70.0.

‡ 3 is 3 twenty-sevenths of 27, or $\frac{1}{9}$ of 27.

§ In each such problem as (e), the number that follows the word *of* always becomes the denominator. 120 is $\frac{120}{140}$ = $\frac{12}{14}$ = $\frac{6}{7}$ of 140.

Percentage.

1 per cent = .01 = $\frac{1}{100}$.

(1)

1. One % of \$435 = (a) Find 7 % of \$435.
2. One % of \$832 = (b) Find 9 % of \$832.
3. One % of \$38.4 = (c) Find 8 % of \$38.4.
4. One % of \$56.30 = (d) Find 6 % of \$56.30.
5. One % of \$8324 = (e) Find 5 % of \$8324.

(2)

6. 63 is 7 % (7 hundredths) of ——.*

(f) 364 is 7 % of what? (g) 635 is 5 % of what?

7. 72 is 9 % (9 hundredths) of ——.

(h) 657 is 9 % of what? (i) 584 is 8 % of what?

8. 54 is 6 % (6 hundredths) of ——.

(j) 456 is 6 % of what? (k) 504 is 7 % of what?

(3)

9. \$12 is ——% of \$200.† 10. \$13 is ——% of \$200.

(l) \$54 is what % of \$200? (m) \$45 is what % of \$200?

11. \$21 is ——% of \$300. 12. \$22 is ——% of \$300.

(n) \$84 is what % of \$300? (o) \$85 is what % of \$300?

13. \$36 is ——% of \$400. 14. \$38 is ——% of \$400.

(p) \$96 is what % of \$400? (q) \$98 is what % of \$400?

15. \$40 is ——% of \$500. 16. \$42 is ——% of \$500.

(r) \$95 is what % of \$500? (s) \$96 is what % of \$500?

* If 63 is 7 hundredths, 1 hundredth is 9, and the whole (100 hundredths) is 900.

† Lead the pupil to observe that he has already learned to solve "Case 3" problems in percentage in two ways. Sometimes he has found what part one number is of another and has changed the fraction thus obtained to hundredths. See problems 10 to 15, page 87. Sometimes he has found one per cent of the base and has used this number as a divisor to obtain the required result. See page 58, problem (o); page 68, problems 5 to 8; page 78, problems 5 to 8. The "Case 3" problems on this page and the greater number of those that follow in Part I., can be most readily solved by the second method. 12 is what % of 200? *One %. of* 200 *is* 2; *therefore* 12 *is* 6 *per cent of* 200: 13 *is* 6½ *per cent*; 54 *is* 27 *per cent, and* 45 *is* 22½ *per cent.*

Percentage.

1. Three % of $620 = (a) Find 9 % of $620.
2. Eighteen is 3 % of ——. (b) $75 is 3 % of what?
3. Eighteen is —% of 600.* (c) 138 is what % of 600?*
4. Eight % of $220 = (d) Find 8 % of $738.
5. Forty-eight is 8 % of ——. (e) $76 is 4 % of what?
6. Thirty-six is —% of 600. (f) 174 is what % of 600?

(g) A farmer had 825 sheep; he sold 8 % of them. How many sheep did he sell? (h) How many sheep did he have left?

(i) A dealer bought some sheep and immediately sold 72 of them; these were 8 % of all he purchased. How many sheep did he purchase? (j) How many sheep did he have left?

(k) Mr. B. had 400 sheep; he sold 48 of them. What % of his sheep did he sell?

(l) A man had $485; he spent 7 % of his money. How much money did he spend? (m) How much money did he have left?

(n) Peter Gray paid $35.50 for a suit of clothes; this was 5 % of all he earned in a year. How much did he earn in a year?

(o) Judge Branson had $600 in the bank; he drew out $198. What % of his deposit did he draw out?

7. A lawyer collected $800; he retained as his commission $56 of this sum and paid the remainder,—— dollars, to the man from whom he collected the money; the lawyer's commission for collecting was —— %.

(p) A lawyer collected $850; he retained as his commission for collecting $51 of this sum.† What per cent did the lawyer charge for making the collection?

* 6 is 1 per cent of 600.

† 1 per cent of $850 is $8.50.

Review.

(a) A creamery received milk as follows: Monday, 8250 lb.; Tuesday, 7640 lb.; Wednesday, 8375 lb.; Thursday, 7230 lb.; Friday, 6953 lb.; Saturday, 8424 lb.; Sunday, 7651 lb. Find the number of pounds received during the week. (b) Find the average daily receipts for the week.

1. Add $\frac{11}{16}$ and $\frac{3}{10}$. (Change to ——ths.)

(c) Find the sum of $375\frac{11}{16}$, $246\frac{1}{8}$, $821\frac{7}{10}$ and 234.

(d) Find the product of 372 multiplied by $6\frac{3}{4}$.

(e) Find the product of 372 multiplied by 6.75.

Compare the answers to problems (d) and (e).

(f) Make a Bill containing the following items and find the amount.

August 2, 24 lb. sugar @ 5¢; Aug. 5, ½ lb. tea @ 70¢; August 10, 2 gal. syrup @ 35¢; Aug. 13, 5 bu. potatoes @ 55¢; August 18, 5 cans peaches @ 18¢; Aug. 18, 5 gal. kerosene @ 15¢.

(g) If I sell a ream of paper at the rate of 6 sheets for 4¢, how much will I receive for it?

2. Draw a diagram of a piece of land 40 rods by 36 rods, scale, ⅛ in. to a rod. The diagram is —— inches long and —— inches wide.

(h) How many square rods in the piece of land described in problem 2? (i) How many acres? (j) The perimeter of the field is how many rods?

3. 3 is — — of 25. 3 twenty-fifths are — hundredths.

4. 4 is — — of 25. 4 twenty-fifths are — hundredths.

5. 7 is — — of 25. 7 twenty-fifths are — hundredths.

(k) 85 is what part of 95? (l) 95 of 85?

(m) 95 is what part of 105? (n) 105 of 95?

(o) 105 is what part ot 115? (p) 115 of 105?

Miscellaneous Problems.

1. Horace spent $\frac{3}{5}$ of his money and had 24¢ left; he spent —— cents. Before spending any money he had —— cents. Three fifths are how many times two fifths? Two fifths are what part of three fifths?

(a) Mr. Green spent $\frac{3}{5}$ of his money and had $56.48 left. How much money did he spend? (b) Before spending any money how much did he have?

2. At $2\frac{1}{2}$¢ an ounce, $\frac{1}{4}$ of a pound of cinnamon costs —— cents.

(c) Find the cost of $5\frac{1}{2}$ pounds of cloves at $1\frac{1}{2}$¢ per ounce.

3. A farmer mixed 10 bushels of oats with 15 bushels of corn; together there were —— bushels of grain. What part of the mixture was oats? What part was corn?

(d) A dealer mixed 250 bushels of oats with 350 bushels of corn. What part of the mixture was oats? (e) What part was corn?

4. Eddie, Willie, and Fay divided a melon among themselves. Eddie had 1 fourth of the melon. Will had $\frac{1}{2}$ of what remained after Eddie had taken his share. What part of the melon did Will have? What part of the melon was left for Fay?

(f) An estate was divided as follows: The widow received 1 third of it, and each of five children received 1 fifth of what remained after the widow had received her share. What part of the whole estate did each child receive? (g) If the estate was worth $9000, what was the value of each child's share?

5. A had 3 sevenths of an estate and B had 4 sevenths of it. A's share was equal to what part of B's share? B's share was how many times A's share?

(h) If the estate mentioned in problem 5 was worth $9660, what was the value of A's share? (i) of B's share?

Simple Numbers.

TIME BOOK, SHOWING HOURS WORKED FOR ONE WEEK.*

NAME	Daily Wages.	M.	T.	W.	T.	F.	S.	Hours	Days.	Am't earn'd
John Miller	$2.00	10	10	8	9	12	10	(a)	(k)	(u)
T. Benjamin	2.50	10	9	10	8	10	15	(b)	(l)	(v)
D. Gordon..	3.00	12	12	12	10	12	12	(c)	(m)	(w)
N. Hanson..	2.00	9	10	12	11	10	9	(d)	(n)	(x)
A. Gillett...	2.40	10	10	10	10	10	10	(e)	(o)	(y)
Wm. Price..	3.50	10	12	12	12	10	11	(f)	(p)	(z)
R. L. Wing	2.60	10	9	9	10	12	12	(g)	(q)	(aa)
J. Carlson...	2.60	10	8	0	10	12	12	(h)	(r)	(bb)
S. Spencer.	3.00	11	10	12	10	10	12	(i)	(s)	(cc)
S. L. Judd..	2.20	10	12	10	0	0	15	(j)	(t)	(dd)
Total hours		(ee)	(ff)	(gg)	(hh)	(ii)	(jj)	(kk)	(ll)	(mm)

The above is from the time-book of a small manufactory.

(a to j) Find the number of hours worked by each employe.

(k to t) Find the number of days (10 hours) worked by each employe.

(u to dd) Find the amount earned by each employe.

(ee to jj) Find the number of hours of work done each day.

(kk) Find the sum of (a) to (j) inclusive; of (ee) to (jj) inclusive.

(ll) Find the sum of (k) to (t) inclusive.

(mm) Find the sum of (u) to (dd) inclusive.

Review.

1. Common multiples of 15 and 20 are ——, ——, etc. The least common multiple of 15 and 20 is ——.

2. What are the prime factors of 62? of 63? of 64?

* Copy this time-book and write numbers in place of the letters; that is, instead of (a), write the number of hours worked by John Miller; in place of (k), write the number of days worked by John Miller, etc. Call 10 hours a day's work.

Common Fractions.

1. Add $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{5}$. (The l. c. m. of 8, 10, and 5 is ——.)

(a) Find the sum of $896\frac{1}{8}$, $507\frac{3}{10}$, and $344\frac{1}{5}$.

2. From $9\frac{1}{8}$ substract 3.2 ($1\frac{1}{8} = \frac{1}{80}$ .2 = $\frac{1}{80}$.)

(b) Find the difference of $1754\frac{3}{8}$ and 592.7.

3. Multiply $2\frac{7}{11}$ by 5. (This means —— 19* ——.)

(c) Find the product of $757\frac{8}{11}$ multiplied by 5.

4. Multiply $1\frac{3}{4}$ by $\frac{1}{4}$. (This means —— 13* ——.)

(d) Find the product of $536\frac{3}{4}$ multiplied by $\frac{1}{4}$. *Story.*

5. Multiply 21 by $\frac{3}{8}$. (This means —— 8* ——.)†

(e) Find the product of 681 multiplied by $\frac{3}{8}$.

6. Multiply 21 by $2\frac{3}{8}$. (This means —— 16* ——.)

(f) Find the product of 681 multiplied by $8\frac{3}{8}$.

7. Multiply $11\frac{1}{2}$ by $2\frac{1}{2}$. (This means —— 22* ——.)

(g) Find the product of $273\frac{1}{2}$ multiplied by $4\frac{1}{2}$.

8. Divide $7\frac{1}{2}$ by $\frac{1}{2}$. (4)* (Change $7\frac{1}{2}$ to ——ths.)

(h) Find the quotient of $97\frac{1}{2}$ divided by $\frac{1}{2}$. *Story.*

9. Divide $\frac{5}{8}$ by $\frac{5}{10}$. (9)* (Change to ——ths.)

(i) Find the quotient of $7\frac{5}{8}$ divided by $\frac{5}{10}$. *Story.*

10. Divide $7\frac{2}{4}$ by $2\frac{1}{2}$. (14)* (Change to ——ths.)

(j) Find the quotient of $86\frac{3}{4}$ divided by $3\frac{1}{2}$.

11. Divide $2\frac{5}{8}$ ($1\frac{1}{6}$) by 3. (This means —— 15* ——.)

(k) Find the quotient of $8\frac{5}{8}$ divided by 9.

12. Divide $21\frac{3}{4}$ by 5. (This means —— 21* ——.)

(l) Find the quotient of $621\frac{3}{4}$ divided by 5.

(m) Add.	(n) Subtract.	(o) Multiply.	(p) Divide.	(q) Divide.
$478\frac{7}{8}$	$539\frac{5}{8}$	$738\frac{5}{6}$	$3\frac{1}{3}$ ft.)$720\frac{1}{3}$ ft.	3)$745\frac{3}{4}$ ft.
$896\frac{3}{8}$	182.3	28		

* These figures refer to notes on pages 6 and 7.

† $\frac{3}{8}$ of 21 = 3 times $\frac{1}{8}$ of 21, or $\frac{1}{8}$ of 3 times 21.

Decimal Fractions.

DIVISION. CASE I.

1. Divide .072 by .012. (This means —— 4 * ——.)

(a) 65.088 ÷ .012 = (b) 47.275 ÷ .025 = *Story.*

2. Divide 6 by .05. (This means —— 14 * ——.)

(c) 468 ÷ .05 = (d) 375.6 ÷ .05 = *Story.*†

3. Divide 8 by .004. (This means ————.)

(e) 75 ÷ .004 = (f) 256.4 ÷ .004 = *Story.*

(g) 38.72 ÷ .004 = (h) .35 ÷ .004 = ‡ *Story.*

(i) 73 ÷ .025 = (j) .35 ÷ .025 = *Story.*

DIVISION. CASE II.

4. Divide $.056 by 8. (This means —— 5 * ——.)

(k) $24.368 ÷ 8 = (l) $35.056 ÷ 8 = *Story.*

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story.

(m) $336.6 ÷ $6 = § (n) $336.6 ÷ 6 =

(o) $336.6 ÷ $.6 = (p) $336.624 ÷ 6 =

(q) $356 × .7 = (r) $356 × 2.7 =

(s) At $.25 a dozen, how many lemons can I buy for $36.75?

(t) I paid $36.75 for 25 barrels of apples. How much did the apples cost per barrel?

(u) If one acre of land is worth $36.75, how much are 5.6 acres worth?

(v) If 2 chairs cost $37.24, how much will 9 chairs cost at the same rate?

* These figures refer to notes on pages 8 and 9.

† Pupils will most readily give a number story in such work as this, by letting the figures represent units of money, thus: At $.05 each, for $375.6, I could buy 7512 tablets. *When the pupils are well grounded in this work*, they may be encouraged to seek variety in their number stories.

‡ Find how many times 4 thousandths are contained in 350 thousandths.

§ First find how many times 36 are contained in $336.

Denominate Numbers.

1. *Twelve dozen are one gross.*
2. At 48¢ a gross, 1 dozen buttons cost —— cents.
3. At 5¢ a dozen, 1 gross of buttons cost —— cents.
4. At 50¢ a gross, 6 dozen buttons cost —— cents.
5. At 36¢ a gross, 3 dozen buttons cost —— cents.
6. George bought pens at 50¢ a gross and sold them at the rate of 2 for 1 cent: on 1 gross he gained —— cents.

(a) Henry bought pens at 60¢ a gross and sold them at the rate of 3 pens for 2 cents. How much did he gain on one gross? (b) How much on 7 gross?

Review.

(c) Make a receipted bill of the following goods sold by yourself to Mr. Frank H. Armstrong:*

Aug. 2, 15 lb. shingle nails @ 4¢; Aug. 10, 6 lb. fence staples @ 5¢; Aug. 20, 130 lb. No. 9 fence wire @ 2½¢; Aug. 25, 1 keg 20d. nails @ $3.25.†

7. One fourth of a ream of paper is —— sheets.
8. One fourth of a ton of coal is —— pounds.
9. One fourth of a mile is —— rods.
10. One fourth of a rod is —— and —— feet.
11. When coal is $6 per ton, 7000 lb. cost —— dollars.

(d) Load of bran; gross weight, 2850 lb.; tare, 1050 lb. Find the value at $7.50 a ton.

(e) Multiply 36 square feet by 24, and divide the product thus obtained by 9 square feet. *Story.*

(f) Multiply 12 cubic feet by 15, and the product thus obtained by 6. Divide the last product by 27 cubic feet. *Story.*

* See page 94.

† Ask a pupil to bring to school a shingle nail, a fence staple, a piece of No. 9 wire, and a 10d. nail; also to learn the current price of the articles named.

Measurements.

1. Draw carefully upon your slate or paper, on a scale of $\frac{1}{8}$ inch to the rod, a diagram of a rectangular piece of land that is 30 rods by 40 rods.

(a) Find the perimeter of the land represented by the diagram you have drawn.

(b) Find its area in square rods.

(c) Find its area in acres.

(d) How much is the land worth at $72.50 an acre?

(e) How much will it cost to build a fence around it at 45¢ a rod?

(f) A strip 4 rods wide across one side of the field contains how many square rods?

(g) A strip 8 rods wide across one side of the field contains how many acres?

2. Land 5 rods by 40 rods contains * ————.

3. Land 7 rods by 40 rods contains ————.

4. Land 9 rods by 40 rods contains ————.

5. Land 17 rods by 40 rods contains ————.

6. Land 25 rods by 40 rods contains ————.

Find the number of acres in each of the following:

(h) 32 rods by 5 rods. (i) 32 rods by 26 rods.

(j) 32 rods by 22 rods. (k) 32 rods by 34 rods.

Find the number of cords in each of the following:

(l) 12 ft. by 8 ft. by 4 ft. (m) 16 ft. by 6 ft. by 4 ft.

(n) 20 ft. by 4 ft. by 4 ft. (o) 21 ft. by 8 ft. by 4 ft.

Find the number of square yards in each of the following:

(p) 30 feet by 40 feet. (q) 25 feet by 33 feet.

(r) 18 feet by 21 feet. (s) 35 feet by 40 feet.

Find the number of cubic yards in each of the following:

(t) 12 ft. by 6 ft. by 9 ft. (u) 15 ft. by 12 ft. by 6 ft.

(v) 15 ft. by 9 ft. by 12 ft. (w) 21 ft. by 6 ft. by 18 ft.

* Note that when a rectangular piece of land is 40 rods long, every rod in width makes $\frac{1}{4}$ of an acre.

Ratio and Proportion.

1. One inch is —— —— of a yard.
2. Two inches are —— —— of a yard.
3. Three inches are —— —— of a yard.
4. Four inches are —— —— of a yard.
5. One foot and 2 inches are —— —— of a yard.
6. One foot and 8 inches are —— —— of a yard.

7. One square rod is —— —— of an acre.
8. Two square rods are —— —— of an acre.

(a) Twelve square rods are what part of an acre?

(b) Twenty-eight square rods are what part of an acre?

(c) Thirty-six square rods are what part of an acre?

9. 10 inches are —— —— of 1 yard. 1 yard is —— —— of 10 inches, or —— and —— —— times 10 inches.

10. 100 square rods are —— —— of one acre. 1 acre is —— —— of 100 square rods, or —— and —— —— times 100 square rods.

11. Two thirds of 33 sq. rd. are 1 half of —— sq. rd.
12. Two thirds of 1 ft. 6 in. are 3 fourths of —— inches.

(d) Two thirds of 6 ft. 9 in. are 3 fourths of how many in.?

(e) Two thirds of 96 sq. rd. are 1 half of how many sq. rd.?

13. If 36 sq. rd. of land is worth $60, 48 sq. rd. is worth —— dollars.

(f) If 36 sq. rd. of land is worth $75.48. how much is 48 sq. rd. worth at the same rate?

14. If 2 ft. 3 in. of silver wire is worth 24¢, 1 yard is worth —— cents.

(g) If 2 ft. 3 in. of platinum wire is worth $1.65, how much is 1 yd. worth at the same rate?

15. If 4 sq. rd. of land is worth $5, 1 acre is worth —— dollars.

(h) If 4 sq. rd. of land is worth $7.25, how much is 1 acre worth at the same rate?

Percentage.

(*1*)

1. One % of 640 =
2. $1\frac{1}{2}$ % of 640 =
3. One % of 420 =
4. $2\frac{1}{2}$ % of 420 =
5. One % of \$972 = (a) Find $7\frac{1}{2}$ % of \$972.
6. One % of \$876 = (b) Find $9\frac{1}{2}$ % of \$876.
7. One % of \$46.80 = (c) Find $8\frac{1}{2}$ % of \$46.80.

(*2*)

8. 84 is 7 % (7 hundredths) of ———.*
(d) 595 is 7 % of what? (e) 876 is 6 % of what?
9. 63 is 9 % (9 hundredths) of ———.
(f) 747 is 9 % of what? (g) 737 is 11 % of what?
10. 72 is 12 % (12 hundredths) of ———.
(h) 572 is 11 % of what? (i) 560 is 10 % of what? †

(*3*)

11. 18 is ———% of 300.
12. 19 is ———% of 300. ‡
(j) 87 is what % of 300? (k) 88 is what % of 300?
13. 24 is ———% of 400.
14. 26 is ———% of 400.
(l) 196 is what % of 400? (m) 380 is what % of 400?
15. 42 is what % of 600?
16. 46 is ———% of 600.
(n) 384 is what % of 600? (o) 576 is what % of 600?

17. Two % of 430 = (p) Find 9 % of 430.
18. Twenty is 4 % of ———. (q) 96 is 4 % of what?
19. Thirty is ———% of 600. (r) 276 is what % of 600?

20. Twenty-five per cent of 24 is what % of 200?
21. Seventy-five per cent of 24 is what % of 300?

* See foot-noot (*), page 97.

† Lead the pupil to see that he may solve this problem as he solved problem (h), or he may solve it by making use of the fact that 10 % = 1 tenth.

‡ See foot-note (†), page 97.

Percentage.

1. A nursery-man set out 200 trees; 8 % of them died; ——— trees were dead and ——— trees were alive.

(a) A nursery-man set out 650 trees; 8 % of them died. How many trees were dead? (b) How many lived?

2. A farmer sold 21 bushels of oats; these were 7 % of all he raised; he raised ——— bushels and had ——— bushels left.

(c) A farmer sold 98 bushels of oats; these were 7 % of all he raised. How many bushels did he raise? (d) How many bushels had he left?

3. A grocer sorted 200 bushels of apples; 12 bushels were "specked" and the remainder were sound; ———% of the apples had begun to decay and ———% of them were sound.

(e) A grocer sorted 400 bushels of apples; 56 bushels were "specked" and the remainder were sound. What % of the apples had begun to decay? (f) What % were sound?

4. Mr. Briggs earns $900 a year; he saves 8 % of his wages; he saves ——— dollars and spends ——— dollars.

(g) Mr. Jones earns $1250 a year; he saves 8 % of his wages. How much money does he save? (h) How much does he spend?

5. Mr. Piper paid $18 for a suit of clothes: this was 12 % of his month's salary; his salary was ——— dollars.

(i) Mr. Brean paid $162 for a horse, buggy, and harness; this was 12 % of his year's salary. How much was his yearly salary?

6. George earned $300; he spent $15; he spent ———% of what he earned.

(j) Henry earned $350; he spent $63. What per cent of what he earned did he spend? *

* If this problem gives the pupil trouble, require him to solve problem 6 with a pencil *after solving it* "*mentally.*" $3)$15

Review.

A manufacturer employed 20 men. The daily wages of these men were as follows: Five received $3.00 each; five received $2.50 each; five received $2.00 each, and five received $1.50 each.

(a) Find the whole amount paid to these men each day.

(b) Find the average daily wages of the men.

(c) Find the amount paid per week to the men if every man worked every day except Sunday.

(d) Find the amount paid for the month of January, if the month began on Monday and every man was present every day except the Sundays of the month.

(e) Divide 336 by $4\frac{1}{2}$. (Change to ——ths.) *Story.*

(f) Divide 336 by 4.8. (336.0 ÷ 4.8 = ?) *Story.*

Compare the answers to problems (e) and (f).

(g) Bought a ream of paper for $2.00 and sold it at 18¢ per quire. How much was my gain?

(h) Bought a box (1 gross) of pens for 75¢ and sold them at the rate of 4 pens for 3 cents. How much did I gain?

(i) If 10 bushels of potatoes cost $3.50, how much will 15 bu. cost at the same rate?

1. Draw a diagram of a piece of land 32 rods by 25 rods, scale $\frac{1}{8}$ in. to a rod. The diagram is —— inches long and —— and —— —— inches wide.

(j) How many square rods in the piece of land described in problem 1? (k) How many acres? (l) The perimeter of the field is how many rods? (m) How much will it cost to build a fence around it at 35¢ a rod? (n) How much will it cost to plow it at $1.25 an acre?

2. 7 is — — of 25. 7 twenty-fifths are — hundredths.

3. 9 is — — of 25. 9 twenty-fifths are — hundredths.

(o) 17 twenty-fifths are how many hundredths?

(p) 19 twenty-fifths are how many hundredths?

Miscellaneous Problems.

1. A woman exchanged 3½ lb. of butter at 20¢ a pound for sugar at 5¢ a pound. She received —— pounds of sugar.

(a) A man exchanged 9 cords of wood @ $4.50 a cord for coal at $6.75 a ton. How many tons of coal did he receive? (b) How many pounds of coal did he receive? (c) How many loads of 3000 lbs. each did he receive?

2. The side of a square measures 5 ft. 8 in.; the perimeter of the square is —— feet —— inches.

(d) The side of a square measures 47 ft. 10 in. What is the measurement of the perimeter of the square?

3. An estate was divided among a widow and four children; the widow received 1 third of the estate; each child received 1 fourth of what remained after the widow had received her share; each child received —— —— of the estate.

(e) The estate mentioned in problem 3 was worth $7140. How much did the widow receive? (f) How much did all the children together receive? (g) What each child received was equal to what part of what the widow received? *

4. The width of an oblong is 8 inches; its length is twice its width; its perimeter is —— inches; its area is —— square inches.

(h) The width of a piece of land in the shape of an oblong is 32 rods; its length is twice its width. How many rods of fence required to enclose it? (i) How much will the fence cost at 35¢ a rod? (j) How many square rods in the piece? (k) How many acres in the piece? (l) What is it worth at $27.50 an acre?

5. At $1.40 a yard, 5½ yd. of carpet cost ——.

(m) At $1.28 a yard, how much will 27½ yd. of carpet cost?

* 1 sixth is what part of 1 third?

Simple Numbers.*

1. The sum of all the odd numbers from 1 to 7 inclusive is ——; of all the even numbers from 2 to 8 inclusive, ——.

(a) Add all the odd numbers from 1 to 37 inclusive.

(b) Add all the even numbers from 2 to 38 inclusive.

2. $5 + 4 + 6 + 7 + 3 + 8 + 6 + 5 + 4 + 3 + 5 + 6 + 3 + 7 + 5 + 9 - 50 =$ ——.

(c) From the sum of 9864, 3792, and 8756, subtract 7598.

(d) Multiply 8754 by 27.

(e) $9328 \times 46 =$

(f) Multiply 5964 by 38.

(g) $3896 \times 52 =$

(h) Multiply 3875 by 45.

(i) $2543 \times 69 =$

(j) Multiply 874 by 126.

(k) $963 \times 245 =$

(l) Multiply 556 by 208.

(m) $831 \times 306 =$

(n) Multiply 734 by 160.

(o) $673 \times 250 =$

(p) Divide 7618 by 52.

(q) $12551 \div 49 =$

(r) Divide 9351 by 54.

(s) $11292 \div 48 =$

(t) Divide 9636 by 55.

(u) $9545 \div 46 =$

(v) Divide 7532 by 56.

(w) $10635 \div 45 =$

(x) Divide 10336 by 57.

(y) $10505 \div 44 =$

(z) Divide 3960 by 27.

(aa) $8757 \div 27 =$

(bb) Divide 6888 by 27.

(cc) $5649 \div 27 =$

(dd) Divide 6848 by 128.

(ee) $4768 \div 128 =$

(ff) Divide 5776 by 128.

(gg) $9696 \div 128 =$

(hh) Divide 2772 by 144.

(ii) $3960 \div 144 =$

(jj) Divide 4860 by 144.

(kk) $7932 \div 144 =$

(ll) Divide 6440 by 160.

(mm) $14280 \div 160 =$

(nn) Divide 11800 by 160.

(oo) $5952 \div 160 =$

* To the Teacher.—If pupils are inaccurate in their work in simple numbers, give them daily practice in exercises similar to those appearing upon this page. Lead the pupils to feel that nothing short of perfect accuracy is highly commendable. A "90% paper" in arithmetic is a *failure*. A "90% accountant" is *worthless*.

Common Fractions.

1. Add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$. (The l. c. m. of 9, 4, and 6 is ———.)

(a) Add $346\frac{2}{3}$, 375, $486\frac{1}{4}$, 296, and $855\frac{5}{6}$.

2. From $12\frac{1}{7}$ subtract $3\frac{1}{2}$. $15\frac{1}{3} - 2\frac{1}{2} =$

(b) From 2381 subtract $154\frac{1}{3}$. (c) $596\frac{3}{8} - 148\frac{1}{2} =$

3. Multiply $3\frac{5}{8}$ by 4. 4. Multiply $7\frac{5}{8}$ by 8.

(d) Multiply $537\frac{1}{3}$ by 7. (e) Multiply $375\frac{5}{8}$ by 9.

5. Multiply 12 by $2\frac{1}{3}$. 6. Multiply 11 by $3\frac{1}{4}$.

(f) Multiply 874 by $2\frac{1}{3}$. (g) Multiply 954 by $3\frac{1}{4}$.

7. Multiply $11\frac{1}{2}$ by $4\frac{1}{3}$. (This means ——— 22* ———.)

(h) Multiply $373\frac{1}{2}$ by $4\frac{1}{3}$. (i) Multiply $525\frac{1}{2}$ by $4\frac{1}{3}$.

8. Divide $8\frac{1}{3}$ by $\frac{5}{6}$. (4)* (Change $8\frac{1}{3}$ to ———ths.)

(j) Divide $48\frac{1}{3}$ by $\frac{5}{6}$. (k) Divide $17\frac{2}{3}$ by $\frac{5}{6}$.

9. Divide $6\frac{2}{3}$ by $1\frac{1}{2}$. ($6\frac{2}{3} = \frac{20}{3}$. $1\frac{1}{2} = \frac{3}{2}$.)

(l) Divide $75\frac{5}{8}$ by $1\frac{1}{2}$. (m) Divide $82\frac{2}{3}$ by $1\frac{1}{2}$.

10. Divide $1\frac{1}{3}$ ($\frac{4}{3}$) by 3. 11. Divide $19\frac{1}{3}$ by 3.†

(n) Divide $577\frac{1}{3}$ by 3. (o) Divide $625\frac{1}{3}$ by 3.

12. James rode his wheel at the rate of $9\frac{3}{4}$ miles an hour:

In 4 hours he rode ——— and ——— ——— miles. $9\frac{3}{4} \times 4 =$

In $\frac{1}{4}$ hour he rode ——— and ——— ——— miles. $9\frac{3}{4} \times \frac{1}{4} =$

In $4\frac{1}{4}$ hours he rode ——— and ——— ——— miles. $9\frac{3}{4} \times 4\frac{1}{4} =$

A locomotive moved at the rate of $49\frac{3}{5}$ miles an hour:

(p) How far did it move in 4 hours? (q) In 5 hours?

(r) How far did it move in $\frac{1}{4}$ of an hour? (s) In $\frac{3}{4}$ of an hour?

(t) How far did it move in $4\frac{1}{4}$ hours? (u) In $5\frac{3}{4}$ hours?

13. If $\frac{5}{6}$ of a yard of print is sufficient for a child's apron, $8\frac{1}{3}$ yards are sufficient for ——— aprons. $8\frac{1}{3} \div \frac{5}{6} =$

(v) $79\frac{1}{3}$ yards divided by $2\frac{1}{3}$ yards.

* These figures refer to notes on pages 6 and 7.

† Do not change $19\frac{1}{3}$ to sixths. Say, rather, 1 *third of* $19\frac{1}{3}$ *is* 6, *with a remainder of* $1\frac{1}{3}$; 1 third of $1\frac{1}{3}$ ($\frac{4}{3}$) is $\frac{4}{9}$.

Decimal Fractions.*

1.	$\frac{1}{2}$ = — tenths.	$\frac{1}{2}$ = — hundredths.	$\frac{1}{2}$ = — thousandths.
2.	$\frac{1}{5}$ = — tenths.	$\frac{1}{5}$ = — hundredths.	$\frac{1}{5}$ = — thousandths.
3.	$\frac{2}{5}$ = — tenths.	$\frac{2}{5}$ = — hundredths.	$\frac{2}{5}$ = — thousandths.
4.	$\frac{1}{4}$ = — tenths.	$\frac{1}{4}$ = — hundredths.	$\frac{1}{4}$ = — thousandths.
5.	$\frac{3}{4}$ = — tenths.	$\frac{3}{4}$ = — hundredths.	$\frac{3}{4}$ = — thousandths.
6.	$\frac{1}{8}$ = — tenths.	$\frac{1}{8}$ = — hundredths.	$\frac{1}{8}$ = — thousandths.
7.	$\frac{3}{8}$ = — tenths.	$\frac{3}{8}$ = — hundredths.	$\frac{3}{8}$ = — thousandths.
8.	$\frac{5}{8}$ = — tenths.	$\frac{5}{8}$ = — hundredths.	$\frac{5}{8}$ = — thousandths.
9.	$\frac{7}{8}$ = — tenths.	$\frac{7}{8}$ = — hundredths.	$\frac{7}{8}$ = — thousandths.
10.	$\frac{1}{6}$ = — tenths.	$\frac{1}{6}$ = — hundredths.	$\frac{1}{6}$ = — thousandths.
11.	$\frac{5}{6}$ = — tenths.	$\frac{5}{6}$ = — hundredths.	$\frac{5}{6}$ = — thousandths.
12.	$\frac{1}{7}$ = — tenths.	$\frac{1}{7}$ = — hundredths.	$\frac{1}{7}$ = — thousandths.
13.	$\frac{1}{3}$ = — tenths.	$\frac{1}{3}$ = — hundredths.	$\frac{1}{3}$ = — thousandths.
14.	$\frac{1}{9}$ = — tenths.	$\frac{1}{9}$ = — hundredths.	$\frac{1}{9}$ = — thousandths.

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story.

(a) Multiply $345 by .7.
(b) Multiply $345 by 5.7.
(c) Multiply $345 by .08.
(d) Multiply $345 by 28.
(e) Multiply $345 by 3.28.
(f) Multiply $345 by .005.
(g) Multiply $345 by .025.
(h) Multiply $345 by .325.
(i) Divide $648 by $9.
(j) Divide $648 by 9.
(k) Divide $648 by $.9.
(l) Divide $648 by $.09.

(m) Find the cost of 7.2 tons of coal at $4.75 a ton.

(n) I paid $38.25 for coal at $4.50 per ton. How many tons did I buy?

(o) I paid $663 for 156 tons of coal. What was the price per ton?

* The pupil will probably need a pencil in the solution of some of the problems 1 to 14. After he is able to fill the blanks, require him to write them as suggested in the following: $\frac{1}{2}$ = .5; $\frac{1}{2}$ = .50; $\frac{1}{2}$ = .500; $\frac{1}{3}$ = .3$\frac{1}{3}$; $\frac{1}{3}$ = .33$\frac{1}{3}$; $\frac{1}{3}$ = .333$\frac{1}{3}$, etc.

Denominate Numbers.

Teach the meaning and use of the expression **hundred weight** and its abbreviation **cwt.**

1. At 50¢ per cwt. 1 ton of bran costs ——— dollars.

2. At 40¢ per cwt. $\frac{1}{2}$ ton of feed costs ——— dollars.

3. At 60¢ per cwt. $\frac{1}{4}$ ton of corn meal costs ——— dollars.

(a) At $2.50 per cwt., how much will $\frac{3}{4}$ of a ton of flour cost? (b) $\frac{9}{10}$ of a ton? (c) $\frac{1}{8}$ of a ton?

4. At $2.40 per cwt., 50 lb. of flour are worth ———.

5. At 50¢ per cwt., 1050 lb. cost ———; 850 lb. cost ———; 1250 lb. cost ———; 950 lb. cost ———; 1150 lb. cost ———.

(d) At $2.70 per cwt., how much will 1450 lb. cost? (e) 1950 lb.? (f) 1350 lb.?

6. At $2.40 per cwt., 25 lb. of flour are worth ———.

7. At 60¢ per cwt., 25 lb. are worth ——— cents; 75 lb. are worth ——— cents; 175 lb. are worth ———.

(g) At $2.60 per cwt., how much will 1225 lb. cost? (h) 1275 lb.? (i) 1525 lb.?

8. At $2.50 per cwt., 20 lb. of flour are worth ——— cents.

9. At 60¢ per cwt., 20 lb. are worth ——— cents; 40 lb. are worth ——— cents; 60 lb. are worth ——— cents; 80 lb. are worth ——— cents.

(j) At $2.50 per cwt., how much will 1220 lb. cost; (k) 1240 lb.? (l) 1260 lb.?

10. At $2.30 per cwt., 10 lb. of flour are worth ——— cents.

11. At 80¢ per cwt., 10 lb. of corn are worth ——— cents; 20 lb. are worth ——— cents; 30 lb. are worth ——— cents; 40 lb. are worth ——— cents; 50 lb. are worth ——— cents; 60 lb. are worth ——— cents; 70 lb. are worth ——— cents.

(m) At $2.50 per cwt., how much will 1210 lb. cost?

Measurements.

1. Draw carefully upon your slate or paper on a scale of $\frac{1}{4}$ in. to the foot, a diagram of the floor of a room that is 17 ft. long and 14 ft. wide.

(a) How many square feet in the floor?

(b) How many feet of flooring will be required for the floor if, on account of the waste and the loss in matching, you must buy 1 fourth more feet of flooring than there are square feet to be covered?

(c) Making no allowance for doors, how many feet in length are the mop-boards of the room?

(d) If this floor is to be covered with a carpet that is 1 yard wide and the strips are to "run lengthwise" of the room, how many feet long will each strip be?

(e) How many strips must be purchased?*

(f) How much of one strip must be turned under or cut off?

(g) How many feet (in length) of carpet must be purchased if there is no waste in matching the strips?

(h) How many yards must be purchased?

(i) If the price of the carpet is 60¢ a yard, how much will it cost?

(j) If the figure of the carpet is such that there is waste in matching, more carpet must be purchased than would otherwise be required. How much more if the waste on each strip (except the first)† is 1 foot?

(k) How much, including the waste, will the carpet cost at 60¢ a yard?

(l) If the same floor is to be covered with carpeting that is 1 yard wide, costing 40¢ a yard, the strips to run lengthwise, and the waste in matching being 6 in. on each strip except the first, how much will the carpet cost?

* Represent the strips of carpet in the diagram.

† The first strip could be cut off without any regard to the figure of the carpet.

Ratio and Proportion.

1. Six inches are ——— ——— of a rod. *
2. Twelve inches are ——— ——— of a rod.
3. One foot and 6 inches are ——— ——— of a rod.
4. Two feet are ——— ——— of a rod.
5. Two feet and six inches are ——— ——— of a rod.
6. Three feet and six inches are ——— ——— of a rod.
7. Four feet and six inches are ——— ——— of a rod.

8. One rod is ——— ——— of a mile. 2 rods are ———.
9. Three rods are ——— ——— of a mile. 4 rods are ———.

(a) Twelve rods are what part of a mile? (b) 10 rods? (c) 20 rods? (d) 30 rods? (e) 40 rods? (f) 60 rods?

10. 200 rods are ——— ——— of a mile. 1 mile is ——— ——— of 200 rods, or ——— and ——— ——— times 200 rods.

11. 12 feet are ——— ——— of a rod. 1 rod is ——— ——— of 12 feet, or ——— and ——— ——— times 12 feet.

12. Two thirds of $7\frac{1}{2}$ feet ($\frac{2}{3}$) are 1 half of ——— feet.
13. Two thirds of $4\frac{1}{2}$ ft. ($\frac{2}{3}$) are 1 half of ——— feet.

(g) Two thirds of $25\frac{1}{2}$ ft. are 1 half of how many feet?

(h) Two thirds of $37\frac{1}{2}$ rd. are 1 half of how many rods?

14. If a piece of rubber hose $5\frac{1}{2}$ feet long is worth 60¢, a piece $16\frac{1}{2}$ feet is worth ———. †

(i) If it is worth $1.65 to make $5\frac{1}{2}$ feet of concrete walk, how much is it worth to make $16\frac{1}{2}$ feet of similar walk?

15. Twenty-five rods are ——— ——— of $37\frac{1}{2}$ rods. $37\frac{1}{2}$ rods are ——— ——— of 25 rods, or ——— and ——— ——— times 25 rods.

(j) If it costs $75.30 to make 25 rods of road, how much will it cost to make $37\frac{1}{2}$ rods at the same rate?

* How many half-feet in a rod?
† $16\frac{1}{2}$ feet are how many times $5\frac{1}{2}$ ft.?

Percentage.

Teach the meaning of the word **base** as used in percentage. Lead the pupils to see that in problems like those on this page, *the base* (the number of which hundredths are taken) *follows the word than.*

(1)

1. 50 % more than 60 is ——* 50 % less than 60 = †
2. 25 % more than 60 is ——. 25 % less than 60 =
3. 20 % more than 60 is ——. 20 % less than 60 =

(a) 50 % more than $846 = (b) 50 % less than $846 =
(c) 25 % more than $576 = (d) 25 % less than $576 =
(e) 20 % more than $475 = (f) 20 % less than $475 =

(2)

4. 60 is 50% more than ——.‡ 60 is 50% less than ——.§
5. 60 is 25% more than ——. 60 is 25% less than ——.
6. 60 is 20% more than ——. 60 is 20% less than ——.

(g) $420 is 50 % more than how many dollars?
(h) $420 is 50 % less than how many dollars?
(i) $420 is 25 % more than how many dollars?
(j) $420 is 25 % less than how many dollars?
(k) $420 is 20 % more than how many dollars?
(l) $420 is 20 % less than how many dollars?

(3)

7. 50 is ——% more than 40.‖ 30 is ——% less than 40.¶
8. 60 is ——% more than 50. 35 is ——% less than 70.
9. 30 is ——% more than 20. 40 is ——% less than 50.

(m) $660 is how many per cent more than $440?
(n) $630 is how many per cent less than $840?
(o) $200 is how many per cent less than $250?

* 50 % of 60 more than 60. 50 % = ½. ½ of 60 = 30. 60 + 30 = 90.

† 50 % of 60 less than 60. 50 % = ½. ½ of 60 = 30. 60 − 30 = 30.

‡ Think of x as standing for the number sought (*the base*). Then 60 = x and 1 half of x, or ½ of x. Since 60 = ½ of x, 1 half of x equals 20, and the whole of x = 2 times 20, or 40.

§ 60 = x less 1 half of x, or ½ of x.

‖ 50 is 10 more than 40, or ⅒ *of* 40 *more than* 40. ⅒ = ¼ = 25 %.

¶ 30 is 10 less than 40, or ⅒ *of* 40 *less than* 40. ⅒ = ¼ = 25 %.

Percentage.

1. Ira earns 80 cents a day; Ernest earns 25 % more than Ira; Ernest earns —— a day. Arthur earns 25 % less than Ira; Arthur earns —— a day.

(a) Mr. James earns $860 a year; Mr. Brown earns 25 % more than Mr. James. How much does Mr. Brown earn? (b) Mr. White earns 25 % less than Mr. James. How much does Mr. White earn?

2. Peter earns $60 a month; this is 25 % more than Paul earns; Paul earns —— a month. *

(c) Mr. Harris earns $1200 a year; this is 25 % more than Mr. Williams earns. How much does Mr. Williams earn?

3. Richard earns $60 a month; this is 25 % less than Harry earns; Harry earns —— a month. †

(d) Mr. Smith earns $840 a year; this is 25 % less than Mr. Rice earns. How much does Mr. Rice earn?

4. George earns $40 a month; Joseph earns $50 a month; George earns —— per cent less than Joseph; Joseph earns —— per cent more than George. ‡

(e) Mr. Dow earns $1000 a year; Mr. Wheeler earns $1250 a year. Mr. Wheeler earns how many per cent more than Mr. Dow? (f) Mr. Dow earns how many per cent less than Mr. Wheeler?

15 is ——% more than 10.	10 is ——% less than 15.
20 is ——% more than 15.	15 is ——% less than 20.
25 is ——% more than 20.	20 is ——% less than 25.
30 is ——% more than 25.	25 is ——% less than 30.
12 is ——% more than 8.	8 is ——% less than 12.
16 is ——% more than 12.	12 is ——% less than 15.

* Lead the pupil to think of x as standing for what Paul earns. Then $60 = x$ and 1 fourth of x, or 5 fourths of x. If $60 is $\frac{4}{5}$ of x, then x is —— dollars.

† Let x stand for what Harry earns. Then $60 = x$ less 1 fourth of x, or $\frac{3}{4}$ of x. If $60 is $\frac{3}{4}$ of x, then x is —— dollars.

‡ Help the pupil to see that in the first part of this problem Joseph's money is the base, and in the second part George's money is the base. Joseph earns $10 more than $40, or $\frac{1}{4}$ of $40 more than $40. George earns $10 less than $50, or $\frac{1}{5}$ of $50 less than $50.

Review.

(a) Find the sum of all the odd numbers from 59 to 73 inclusive.

(b) Find the sum of all the even numbers from 64 to 86 inclusive.

(c) Divide 12 by $\frac{3}{4}$. (d) Multiply 12 by $\frac{1}{3}$.

Compare the answers to problems (c) and (d).

1. $\frac{1}{8}$ = ——— hundredths. $\frac{1}{8}$ = ——— thousandths.

(e) Change $\frac{3}{8}$ to hundredths. (f) Change $\frac{3}{8}$ to thousandths.

(g) Change $\frac{5}{8}$ to hundredths. (h) Change $\frac{5}{8}$ to thousandths.

(i) Change $\frac{7}{8}$ to hundredths. (j) Change $\frac{7}{8}$ to thousandths.

2. At \$3 per cwt., 500 lb. of pork cost ——— dollars; 550 lb. cost ———; 510 lb. cost ———; 520 lb. cost ———.

(k) At \$3.60 per cwt., what is the value of 500 lb. of pork? (l) Of 550 lb.? (m) Of 510 lb.? (n) Of 520 lb.?

3. At 60¢ a gross, 1 dozen pens cost ——— cents; 3 dozen cost ——— cents; 7 dozen cost ——— cents.

(o) At \$5.28 a gross, what is the value of 1 dozen fruit cans? (p) Of 8 dozen?

(q) At \$7.20 a gross, what is the cost of 3 dozen fruit cans? (r) Of 3 cans?

4. A floor 15 feet by 17 feet is to be covered with carpet that is 1 yard wide. If the strips are laid lengthwise of the room, and there is no waste in matching the figure, ——— strips will be required, each of which is ——— feet long.

(s) How many yards of carpet will be required for the room described in problem 4? (t) How much will it cost at 90¢ a yard?

5. Seven feet and 6 inches are ——— ——— of a rod.

Miscellaneous Problems.

1. The state of Illinois is about 380 miles long and 225 miles wide at its widest part. A map of Illinois drawn upon a scale of 10 miles to the inch must be —— inches long and —— and —— —— inches wide.

2. A certain county of Illinois appears upon the map described in problem 1 as a rectangular figure 1.8 inches by 3 inches. The county is —— miles wide and —— miles long.

(a) How many square miles in the county described in problem 2?

Tell the amount of change that should be given in each of the following instances:

3. Bought $4\frac{1}{2}$ lb. meat @ 12¢; gave the salesman $1.*
4. Bought $3\frac{1}{4}$ lb. cheese @ 16¢; gave the salesman $1.
5. Bought $2\frac{3}{4}$ lb. cheese @ 16¢; gave the salesman 50¢.
6. Bought $2\frac{1}{2}$ dozen eggs @ 14¢; gave the salesman $2.
7. Bought $3\frac{1}{2}$ lb. butter @ 22¢; gave the salesman $1.

(b) Bought 2.3 tons coal @ $6.50; gave the salesman $20.

(c) Bought 1.4 tons hay @ $9.50; gave the salesman $15.

(d) Bought $3\frac{1}{2}$ cords wood @ $4.50; gave the salesman $16.

8. Byron bought 8 melons for 50¢; he sold one half of them at 10¢ each and the other half at 5¢ each: he gained —— cents.

(e) A drover bought 36 sheep for $125; he sold one half of them at $4 each and the other half at $4.50 each. How much did he gain?

9. A house rents for $10 per month; the rent for one year and six months is —— dollars.

(f) A house rents for $32 per month. How much is the rent for 2 years and 4 months?

* Lead the pupil to think how much must be added to 54¢ to make $1.

Simple Numbers.*

TICKETS OF ADMISSION SOLD AT A COUNTY FAIR.

	Price.	Wed.	Thur.	Frid.	Sat.	Total.
Children's Tickets	15¢	1645	1154	3561	1424	(e)
Adults' Tickets	25¢	2243	1754	3871	2124	(f)
Single Carriage Tickets	25¢	143	174	186	75	(g)
Double Carriage Tickets	50¢	123	175	162	137	(h)
		(a)	(b)	(c)	(d)	

(a to d) Find the number of tickets sold each day.

(e) Find the whole number of children's tickets sold.

(f) Find the whole number of adults' tickets sold.

(g) Find the whole number of single carriage tickets sold.

(h) Find the whole number of double carriage tickets sold.

(i) Find the sum of (a) to (d) inclusive, and (e) to (h) inclusive.

(j) Find the receipts for tickets sold Wednesday; (k) Thursday; (l) Friday; (m) Saturday; (n) total receipts for tickets sold.

(o) Find the receipts for all the children's tickets sold during the week; (p) adults' tickets; (q) single carriage tickets; (r) double carriage tickets.

(s) Find the sum of (o), (p), (q), and (r), and compare this sum with the answer to problem (n).

1. From June 28th to July 3d it is —— days. †
2. From June 28th to Aug. 3d it is —— days. ‡
3. From June 28th to Sept. 3d it is —— days.

(t) How many days from June 28th to Dec. 25th? (u) From April 20th to Dec. 10th?

4. From Feb. 25th of a leap year to March 6th it is —— days.
5. From Feb. 26th of a common year to March 10th it is —— days.

* Insist upon accuracy; "90 per cent of accuracy" is failure.
† To June 30th is 2 days; to July 3d, 3 more days.
‡ Two and 31 and 3.

Common Fractions.*

Reduce the following fractions to their lowest terms:

1. $\frac{14}{35}$ = $\frac{18}{27}$ = $\frac{21}{56}$ = $\frac{24}{40}$ = $\frac{15}{35}$ = $\frac{18}{42}$ =

(a) $\frac{140}{350}$ = (b) $\frac{170}{272}$ = (c) $\frac{924}{1232}$ = (d) $\frac{936}{2574}$ =

Reduce the following improper fractions to whole or mixed numbers:

2. $\frac{27}{7}$ = $\frac{34}{8}$ = $\frac{51}{6}$ = $\frac{52}{8}$ = $\frac{46}{12}$ = $\frac{37}{11}$ =

(e) $\frac{276}{8}$ = (f) $\frac{476}{7}$ = (g) $\frac{242}{5}$ = (h) $\frac{278}{6}$ =

Reduce the following mixed numbers to improper fractions:

3. $7\frac{1}{3}$ = $9\frac{3}{7}$ = $8\frac{5}{6}$ = $12\frac{3}{4}$ = $11\frac{2}{3}$ = $10\frac{7}{8}$ =

(i) $56\frac{5}{7}$ = (j) $74\frac{3}{5}$ = (k) $86\frac{3}{5}$ = (l) $73\frac{5}{8}$ =

Reduce the following fractions to equivalent fractions having a common denominator:

4. $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{6}$. The l. c. m. of 8, 4, and 6 is ———.

(m) $\frac{5}{6}$ and $\frac{7}{8}$. (n) $\frac{6}{11}$ and $\frac{3}{7}$. (o) $\frac{5}{13}$ and $\frac{3}{10}$.

5. Change 8 to a fraction whose denominator is 5.†

(p) Change 47 to a fraction whose denominator is 6.

6. The sum of two numbers is 78; one of the numbers is 32; the other number is ———.

(q) The sum of two fractions is $\frac{8}{9}$; one of the fractions is $\frac{2}{9}$. What is the other fraction?

7. If it takes $\frac{3}{4}$ of a yard of cloth to make one vest, from three yards ——— vests can be made.

(r) How many vests each containing $\frac{3}{4}$ of a yard can be made from 36 yards?

* Review, if necessary, pages 12, 22, 32, and 42.

† That is, *change 8 to fifths.*

Decimal Fractions.

Change the following common fractions to decimal fractions: *

1. $\frac{1}{2}$ = $\frac{1}{4}$ = $\frac{3}{4}$ = $\frac{2}{5}$ = $\frac{3}{5}$ = $\frac{4}{5}$ =

(a) $\frac{1}{8}$ = † (b) $\frac{2}{3}$ = (c) $\frac{1}{6}$ = (d) $\frac{5}{6}$ = (e) $\frac{1}{7}$ =

(f) $\frac{2}{7}$ = (g) $\frac{3}{7}$ = (h) $\frac{4}{7}$ = (i) $\frac{5}{7}$ = (j) $\frac{6}{7}$ =

(k) $\frac{1}{9}$ = (l) $\frac{2}{9}$ = (m) $\frac{4}{9}$ = (n) $\frac{5}{9}$ = (o) $\frac{7}{9}$ =

Change the following decimal fractions to common fractions and reduce to their lowest term:

2. .8 = .5 = .4 = .25 = .75 = .12½ ‡ .3⅓ =

(p) .125 = (q) .375 = (r) .625 = (s) .875 =

(t) .175 = (u) .225 = (v) .325 = (w) .675 =

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story: ‖

(x) Multiply $475 by .9. (y) Multiply $475 by 2.9.

(z) Multiply $534 by .07. (aa) Multiply $534 by .37.

(bb) Multiply $534 by 2.37. (cc) Multiply $534 by .003.

(dd) Multiply $534 by .043. (ee) Multiply $534 by .243.

(ff) Divide $724 by $8. (gg) Divide $724 by 8.

(hh) Divide $724 by $.8. (ii) Divide $724 by $.08.

(jj) At $12.50 per ton, find the cost of 3.7 tons of hay.

* Review page 113.

† Change these to thousandths. There are 1000 thousandths in a whole (a unit); in ⅓ of a unit there are ⅓ of 1000 thousandths. This answer may be written, .333⅓ or .333 +.

‡ Observe that you can divide both the numerator and denominator of 12½ hundredths by 12½.

‖ Do not omit the number story, or business problem. The number story for problem (*x*) might be: *Mr. Hoyt bought .9 of an acre of land at $475 an acre; the land cost him —— dollars.*

Denominate Numbers.

Pork and *beef* are usually bought by the cwt.; *hay* and *coal* by the ton, and *lath* and *brick* by the thousand. Teach the abbreviation *M.* for Latin *mille*, one thousand.

Find the cost:

1. 3000 lb. hay at $6 per ton.
2. 500 lb. hay @ $6.
3. 1500 lath at $3 per M.
4. 250 lath @ $3.
5. 1250 lb. pork at $4 per cwt.
6. 125 lb. pork @ $4.
7. 2500 lb. coal at $5 per ton.
8. 400 lb. coal @ $5.
9. 650 lb. beef @ $5 per cwt.
10. 25 lb. beef at $5.
11. 2500 brick @ $8 per M.
12. 250 brick @ $8.

Find the value:

13. 1 brick at $8 per M.
14. 100 brick @ $8.
15. 1 lb. hay at $10 per ton.
16. 100 lb. hay @ $10.
17. 1 lb. of beef @ $4 per cwt.
18. 10 lb. beef @ $4.

(a) 2150 brick at $8 per M. (b) 625 brick @ $8.
(c) 2150 lb. pork at $4 per cwt. (d) 625 lb. pork @ $4.
(e) 2150 lb. hay at $10 per ton. (f) 650 lb. hay @ $10.
(g) 3240 lath at $3 per M. (h) 750 lath @ $3.
(i) 3240 lb. coal @ $5.50 per ton. (j) 760 coal @ $5.50.
(k) 3240 lb. beef @ $6.50 per cwt. (l) 86 lb. beef @ $6.50.

19. At 60¢ a gross, 2 doz. buttons cost ——— cents.

(m) At $1.60 per gross, 75 doz. buttons cost how much?

(n) Make a receipted bill of the following goods sold by yourself to Christopher Columbus:

Sept. 1, 1230 lb. pork @ $4.60 per cwt., and 1080 lb. beef @ $6.75 per cwt.

20. At $2.40 per ream, 1 quire of paper costs ——— cents; 12 sheets cost ——— cents; 6 sheets cost ——— cents.

(o) Bought paper @ $2.40 per ream and sold it at 15¢ a quire. What was the gain on 3 reams?

(p) At 7¢ a quire, how much will 1440 sheets of paper cost?

Measurements.

1. Draw carefully upon your slate or paper, on a scale of $\frac{1}{8}$ in. to the foot, a diagram of a rectangular grass plot 50 ft. by 40 ft. Represent a gravel walk 5 feet wide just outside the grass plot and extending entirely around it. The diagram of the grass plot is —— and —— —— inches long and —— inches wide. The width of the walk as shown in the diagram is —— —— of an inch. The entire length of the figure including the diagram of the grass plot and walk is —— and —— —— inches. The entire width of the diagram is —— and —— —— inches.

(a) How many square feet in the grass plot described in problem 1? (b) How many square yards?

(c) The perimeter of the grass plot described in problem 1 is how many feet? (d) How many yards?

(e) The perimeter of the outside of the gravel walk described in problem 1, is how many feet? (f) How many yards?

(g) The outside perimeter of the gravel walk described in problem 1, is how much greater than the inside perimeter? (h) How many square feet in the walk? (i) How many square yards in the walk? (j) How much did it cost to make the walk at 54¢ per square yard?

2. A floor 14 ft. by 17 ft. is to be covered with a carpet that is one yard wide. If the breadths are 17 feet long, —— breadths will be required. If the breadths are 14 ft. long, —— breadths will be required.

(k) If there is no waste in matching, how many yards of carpet will be required for the room described in problem 2, if 17-foot strips are used? (l) How many yards if 14-foot strips are used?

Ratio and Proportion.

1. One cwt. is —— —— of a ton. 2 cwt. =

2. Three cwt. are —— —— of a ton. 4 cwt. =

3. Five cwt. are —— —— of a ton. 6 cwt. =

4. Seven cwt. are —— —— of a ton. 8 cwt. =

5. Nine cwt. are —— —— of a ton. 10 cwt. =

6. Fifteen cwt. are —— —— of a ton. One ton is —— —— of 15 cwt., or —— and —— —— times 15 cwt.

7. Sixteen cwt. are —— —— of a ton. One ton is —— —— of 16 cwt., or —— and —— —— times 16 cwt.

8. Eight cwt. are —— ——of a ton. One ton is —— —— of 8 cwt., or —— and —— —— times 8 cwt.

9. Twelve cwt. of hay is —— —— of a ton of hay.

(a) If one ton of hay is worth $12.75, how much is 12 cwt. of hay worth at the same rate?

10. Eighteen cwt. of coal is —— —— of a ton of coal.

(b) If one ton of coal is worth $6.50, how much is 18 cwt. of coal worth at the same rate?

11. Six cwt. of flour is —— —— of a ton of flour. One ton of flour is —— —— of 6 cwt. of flour, or —— and —— —— times 6 cwt.

(c) If 6 cwt. of flour is worth $12.30, how much is 1 ton of flour worth at the same rate?

12. Forty pounds are —— —— of 1 cwt.

(d) If 1 cwt. of sugar is worth $5.25, how much is 40 lb. of sugar worth?

13. Eighty pounds are —— —— of 1 cwt. 1 cwt. is —— —— of 80 lb., or —— and —— —— times 80 lb.

(e) If 80 lb. of beef is worth $5.60, how much is 1 cwt. of beef worth at the same rate?

Percentage.*

1. 40 % more than 25 is ——.† 40 % less than 25 is ——.

2. $16\frac{2}{3}$ % more than 30 is ——. $16\frac{2}{3}$ % less than 30 is ——.

3. $33\frac{1}{3}$ % more than 36 is ——. $33\frac{1}{3}$ % less than 36 is ——.

(a) 40 % more than \$570 = (b) 40 % less than \$570 =

(c) $16\frac{2}{3}$ % more than \$834 = (d) $16\frac{2}{3}$ % less than \$834 =

(e) $33\frac{1}{3}$ % more than \$726 = (f) $33\frac{1}{3}$ % less than \$726 =

(*2*)

4. 49 is 40 % more than ——.‡ 36 is 40 % less than ——.§

5. 42 is $16\frac{2}{3}$ % more than ——. 40 is $16\frac{2}{3}$ % less than ——.

6. 48 is $33\frac{1}{3}$ % more than ——. 30 is $33\frac{1}{3}$ % less than ——.

(g) \$245 is 40 per cent more than how many dollars?

(h) \$831 is 40 per cent less than how many dollars?

(i) \$658 is $16\frac{2}{3}$ per cent more than how many dollars?

(j) \$645 is $16\frac{2}{3}$ per cent less than how many dollars?

(*3*)

7. 70 is ——% more than 50.¶ 50 is ——% less than 60.‖

8. 40 is ——% more than 30. 30 is ——% less than 50.

9. 70 is ——% more than 60. 20 is ——% less than 30.

(k) \$450 is how many per cent less than \$750?

(l) \$400 is how many per cent less than \$480?

(m) \$480 is how many per cent more than \$360?

* Review page 117. † 40 % of what? 40 % = $\frac{2}{5}$. $\frac{2}{5}$ of 25 = 10. 25 + 10 = 35.

‡ Think of x as standing for the number sought (the base). Then 49 = x and 2 fifths of x, or $\frac{2}{5}$ of x. Since 49 is $\frac{2}{5}$ of x, 1 fifth of x is 7, and the whole of x = 5 times 7, or 35.

§ 36 = x less 2 fifths of x, or $\frac{3}{5}$ of x.

¶ 70 is 20 more than 50, or $\frac{20}{50}$ of 50 more than 50. $\frac{20}{50}$ = $\frac{2}{5}$. $\frac{2}{5}$ = 40 %.

‖ 50 is 10 less than 60, or $\frac{10}{60}$ of 60 less than 60. $\frac{10}{60}$ = $\frac{1}{6}$, or $16\frac{2}{3}$ %.

Percentage.

1. Hiram had $50; Samuel had $40; Hiram had ——% more than Samuel; Samuel had ——% less than Hiram.*

(a) Mr. Cooper has 880 bushels of corn; Mr. Judd has 660 bu. Mr. Judd has how many per cent less than Mr. Cooper? (b) Mr. Cooper has how many per cent more than Mr. Judd?

2. In 1896 a farmer raised 60 bu. of oats to the acre; this was a 50 % better yield than he had in 1895; in 1895, he raised —— bushels to the acre.†

(c) In 1896 a farmer sold 726 bushels of wheat; this was 50 % more than he sold in 1895. How many bushels did he sell in 1895?

3. William earns $30 a month; Benjamin earns 40 % more than William; Benjamin earns —— dollars a month. Joseph earns 50 % more than Benjamin; Joseph earns—— dollars a month.

(d) Isaac earns $27.50 a month; Ralph earns 40 % more than Isaac. How much does Ralph earn per month? (e) Clinton earns 50 % more than Ralph. How much does Clinton earn per month?

4. Good cheese is worth 18¢ a pound; this is 50 % higher than it was 1 year ago; a year ago it was worth —— cents.

(f) A certain brand of flour is worth $5.55 per barrel. If this is 50 % more than it was worth a year ago, what was it then worth?

18 is ——% more than 12.	12 is ——% less than 18.
24 is ——% more than 18.	18 is ——% less than 24.
30 is ——% more than 24.	24 is ——% less than 30.
36 is ——% more than 30.	30 is ——% less than 36.
30 is ——% more than 18.	18 is ——% less than 30.

* Lead the pupil to see that in one part of this problem he is asked to find what % 10 is of 40; in the other part to find what % 10 is of 50.

† Lead the pupil to think of x as standing for the yield of 1895. Then 60 bu. = x and 1 half of x, or ½ of x. Since 60 bu. = ½ of x, x = —— bushels.

Review.

1. From April 25th to May 5th it is —— days.

(a) How many days from April 25th to August 5th?

2. From Feb. 20th of a leap year to March 10th, it is —— days.

(b) How many days from Feb. 20th of a leap year to July 4th?*

(c) Divide 64 by $\frac{1}{2}$. (d) Multiply 64 by $\frac{1}{2}$.

Compare the answers to problems (c) and (d).

3. $\frac{1}{2}$ = —— hundredths. $\frac{1}{2}$ = —— thousandths.

(e) Change $\frac{3}{5}$ to hundredths. (f) Change $\frac{3}{5}$ to thousandths.

(g) Change $\frac{4}{5}$ to hundredths. (h) Change $\frac{4}{5}$ to thousandths.

(i) Change $\frac{5}{5}$ to hundredths. (j) Change $\frac{5}{5}$ to thousandths.

Change the following decimal fractions to common fractions and reduce them to their lowest terms:

4. $.37\frac{1}{2}$ = † $.62\frac{1}{2}$ = $.87\frac{1}{2}$ = $.16\frac{2}{3}$ = $.33\frac{1}{3}$ =

(k) .15 = (l) .85 = (m) .65 = (n) .75 = (o) .55 =

Find the cost:

5. 5500 brick at $8 per M. 6. 250 brick @ $8.

(p) 7600 brick at $6 per M. (q) 520 brick @ $6.

(r) There is a 4-foot gravel walk around a rectangular grass plot that is 30 ft. by 40 ft. How many feet in the perimeter of the grass plot? (s) How many feet in the perimeter of the outside of the walk? (t) How many square feet in the grass plot? (u) How many square feet in the gravel walk?

(v) If one ton of coal is worth $7.50, how much is 14 cwt. of coal worth? ‡

(w) If one hundredweight of corned beef is worth $4.50, 90 lb. of beef is worth how much?

* See Book I, pages 124, 134, and 144.

† Observe that you can divide both the numerator and the denominator of $\frac{7\frac{1}{2}}{100}$ by $12\frac{1}{2}$. ‡ 1 tenth of a ton is worth ——. 14 cwt. is —— tenths of ton.

Miscellaneous Problems.

1. I am thinking of the surface of a box that is 6 in. long, 4 in. wide, and 3 in. high. The surface of the bottom is —— square inches. The surface of one side is —— square inches. The surface of one end is —— square inches. The surface of the entire box—top, bottom, sides, and ends, is —— square inches.

(a) Find the entire surface of a box that is 15 in. long, 12 inches wide, and 8 inches high.

(b) Make a receipted bill of the following goods sold by yourself to your teacher: *

March 6, 150 lb. sugar @ 4½¢; 7 lb. coffee @ 35¢;
March 18, 4 lb. cheese @ 15¢; 1 vinegar barrel, $1.00;
March 25, 6 gal. kerosene @ 12¢; 3 gal. molasses @ 45¢.

(c) A farmer sold 15 head of cattle weighing 19650 lb. at $5.40 cwt. How much did he receive for them? (d) What was the average weight per head?

(e) If there is 1 half a ream in a package of paper, how many sheets in 6 such packages?

(f) A merchant had $275 in the bank Monday morning. His deposits for the week were as follows: Monday, $86; Tuesday, $55; Wednesday, $72; Thursday, $64; Friday, $83; Saturday, $124. He drew from the bank during the week $354.24. How much remained on deposit Saturday night?

2. I gave 3 doz. eggs, worth 15¢ a dozen, in part payment for 20 lb. sugar at 4¢ a pound. There remains unpaid —— cents.

(g) I gave 2250 lb. hay at $10 per ton in part payment for 2 tons of coal @ $6.50 per ton. How much remains unpaid?

* See page 94.

Simple Numbers.

1. The sum of two numbers is 38; one of the numbers is 12; the other number is ——.

(a) The sum of two numbers is twelve thousand three hundred forty-six; one of the numbers is four thousand seven hundred thirty-four. What is the other number?

2. The difference of two numbers is 17; the less number is 45; the greater number is ——.

(b) The difference of two numbers is five hundred forty-seven; the less number is three thousand four hundred seventy-six. What is the greater number?

3. The difference of two numbers is 14; the greater number is 45; the less number is ——.

(c) The difference of two numbers is six hundred seven; the greater number is four thousand forty-five. What is the less number?

4. In a problem the multiplier is 6 and the product is $42; the multiplicand is — —.

? multiplicand.
6 multiplier.
$42 product.

(d) In a problem the multiplier is fifteen and the product is nine hundred forty-five. What is the multiplicand?

5. In a problem the multiplicand is $25 and the product is $125; the multiplier is ——.

$25 multiplicand.
? multiplier.
$125 product.

(e) In a problem the multiplicand is two hundred thirty-five dollars and the product is five thousand four hundred five dollars. What is the multiplier?

6. In a problem the divisor is $12 and the quotient is 8; the dividend is —— ——.

$12)?
8

(f) In a problem the divisor is one hundred forty-six dollars and the quotient is twenty-seven. What is the dividend?

7. In a problem the quotient is 12 and the divisor is $9; the dividend is —— ——.

$9)
12

(g) In a problem the quotient is forty-six and the divisor is 24 dollars. What is the dividend?

Common Fractions.

1. Three fourths of 36 feet are —— feet. $\frac{3}{4}$ of 37 =

(a) Three fourths of 984 feet are how many feet?

(b) Three fourths of 985 = (c) Three fourths of 986 =

2. 36 feet are three fourths of —— feet. 37 ft. are three fourths of —— and —— —— feet. 38 ft. are $\frac{3}{4}$ of ——.

(d) 576 ft. are three fourths of how many feet?

(e) 577 is three fourths of what? (f) 578 is $\frac{3}{4}$ of what?

3. Paul spent $\frac{1}{2}$ of the money his father gave him for a book and $\frac{1}{3}$ of it for a knife, and had 12¢ left. Before he spent any money he had —— cents.*

(g) A man spent $\frac{1}{2}$ of his month's wages for fuel and $\frac{1}{3}$ of it for groceries, and had $17.25 left. How much was his wages? (h) How much did he spend for fuel? (i) How much did he spend for groceries?

4. Harris paid $4 for chickens at $\frac{2}{5}$ of a dollar each. He bought —— chickens. $4 \div \frac{2}{5}$ means, *find how many times 2 fifths are contained in* 4 (20 *fifths*).

(j) A man paid $375 for wheat at $\$\frac{3}{5}$ a bushel. How many bushels did he buy?

(k) A man paid $368 for apples at $\$1\frac{3}{5}$ a barrel. How many barrels did he buy?

5. Ellis sold $2\frac{1}{2}$ cords of wood at $\$4\frac{1}{2}$ a cord; he should receive for the wood —— and —— —— dollars. $2\frac{1}{2}$ times $4\frac{1}{2}$ means, 2 *times* $4\frac{1}{2}$ *and* 1 *half of* $4\frac{1}{2}$. 2 times $4\frac{1}{2}$ are ——, and 1 half of $4\frac{1}{2}$ is ——.

(l) A man sold $12\frac{1}{2}$ acres of land at $\$56\frac{1}{2}$ an acre. How much should he receive for the land?

* Lead the pupil to think of x as standing for the money his father gave him. Then he spent 1 half of x and 1 third of x, or 5 sixths of x, and must have had —— —— of x remaining.

Decimal Fractions.

1. At $60 an acre, 3.2 acres of land are worth —— dollars. 3.2 times 60 means, 3 *times* 60, *plus* .2 *of* 60.

(a) At $85 an acre, how much are 6.7 acres of land worth?
(b) 6.2 acres at $75 an acre?

2. At $70 an acre, 3.02 acres of land are worth ——. 3.02 times 70 means, 3 *times* 70, *plus* 2 *hundredths of* 70.

(c) At $65 an acre, how much are 5.03 acres of land worth?
(d) 8.06 acres at $95 an acre?

3. At $40 an acre, 2.24 acres of land are worth —— dollars. 2.24 times 40 means, 2 *times* 40, *plus* 2 *tenths of* 40, *plus* 4 *hundredths of* 40. 2 times $40 = 2 tenths of $40 = 4 hundredths of $40 =

(e) At $45 an acre, how much are 4.35 acres of land worth?
(f) 7.26 acres at $64 an acre?

Find the cost:

(g) 5.34 acres @ $265.
(h) 7.34 acres @ $465.
(i) 6.23 acres @ $52.5.
(j) 7.03 acres @ $325.
(k) 3.27 acres @ $43.5.
(l) 5.37 acres @ $54.6.
(m) 1.56 acres @ $276.
(n) 24.3 acres @ $342.
(o) 32.6 acres @ $41.6.
(p) 32.5 acres @ $4.25.
(q) 3.64 acres @ $74.2.

$265	Price per acre.
5.34	Number of acres.
$10.60	Value of .04 of an acre.
$79.5	Value of .3 of an acre.
$1325.	Value of 5 acres.
$1415.10	Value of 5.34 acres.

NOTE.—While the pupil is multiplying by 4, a *separatrix* may stand between the 2 and 6 of the multiplicand; thus, $2˘65. This will help him to remember that he is really multiplying $2.65, the value of 1 hundredth of an acre, by 4. When he is ready to multiply by 3, the separatrix should be erased and written thus: $26˘5. This will help him to remember that he is really multiplying $26.5, the value of 1 tenth of an acre, by 3. After a time he can simply imagine the separatrix in its place. Require the pupil to write the decimal point in each partial product when, in the process of multiplication, he reaches the place where it belongs. The pupil may now be taught that when he has solved a problem in multiplication of decimals, if he has "pointed off" correctly, *the decimal places in the product will be equal to those in the multiplicand and multiplier counted together.*

Denominate Numbers.

1. From March 26th to April 2d, it is —— days. If March 26th is Monday, April 2d is ——.

2. From April 20th to May 5th, it is —— days, or —— weeks and —— day. If April 20th is Friday, May 5th is ——.

3. From April 20th to May 12th, it is —— days, or —— weeks and —— day. If April 20th is Friday, May 12th is ——.

(a) How many days from April 20th to June 9th?

(b) If April 20th is Friday, what day of the week is June 9th? (c) June 12th?

4. In a year in which there is a Feb. 29th, there are —— days, or —— weeks and —— days.*

5. In a year in which there is no Feb. 29th, there are —— days, or —— weeks and —— day.*

6. If the first day of February of a common year is Monday, the first day of February of the next year is ——.

7. If the first day of February of a leap year is Monday, the first day of February of the next year is ——.

8. If the tenth day of February of a leap year is Saturday, the tenth day of February of the next year is ——.

9. If the 17th day of April of a leap year is Wednesday, the 17th day of April of the next year is ——.

Find how many weeks and days:

(d) Apr. 10 to July 4. †

(e) May 5 to July 10.

(f) May 18 to Aug. 5.

(g) June 15 to Oct. 4.

(h) July 12 to Sept. 1.

(i) Aug. 1 to Sept. 25.

(j) Sept. 8 to Dec. 25.

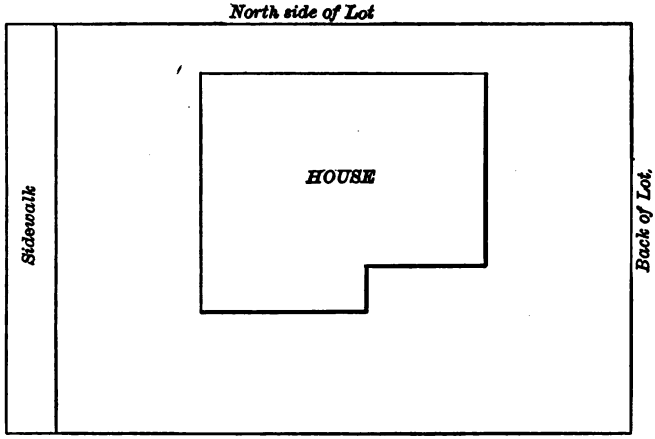
(k) Dec. 15 to Feb. 22.

* See Werner Arithmetic, Book I., pp. 124, 134, 144.

† Lead the pupil to think as suggested in the following: April 10 to Apr. 30, 20 days; Apr. 30 to May 31, 31 days; May 31 to June 30, 30 days; June 30 to July 4, 4 days. 20 days + 31 days + 30 days + 4 days = ?

Measurements.

The following diagram of a house and lot is drawn on a scale of 24 feet to an inch.



(a) How many feet long is the lot not including the walk?*

(b) How many feet wide is the lot?

(c) How many feet from the sidewalk to the house?

(d) How many feet from the house to the back of the lot?

(e) How far from the house to the north side of the lot?

(f) How many feet long is the house?

(g) How many feet wide is the front of the house?

(h) How many feet wide is the rear of the house?

(i) How far from the south side of the lot to the house?

(j) How wide is the sidewalk? (k) How much will the sidewalk cost at 12¢ per square foot?†

(l) How much is the lot worth at $25 a foot front?‡

* Dividers will be found very convenient in making these measurements. Apply the dividers first to the figure, then to a rule.

† There are 306 square feet in the walk. In finding the cost the pupil may think that at 1¢ a foot it would cost $3.06 and at 12¢ a foot, 12 times $3.06; or he may think that if 1 foot costs 12¢, 306 feet costs 306 times 12¢. Do not allow him to think that he multiplies by 12 cents.

‡ Explain to the pupil if necessary that a "foot front" is 1 foot of the width of the lot. The expression "foot front" stands for a strip 1 foot wide and as long as the lot is "deep."

Ratio and Proportion.

1. One ton is ——— and ——— ——— times 800 lb. If 800 lb. of hay is worth $3.00, 1 ton is worth ——— dollars.

(a) If 800 lb. of flour is worth $18.50, how much is 1 ton worth at the same rate?

2. One mile is ——— times 80 rd. If it costs $30 to build 80 rd. of fence, to build one mile of similar fence will cost ——— dollars.

(b) If it costs $375 to make 80 rd. of road, how much will it cost to make 1 mile of road at the same rate?

3. One hour is ——— and ——— ——— times 24 minutes. Harry rode 5 miles in 24 minutes; this was at the rate of ——— an hour.

(c) A locomotive moved 19 miles in 24 minutes. This was equal to what rate per hour?

4. One minute is ——— times 20 seconds. A pump rod made 18 strokes in 20 seconds; this was at the rate of ——— strokes a minute.

(d) A certain pulley revolves 106 times in 20 seconds. What is its rate of revolution per minute? (e) How many times does it revolve in one hour?

(f) Find how many times your pulse beats in 20 seconds. This is at what rate per minute? Per hour?

5. One pound is ——— and ——— ——— times 12 ounces. If 12 ounces of sage cheese is worth 15¢, 1 lb. is worth ——— cents.

(g) If 12 ounces of onion seed is worth 72¢, how much is 1 pound worth?

6. One acre is ——— and ——— ——— times 50 sq. rd. If 50 sq. rods of land is worth $20, at the same rate 1 acre is worth ——— dollars.

(h) If 50 square rods of land is worth $345, how much is 1 acre worth at the same rate?

Percentage.*

(1)

1. $12\frac{1}{2}$ % more than 56 is ———.† $12\frac{1}{2}$ % less than 56 =
2. 10 % more than 60 is ———. 10 % less than 60 =
3. 30 % more than 50 is ———. 30 % less than 50 =

(a) $12\frac{1}{2}$ % more than \$972 = (b) $12\frac{1}{2}$ % less than \$972 =
(c) 10 % more than \$845 = (d) 10 % less than \$845 =
(e) 30 % more than \$650 = (f) 30 % less than \$650 =

(2)

4. 45 is $12\frac{1}{2}$ % more than ———.‡ 49 is $12\frac{1}{2}$ % less than ———.§
5. 77 is 10 % more than ———. 54 is 10 % less than ———.
6. 39 is 30 % more than ———. 28 is 30 % less than ———.

(g) \$747 is $12\frac{1}{2}$ per cent more than how many dollars?
(h) \$868 is $12\frac{1}{2}$ per cent less than how many dollars?
(i) \$572 is 10 per cent more than how many dollars?
(j) \$729 is 10 per cent less than how many dollars?
(k) \$299 is 30 per cent more than how many dollars?
(l) \$595 is 30 % less than how many dollars?

(3)

7. 88 is ———% more than 80.‖ 56 is ———% less than 64.
8. 63 is ———% more than 56. 35 is ———% less than 50.
9. 52 is ———% more than 40. 54 is ———% less than 60.

(m) \$350 is how many per cent less than \$500?
(n) \$450 is how many per cent more than \$400?
(o) \$660 is how many per cent more than \$600?

* Review pp. 117 and 127. † $12\frac{1}{2}$ % of what? $12\frac{1}{2}$ % = $\frac{1}{8}$. $\frac{1}{8}$ of 56 = 7. 56 + 7 = 63.

‡ Think of x as standing for the number sought. Then 45 = x and 1 eighth of x, or $\frac{1}{8}$ of x. Since 45 is 9 eighths of x, 1 eighth of x is 5, and the whole of x is 8 times 5 = 40.

§ 49 = x less 1 eighth of x, or $\frac{1}{8}$ of x. Since 49 is $\frac{7}{8}$ of x, 1 eighth of x is 7, and the whole of x is 8 times 7 = 56.

‖ 88 is 8 more than 80, or $\frac{8}{80}$ of 80 more than 80. $\frac{8}{80}$ = $\frac{1}{10}$ = 10 %.

Percentage.

1. Five years ago the population of a certain city was 6000; it has increased $33\frac{1}{3}$ %; its present population is ——.

(a) Eight years ago the population of a certain city was 9750; it has increased $33\frac{1}{3}$ %. What is it present population?

2. A man sold a horse for $100 which was 25 % more than he gave for it; he gave —— dollars for the horse.*

(b) A man sold a farm for $6825 which was 25 % more than he gave for it. How much did he give for the farm?

3. Alice has 50¢; Jane has 60¢; Jane has ——% more than Alice; Alice has ——% less than Jane.†

(c) Mr. Lyon has 600 acres of land; Mr. Whitney has 400 acres. Mr. Lyon has how many per cent more than Mr. Whitney? (d) Mr. Whitney has how many per cent less than Mr. Lyon?

4. By selling a horse for $60, the owner would lose 25 %; the horse cost —— dollars.

(e) By selling a farm for $4320, the owner would lose 25 %. How much did the farm cost him?

5. By selling a horse for $60, the owner would gain 25 %;‡ the horse cost —— dollars.

(f) By selling a farm for $5300, the owner would gain 25 %. How much did the farm cost him?

6. 25 % of the vinegar in a cask leaked out and 36 gal. remained; before the leakage there were —— gallons.

(g) 25 % of the water in a tank leaked out and 465 gallons remained. How many gallons in the tank before the leakage?

* Think of x as standing for what he gave for the horse. Then $100 = x + 1 fourth of x, or $\frac{4}{4}$ of x. Since $100 = 5 fourths of x, 1 fourth of x is 1 fifth of $100, or $20, and the whole of x is 4 times $20, or $80.

† In one part of this problem 50 is the base; in the other part, 60 is the base.

‡ 25 % of what?

Review.

1. The sum of three numbers is 27; two of the numbers are 8 and 10; the other number is ——.

(a) The sum of three numbers is 2756; two of the numbers are 784 and 975. What is the other number?

2. Arthur rode on his bicycle three consecutive hours; the first hour he rode 12 miles; the second hour, 10 miles, and the third hour, 8 miles; his average speed per hour was —— miles. $12 + 10 + 8 =$ ——. $30 \div 3 =$ ——.

(b) The attendance at a certain school for one week was as follows: Monday, 35; Tuesday, 38; Wednesday, 37; Thursday, 36; Friday, 34. What was the average daily attendance?

3. At \$7½ per ton, 2½ tons of coal cost ——.*

(c) At \$348½ an acre, how much will 5½ acres of land cost?

(d) At \$348.50 an acre, how much will 6.5 acres of land cost?

(e) Compare the answers to (c) and (d). How much is their difference?

4. The Fourth of July, 1896 was Saturday; the Fourth of July, 1897 was ——.

5. The first day of January, 1897 was Friday. Tell the day of the week of the first day of January of each of the following years: 1898, ——; 1899, ——; 1900, ——; 1901,† ——; 1902, ——; 1903, ——; 1904, ——; 1905, ——; 1906, ——; 1907, ——; 1908, ——; 1909, ——; 1910, ——.

(f) Upon what day of the week will the first day of January 1925 occur?

6. A plot of a certain garden is drawn on a scale of 20 feet to an inch. A line 3¾ inches long represents —— feet.

(g) A certain map is drawn on a scale of 25 miles to an inch. A line 15½ inches long represents how many miles?

* 2 times 7½ and 1 half of 7½. † Remember that the year 1900 is not a leap year.

Miscellaneous Problems.

1. In a pane of glass 9 in. by 12 in., there are ——— sq. inches.

(a) How many square inches in 36 panes of glass each 9 in. by 12 in.? (b) How many square feet?

2. Mr. Black received $30 per month as rent for a house In one year he received ——— dollars.

(c) At $35 per month, how much is the rent of a house for 2 years and 6 months?

3. I paid $2.00 for coffee at 25¢ a pound; I purchased ——— pounds.

(d) Paid $34.75 for coffee at 25¢ a pound. How many pounds were purchased?

4. In a floor 12 ft. by 12 ft. there are ——— square feet; there are ——— square yards.

(e) In a lot 24 feet by 96 feet, there are how many square feet? (f) How many square yards?

5. If a train moves at the rate of 20 miles an hour, to move 110 miles will require ——— hours.

(g) If a train moves at the rate of 35 miles an hour, how long will it take to go from Chicago to Denver, a distance of 1000 miles?

6. A boy bought 10 chickens for 25¢ each, and 10 for 35¢ each; the average price paid was ——— cents.

(h) A man bought 10 horses at $135 each and 10 at $124.50 each. What was the average price?

7. A man sold a horse at $\frac{4}{5}$ of what it cost him, thereby losing $10; the horse cost him ——— dollars; he sold it for ——— dollars.

(i) A man sold a farm for $\frac{4}{5}$ of what it cost him, thereby losing $1275. How much did the farm cost him? (j) For how much did he sell it?

Simple Numbers.

Review page 11.

1. Name five odd numbers; five even numbers.

2. Name three exact divisors of 36; of 48; of 75.

Review page 21.

3. Name five integral numbers; five fractional numbers; five mixed numbers.

4. $\frac{1}{2}\frac{3}{0}$ and .7 are ——— numbers. $4\frac{1}{2}$ and 7.2 are ———.

Review page 31.

5. Name five prime numbers; five composite numbers.

6. Which of the following are prime and which are composite? 2, 22, 5, 37 45, 49, 53, 72, 87.

Review page 41.

7. What are the prime factors of 36? of 35? of 34? of 33?

8. Of what number are 2, 2, 3, and 5 the prime factors? 7, 2, and 5? 3, 5, and 7? 2 and 11?

Review page 51.

9. Name three multiples of 5; of 8; of 12; of 15.

10. Name three common multiples of 4 and 6; of 6 and 8.

11. What is the least common multiple of 4 and 6? of 6 and 8?

Review page 61.

(a) Multiply 746 by 20. (b) Multiply 394 by 80.

(c) Multiply 547 by 300. (d) Multiply 834 by 700.

Review page 71.

(e) Divide 891 by 30. (f) Divide 1265 by 50.

(g) Divide 728 by 40. (h) Divide 2478 by 70.

Review page 81.

(i) A farmer bought 30 sheep; for 5 of them he paid $6 per head; for 10 he paid $5 per head; for the remainder he paid $70. How much did the 30 sheep cost him? (j) What was the average price per head?

Common Fractions.

Review page 12.

1. Name three fractions that have a common denominator; three that do not have a common denominator.

2. Change the following to equivalent fractions having a common denominator: $\frac{3}{4}$ and $\frac{1}{2}$.

Review page 22.

3. Tell the terms of each of the following: $\frac{3}{7}$, $\frac{5}{9}$, $\frac{7}{11}$.

4. Reduce each of the following to its lowest terms; $\frac{12}{24}$, $\frac{18}{12}$, $\frac{15}{10}$, $\frac{44}{11}$, $\frac{27}{24}$. (a) $\frac{240}{360}$. (b) $\frac{525}{625}$.

Review page 32.

5. Reduce each of the following to a whole or mixed number: $\frac{18}{3}$, $\frac{25}{5}$, $\frac{24}{7}$, $\frac{32}{10}$, $\frac{91}{13}$. (c) $\frac{238}{8}$. (d) $\frac{476}{9}$.

Review page 42.

6. Reduce each of the following to an improper fraction: $9\frac{2}{3}$, $7\frac{5}{8}$, $11\frac{3}{5}$, $5\frac{1}{9}$. (e) $28\frac{4}{5}$. (f) $47\frac{5}{6}$. (g) $94\frac{3}{7}$. (h) $86\frac{5}{12}$.

Review page 52.

(i) $\frac{7}{8} + \frac{5}{12} =$* (j) $\frac{4}{15} + \frac{7}{10} =$ (k) $\frac{9}{10} + \frac{5}{12} =$

(l) $\frac{5}{6} - \frac{7}{9} =$* (m) $\frac{8}{15} - \frac{3}{10} =$ (n) $\frac{9}{10} - \frac{7}{12} =$

(o) $\frac{5}{8} \times \frac{3}{5} =$ (p) $\frac{5}{11} \times \frac{8}{4} =$ (q) $\frac{3}{7} \times \frac{4}{5} =$

(r) $\frac{11}{12} \div \frac{5}{8} =$* (s) $\frac{8}{15} \div \frac{1}{10} =$ (t) $\frac{9}{10} \div \frac{5}{12} =$

Review pages 62 and 72.

(u) Find the product of $794\frac{1}{2}$ multiplied by $6\frac{1}{2}$.†

(v) Find the quotient of $835\frac{1}{2}$ bu. divided by $2\frac{1}{4}$ bu.†

(w) Find the quotient of $654\frac{1}{8}$ bu. divided by 9.†

* Solve and explain as indicated on page 52.

† Solve and tell a suggested number story.

Decimal Fractions.

$$\begin{array}{r} {}^{v}6.40 \\ 3.7 \\ \hline 4.480 \\ 19.20 \\ \hline 23.680 \end{array}$$

$$\begin{array}{r} {}^{v}73.42 \\ 3.56 \\ \hline 4.4052 \\ 36.710 \\ 220.26 \\ \hline 261.3752 \end{array}$$

$$.005\overline{)38.455^{v}} \quad * \qquad 7691.$$

$$.05\overline{)38.47^{v}5} \quad \dagger \qquad 769.5$$

$$.5\overline{)38.4^{v}75} \quad \ddagger \qquad 76.95$$

$$5\overline{)38.^{v}475} \quad \S \qquad 7.695$$

$$.5\overline{)78.0^{v}} \quad \| \qquad 156.$$

$$.05\overline{)78.00^{v}} \quad \P \qquad 1560.$$

Review pp. 13, 23, 33, 43, 53, 63, 73, and 133.

The pupil should be led again to observe the fact that when he has thoughtfully and accurately solved a problem in multiplication of decimals, *the number of decimal places in the product is equal to the number of decimal places in the multiplicand and multiplier counted together.* He should use this fact as a test of the accuracy of his work rather than as a rule for "pointing off."

Lead him also to observe that when he has correctly located the decimal point in the first partial product, the remainder of the "pointing off" *may* be done mechanically by placing the point in each of the other partial products and in the complete product directly under the one in the first partial product.

Review pages 83, 93, and 103.

All abstract work in division of decimals may be regarded as belonging to Case I.; that is, the pupil may consider that he is to find how many times the divisor is contained in the dividend.

In the process of division, the integral part of the quotient is obtained when that figure of the dividend has been used that corresponds in denomination to the right-hand figure of the divisor. Therefore, *before beginning to divide, place a separatrix* (ᵛ) *in the dividend immediately after that figure in the dividend that is of the same denomination as the right-hand figure of the divisor. When in the process of division this separatrix is reached, the decimal point must be written in the quotient.* **

* Find how many times 5 thousandths are contained in 38455 thousandths.
† Find how many times 5 hundredths are contained in 3847 hundredths.
‡ Find how many times 5 tenths are contained in 384 tenths.
§ Find how many times 5 units are contained in 38 units.
‖ Find how many times 5 tenths are contained in 780 tenths.
¶ Find how many times 5 hundredths are contained in 7800 hundredths.
** This rule may be mechanically applied to all problems in division of decimals, but the reason for its use in problems belonging to Case II. is not so easily apparent. In Part II., of this book, this matter is further treated. See pp. 163, 173, 183, 193, etc.

Denominate Numbers.

Review page 14.

1. One half a ton is ——— lb. 1 tenth of a ton = ——— lb. 1 hundredth of a ton = ——— lb. 1 thousandth of a ton = ——— lb.

Review page 24.

2. A bushel of wheat weighs ——— pounds. 9 bu. wheat weigh ———.

(a) 75 bushels of wheat weigh how much more than 2 tons?

Review page 34.

(b) Change 3.26 tons to pounds.

(c) 4.7 tons are how many pounds?

(d) Change 3264 lb. to tons. (e) 5624 lb. are how many tons?

Review page 44.

(f) Find the cost of 7360 lb. coal at $7.25 per ton.

(g) Find the cost of 5360 lb. hay at $9.50 per ton.

Review page 54.

(h) Change 28 rods to feet. (i) Change 7 miles to rods.

(j) Change 506 yd. to rods. (k) Change 2880 rods to miles.

Review page 64.

(l) The gross weight of a load of bran was 2850 lb.; tare 1275 lb. Find the cost at $8 per ton?

(m) The gross weight of a load of oats was 2970 lb.; tare 1050 lb. How many bushels?

(n) Find the cost at 25¢ a bushel.

Review page 74.

(o) A mountain is 11000 ft. high. How many feet more than 2 miles high is it?

(p) A mountain is 5 miles high. How many feet high is it?

Review page 84.

(q) How many sheets of paper in 8 reams? (r) In 16 quires?

(s) How many quires in $9\frac{1}{2}$ reams? (t) In $6\frac{3}{4}$ reams?

Measurements.

Review pages 15 and 25.

1. Which is the larger, a five foot square or an oblong 4 feet by 6 feet?

(a) Which is the larger, a 25 ft. square or an oblong 26 ft. by 24 ft.?

(b) Find the area of a 15 ft. square.

(c) Find the solid content of a 15 ft. cube.

Review page 35.

2. Every rectangular figure has —— sides and —— right angles. If the sides are equal, the figure is a ——. If two of the sides are longer than the other two, the figure is an ——.

3. Draw a 4-sided figure that is neither a square nor an oblong. Is the figure you have drawn rectangular?

4. All the angles of a square are —— angles.

5. All the angles of an oblong are —— angles.

6. Angles that are not right angles are either —— angles or —— angles.

Review page 45.

7. Every rectangular solid has —— faces. These faces may be either squares or ——. If they are all squares the solid is a ——.

8. All the angles of a rectangular solid are —— angles.

9. Cut from a potato or a turnip a solid with six faces, some of which are not rectangular. Observe the acute angles and the obtuse angles.

Review pages 55, 65, 75, and 85.

(d) In $2\frac{1}{2}$ cords there are how many cubic feet?

(e) In $2\frac{1}{2}$ acres there are how many square rods?

(f) In $2\frac{1}{3}$ cubic yd. there are how many cubic feet?

Ratio and Proportion.

Review pages 16, 26, 36, 46, 56, 66, 76, 86, and 96.

1. A 1-foot square equals what part of a 2-foot square?
2. A 1-foot cube equals what part of a 2-foot cube?
3. A 2-foot square equals what part of a 3-foot square?
4. A 2-foot cube equals what part of a 3-foot cube?
5. A 2-yd. square equals how many times a 1-yd. square?
6. A 2-yd. cube equals how many times a 1-yd. cube?
7. A 3-yd. square equals how many times a 2-yd. square?
8. A 3-yd. cube equals how many times a 2-yd. cube?
9. A 3-yd. square equals what part of a 4-yd. square?

(a) A 10-rod square equals what part of a 12-rd. square?
(b) A 12-rd. square equals what part of a 16-rd. square?
(c) A 12-rd. square equals what part of a 24-rd. square?
(d) A 12-rd. square equals what part of a 36-rd. square?

10. A 3-ft. cube equals what part of a 4-ft. cube?

(e) A 3-ft. cube equals what part of a 5-ft. cube?
(f) A 3-ft. cube equals what part of a 6-ft. cube?
(g) A 3-ft. cube equals what part of a 9-ft. cube?

11. A $\frac{1}{2}$-ft. square equals what part of a 1-ft. square?

(h) A $\frac{1}{2}$-ft. cube equals what part of a 1-ft. cube?

12. A 1-ft. square equals how many times a $\frac{1}{4}$-ft. square?

(i) A 1-ft. cube equals how many times a $\frac{1}{4}$-ft. cube?

13. The surface of a 1-foot cube equals what part of the surface of a 2-foot cube?

(j) The surface of a 2-ft. cube equals what part of the surface of a 3-ft. cube?

(k) The surface of a 1-ft. cube equals what part of the surface of a 1-yd. cube?

(l) If a 1-inch cube of silver is worth $3.60, how much is a 3-inch cube of the same metal worth?

Percentage.

Review pages 17 and 18.

1. 25 % of 24 is ——. (a) Find 25 % of $3479.
2. 24 is 25 % of ——. (b) $3479 is 25 % of what?
3. 8 is ——% of 24. (c) 75 is what per cent of 375?*

Review pages 27 and 28.

4. 12½ % of 72 is ——. (d) Find 12½ % of $650.
5. 72 is 12 % of ——. (e) $650 is 12½ % of what?
6. 12 is ——% of 72. (f) 35 is what per cent of 245?†

Review pages 37 and 38.

7. 10 % of 45 is ——. (g) Find 10 % of $725.
8. 45 is 10 % of ——. (h) $725 is 10 % of what?
9. 5 is —— % of 45. (i) 55 is what per cent of 440?

Review pages 47 and 48.

10. 66⅔ % of 48 is ——. (j) Find 66⅔ of $756.
11. 48 is 66⅔ % of ——. (k) $756 is 66⅔ % of what?
12. 36 is ——% of 48. (l) $450 is what per cent of $600?

Review pages 57 and 58.

13. 60 % of 75 is ——. (m) Find 60 % of $810.
14. 75 is 60 % of ——. (n) $810 is 60 % of what?
15. 20 is ——% of 40. (o) $84 is what per cent of $210?

Review pages 67 and 68.

16. 80 % of 60 is ——. (p) Find 80 % of $640.
17. 60 is 80 % of ——. (q) $640 is 80 % of what?
18. 50 is ——% of 60. (r) $550 is what per cent of $660?

Review pages 77, 78, 87 and 88.

19. 37½ % of 24 is ——. (s) Find 37½ % of $576.
20. 24 is 37½ % of ——. (t) $576 is 37½ % of what?
21. 24 is ——% of 80. (u) $675 is what per cent of $750?

* 75 is what part of 375? † 35 is what part of 245?

Percentage.

Review pages 97 and 98.

1. One % of 357 = (a) Find 13 % of 357.
2. 39 is 3 % of ———. (b) 264 is 8 % of what?
3. 15 is ———% of 300.* (c) 60 is what per cent of 750? †

Review pages 107 and 108.

4. One % of 736 = (d) Find $8\frac{1}{2}$ % of 736.
5. 108 is 9 % of ———. (e) 375 is 5 % of what?
6. 57 is ———% of 300.* (f) 41.5 is what per cent of 830? ‡

Review pages 117 and 118.

7. 25 % more than 80 = 25 % less than 80 =
8. 40 is 25 % more than —. § 45 is 25 % less than —. ‖
9. 75 is —% more than 60.¶ 60 is —% less than 75.**

10. Alice has $40; Jane has $50; Mary has $60.

(g) Jane has what per cent more than Alice?
(h) Mary has what per cent more than Jane?
(i) Mary has what per cent more than Alice?
(j) Jane has what per cent less than Mary?
(k) Alice has what per cent less than Jane?
(l) Alice has what per cent less than Mary?
(m) Alice's money equals what % of Jane's money?
(n) Alice's money equals what % of Mary's money?
(o) Jane's money equals what % of Alice's money?
(p) Jane's money equals what % of Mary's money?
(q) Mary's money equals what % of Alice's money?
(r) Mary's money equals what % of Jane's money?

* First find 1 % of 300. † First find 1 % of 750.
‡ First find 1 % of 830.
§ Let x = the number sought, the base; then 40 = x and 1 fourth of x, or $\frac{1}{4}$ of x.
‖ 45 is —— fourths of the number sought.
¶ 75 is how many more than 60? 15 is what % of 60?
** 60 is how many less than 75? 15 is what % of 75?

Review pages 113 and 123.

1. $\frac{3}{4}$ = ——— hundredths. (a) $\frac{3}{4}$ = ——— thousandths.

2. $\frac{7}{8}$ = ——— hundredths. (b) $\frac{7}{8}$ = ——— thousandths.

(c) Change .275 to a common fraction and reduce it to its lowest terms. (d) .375 (e) .425 (f) .575 (g) .625

Review pages 133 and 143.

(h) Find the cost of 6.28 acres of land at $2.75 an acre.*

(i) Find the cost of 3.46 tons of coal at @ $6.75 a ton.

(j) Divide 6.25 by 5. (6.$^{\vee}$25 ÷ 5 units.)

(k) Divide 6.25 by .5 (6.2$^{\vee}$5 ÷ 5 tenths.)

(l) Divide 6.25 by .05 (6.25$^{\vee}$ ÷ 5 hundredths.)

(m) Divide 36 by 5. (36$^{\vee}$ divided by 5 units.)

(n) Divide 36 by .5 (36.0$^{\vee}$ ÷ 5 tenths.)

(o) Divide 36 by .05 (36.00$^{\vee}$ ÷ 5 hundredths.)

(p) Divide 36 by .005 (36.000$^{\vee}$ ÷ 5 thousandths.)

(q) Divide 57.26 by 7. (57.$^{\vee}$26 divided by 7 units.)

(r) Divide 57.26 by .7 (57.2$^{\vee}$6 ÷ 7 tenths.)

(s) Divide 57.26 by .07 (57.26$^{\vee}$ ÷ 7 hundredths.)

(t) Divide 57.26 by .007 (57.260$^{\vee}$ ÷ 7 thousandths.)

(u) Divide 67.5 by 25. (67.$^{\vee}$5 divided by 25 units.)

(v) Divide 67.5 by 2.5 (67.5$^{\vee}$ ÷ 25 tenths.)

(w) Divide 67.5 by .25. (67.50$^{\vee}$ ÷ 25 hundredths.)

(x) Divide 67.5 by .025 (67.500$^{\vee}$ ÷ 25 thousandths.)

(y) Divide 6.75 by 25. (25 units in 6 units = 0., etc.)

(z) Divide 6.75 by 2.5 (6.7$^{\vee}$5 ÷ 25 tenths.)

* Require the pupil to put the work on the blackboard and to explain by telling (1) the cost of 1 hundredth of an acre; (2) of 8 hundredths; (3) of 1 tenth; (4) of 2 tenths; (5) of 1 acre; (6) of 6 acres; (7) of 6.28 acres. How many decimal places in the product? How many in the multiplicand? How many in the multiplier?

BOOK TWO — PART II.

Simple Numbers.

1. The prime factors of 18 are ——, ——, and ——.

(a) What are the prime factors of 190? (b) Of 42? *

2. Eleven is a prime number. 35 is a composite number; its prime factors are —— and ——.

(c) Write five composite numbers; resolve each into the prime factors of which it is composed. †

3. The number 17 is ——. The number 51 is ——; it is composed of the prime factors —— and ——.

4. Some odd numbers are prime and some are ——. Name three odd prime numbers; three odd composite numbers.

5. Write all the odd numbers from 1 to 31 inclusive and tell which are prime and which are composite.

6. There is only one even prime number. Can you tell what it is?

(d) Find the sum of all the odd prime numbers from 1 to 27 inclusive.

(e) Find the sum of all the odd composite numbers from 1 to 27 inclusive.

(f) Find the sum of all the prime numbers from 100 to 111 inclusive.

(g) Find the sum of all the composite numbers from 100 to 111 inclusive.

* See pages 31 and 41. † Note the similarity of the words *composite* and *composed*.

Common Fractions.*

1. If tea is worth $\frac{1}{2}$ a dollar a pound, for $\frac{3}{4}$ of a dollar, I can buy —— pounds. $\frac{3}{4} \div \frac{1}{2} =$

(a) If potatoes are worth $\frac{1}{2}$ a dollar a bushel, how many bushels should I receive for $15\frac{3}{4}$ dollars? $15\frac{3}{4} \div \frac{1}{2} =$

2. If $\frac{3}{4}$ of a yard of cloth is worth 45¢, 1 yard is worth ——.

(b) If $\frac{3}{4}$ of an acre of land is worth 252 dollars, how much is one acre worth at the same rate?

(c) If $\frac{3}{4}$ of an acre of land is worth 200 dollars, how much is one acre worth at the same rate?

3. If $\frac{3}{4}$ of a pound of maple sugar is worth 15¢, $7\frac{1}{2}$ pounds are worth ——.

(d) If $\frac{3}{4}$ of a cord of wood is worth $3.48, how much are $7\frac{1}{2}$ cords worth at the same rate?

(e) Add $275\frac{5}{12}$ ft., $326\frac{3}{4}$ ft., $175\frac{2}{3}$ ft. and $249\frac{5}{6}$ ft. *

(f) $8\frac{1}{4}$ in. + $7\frac{1}{8}$ in. + $9\frac{3}{8}$ in. + $5\frac{11}{16}$ in. + $10\frac{1}{2}$ in. + $3\frac{3}{4}$ in.

(g) $275\frac{1}{2}$ ft. − $136\frac{5}{6}$ ft. =

(h) $42\frac{1}{3}$ ft. − $27\frac{3}{4}$ ft. =

(i) $28\frac{1}{4}$ in. − $14\frac{5}{16}$ in. =

(j) $31\frac{1}{16}$ in. − $15\frac{3}{8}$ in. =

(k) $27\frac{5}{12}$ ft. × 4 =

(l) $43\frac{2}{3}$ ft. × $3\frac{1}{2}$ = †

(m) $34\frac{5}{8}$ in. × 5 =

(n) $61\frac{3}{8}$ in. × 6 =

(o) $32\frac{1}{2}$ ft. ÷ $6\frac{1}{4}$ ft. = ‡

(p) $65\frac{1}{3}$ ft. ÷ $9\frac{1}{3}$ ft. =

(q) $28\frac{1}{2}$ in. ÷ $7\frac{1}{8}$ in. = §

(r) $35\frac{1}{16}$ in. ÷ $6\frac{3}{8}$ in. =

(s) $27\frac{3}{4}$ ft. ÷ 5 = ‖

(t) $45\frac{2}{3}$ ft. ÷ 6 =

(u) $34\frac{1}{4}$ in. ÷ 3 =

(v) $29\frac{3}{8}$ in. ÷ 4 =

* Require the pupil to tell a suggested number story for at least one problem in each of the last five subdivisions on this page (e to v).

† Do not reduce $43\frac{1}{3}$ to an improper fraction. Rather, take 3 times $43\frac{1}{3}$, and $\frac{1}{2}$ of $43\frac{1}{3}$ and add them together.

‡ Change the dividend and the divisor to halves.

§ Change the dividend and the divisor to eighths.

‖ Do not change the dividend to an improper fraction. One fifth of 27 ft. is —— feet with a remainder of —— feet. $2\frac{3}{4}$ ft. = $\frac{11}{4}$ ft. 1 fifth of $\frac{11}{4}$ ft. = ————.

Decimal Fractions.*

1.	2.	3.
$2\overset{\vee}{.}34$	$.2\overset{\vee}{4}$	$.02\overset{\vee}{4}$
4.	**5.**	**6.**
$.2\overset{\vee}{3}4$	$.02\overset{\vee}{4}$	$.002\overset{\vee}{4}$
7.	**8.**	**9.**
$2\overset{\vee}{.}0234$	$.2\overset{\vee}{0}234$	$.02\overset{\vee}{3}4$
10.	**11.**	**12.**
$.002\overset{\vee}{4}$	$.0002\overset{\vee}{4}$	$2\overset{\vee}{.}$
13.	**14.**	**15.**
$.2\overset{\vee}{0}$	$.02\overset{\vee}{0}0$	$.002\overset{\vee}{0}00$
(a)	(b)	(c)
.4)6.24†	4)6.24†	.04)6.24†
(d)	(e)	(f)
.04).624	.4).624	.004).624
(g)	(h)	(i)
.4).0624	.04).0624	4).0624
(j)	(k)	(l)
.004).0624	.0004).0624	4)624
(m)	(n)	(o)
.4)624	.04)624.	.004)624

* Review the last part of page 143. Each problem on this page *may* mean, find how many times the divisor is contained in the dividend.

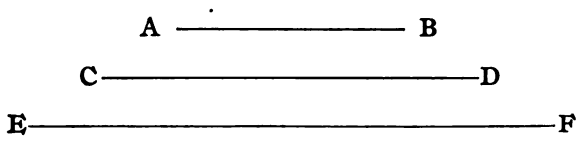
† In solving problems (a) to (o), require the pupil *first* to write the separatrix as suggested on page 143 and as shown in problems 1 to 15 on this page; then to divide, writing the decimal point in the quotient when, in the process of division, the separatrix is reached. Observe that *the figure to the left of the separatrix is always of the same denomination as the right-hand figure of the divisor.*

Denominate Numbers—Metric System.

A meter is a little more than one yard.*

One tenth of a meter is called a decimeter.

One tenth of a decimeter is called a centimeter.



1. From A to B is —— centimeters.
2. From C to D is —— centimeters.
3. From E to F is —— centimeters.
4. The difference between the lines EF and AB is —— centimeters.
5. The sum of the lines AB and CD is —— centimeters; their difference is —— centimeters.
6. Hold up your hands three decimeters apart. Three decimeters equal —— centimeters.
7. Draw a line upon the blackboard 25 centimeters long. Twenty-five centimeters equal —— and —— tenths decimeters.
8. How wide is your desk? How long?
9. Is the door-way two meters high?
10. How much more than 1 meter wide is the blackboard?
11. How tall is the teacher?
12. How wide is the school-room? How long?
13. Two meters equal —— decimeters. 30 decimeters =
14. Four decimeters equal —— centimeters.

10 centimeters = 1 decimeter.

* A meter = 39.37 inches. Provide a stick one meter long. Divide it into tenths or decimeters. Then divide each decimeter into tenths, or centimeters. Require each pupil to make a rule three or four decimeters in length and to divide it carefully into decimeters and centimeters. Require him to use this in measurements, *without relating it in his thought to the inch or foot*, until he is thoroughly familiar with these new terms and their use.

Measurements—Lumber.

A piece of lumber, 1 *foot long*, 1 *foot wide, and* 1 *inch thick, or its equivalent*, is called 1 *foot of lumber.** A piece of lumber 12 *feet long*, 1 *in. wide, and* 1 *inch thick*, is 1 foot of lumber.

1. A piece of board 1 ft. by 1 ft. by 1 inch is ——.

2. A piece of board 12 ft. by 1 inch by 1 inch is ——.

3. A piece of board 12 ft. long, 1 inch thick, and 2 inches wide, is —— feet of lumber.

4. A piece of board 12 ft. long, 1 in. thick, and 7 inches wide, is —— feet of lumber.

5. A piece of board 12 ft. long, 1 in. thick, and 13 inches wide, is —— feet of lumber.

6. I have two 12-foot boards,† each 1 in. in thickness; one is 8 inches wide and the other is 11 inches wide; I have —— feet of lumber.

7. I have three 12-ft., 1-in. boards; their widths are 9 inches, 12 inches, and 10 inches; I have —— feet of lumber.

In each of the following columns of figures, the widths of eight 12-ft. boards are given. How many feet of lumber in each lot? ‡

(a)	(b)	(c)	(d)	(e)
13 in.	12 in.	15 in.	9 in.	15 in.
9 in.	10 in.	13 in.	7 in.	9 in.
15 in.	11 in.	14 in.	7 in.	9 in.
8 in.	9 in.	12 in.	11 in.	14 in.
12 in.	8 in.	13 in.	8 in.	7 in.
11 in.	13 in.	16 in.	7 in.	12 in.
10 in.	12 in.	17 in.	10 in.	5 in.
8 in.	10 in.	12 in.	9 in.	14 in.

* For exception to this, see page 185. The work on this page should not be attempted until the following pieces of lumber have been provided: A piece of board, 1 ft. by 1 ft. by 1 in., and a piece 12 ft. by 1 in. by 1 in.

† Boards 12 feet long.

‡ In measuring the widths of common boards for the purpose of finding how many feet of lumber they contain, it is not customary to take into the account fractions of inches. If a board is 8¾ inches wide, call it 9 inches; if a board is 10¼ inches wide, call it 10 inches.

Ratio and Proportion.

Ratio is the *quotient* obtained by dividing one number by another of the same kind. The number mentioned first is regarded as the dividend. The ratio of $6 to $3 is 2.

1. The ratio of 20 feet to 5 feet is ———.*
2. The ratio of 20 inches to 10 inches is ———.
3. The ratio of 20 miles to 20 miles is ———.
4. The ratio of 20 acres to 40 acres is ———.†
5. The ratio of 20 pounds to 30 pounds is ———.
6. The ratio of 15 bushels to 20 bushels is ———.
7. The ratio of 50 dollars to 20 dollars is ———.
8. The ratio of 25 meters to 30 meters is ———.‡
9. The ratio of 8 centimeters to 2 centimeters is ———.§
10. The ratio of 4 decimeters to 9 decimeters is ———.‖
11. The ratio of 48 feet of lumber to 36 feet of lumber is ———.
12. The ratio of 36 feet of lumber to 48 feet of lumber is ———.

The two numbers of which the ratio is to be found are called the **terms of the ratio.** The first term is called the **antecedent** and the second the **consequent.** The ratio is the quotient of the antecedent divided by the consequent.

13. In problem 1 on this page, the terms are ——— and ———; the antecedent is ———; the consequent is ———.
14. In problem 2, the terms are ——— and ———; 10 inches is the ———, and 20 inches is the ———.
15. If the antecedent is *greater* than the consequent, the ratio is an improper fraction which may be reduced to a ——— or ——— number.
16. If the antecedent is *less* than the consequent, the ratio is a ——— fraction which may or may not be expressed in lower terms than the terms of the ratio.

* Not 4 *feet*, but simply 4. † $\frac{2}{4} = \frac{1}{2}$.

‡ Go out on the play-ground and put up two stakes 25 meters apart, and two that are 30 meters apart.

§ Draw a line on your slate 8 centimeters long; a line 2 centimeters long.

‖ Hold up your hands 4 decimeters apart; 9 decimeters apart.

Percentage.

Case I. Find $17\frac{1}{2}$ % of 496.

Operation.		*Explanation.*
496		
$.17\frac{1}{2}$		
34.72	(7 % of 496.)	One % of 496 = 4.96.
49.6	(10 % of 496.)	$17\frac{1}{2}$ % of 496 = $17\frac{1}{2}$ times 4.96,
2.48	($\frac{1}{2}$ % of 496.)	or 86.80.
86.80	($17\frac{1}{2}$ % of 496.)	

Case II. 352 is 11 % of what?

Operation.		*Explanation.*
Let x = the number sought.		
11)352	(11 % of x.)	Since 352 is 11 % of the number sought, 1 % is $\frac{1}{11}$ of 352, or 32. Since 32 is 1 %, 100 % is 100 times 32, or 3200.
32	(1 % of x.)	
100		
3200	(x)	

Case III. 221 is what % of 1700?

Operation.	*Explanation.*
1700 ÷ 100 = 17.	
17)221(13 times.	17 is 1 % of 1700.
17	221 is as many % of 1700 as 17 is contained times in 221. It is contained 13 times; therefore 221 is 13 % of 1700.
51	
51	

(a) Find $23\frac{1}{2}$ per cent of \$874.
(b) \$252 is 9 per cent of what?
(c) \$368 is what per cent of \$1600?

(d) Find $34\frac{1}{2}$ per cent of \$732.
(e) \$371 is 7 per cent of what?
(f) \$198 is what per cent of \$1800?

Percentage—Selling on Commission.

Commission, as the word is commonly used in arithmetic, is the amount paid an agent for buying or selling goods, collecting money, or transacting other business.

When an agent *sells goods* on commission, his pay for doing this (his commission) is reckoned on the *selling price* as a *base.*

1. Sold books for $20; my commission was 40 %.* How much should I retain and how much "pay over" to my employer?

(a) Sold books for $175.40; my commission was 40 %. How much should I retain? (b) How much should I remit to my employer?

2. At 40 %,* my commission for selling a lot of books amounted to $20. How many dollars' worth of books did I sell? How many dollars should I send to my employer?

(c) At 40 %, my commission for selling a lot of books amounted to $134.60. How much did I receive for the books? (d) How much must I pay to my employer?

3. I sold books for $35; of this amount I was allowed for my services $7. What % commission was I paid?

(e) I sold books for $224; of this sum I was allowed for my services $56. What % commission was I paid?

(f) Selling price, **$275.50;** rate of commission, 6 %. Commission? †

(g) Commission, **$27.60;** rate of commission, 6 %. Selling price?

(h) Commission, $16.80; selling price, **$560.** Rate of commission?

(i) Selling price, **$356.40;** rate of commission, 5 %. Commission?

(j) Commission, **$31.50;** rate of commission, 5 %. Selling price?

(k) Commission, $16.50; selling price, **$330.** Rate of commission? ‡

* 40 % of what? † Explain the meaning of the word *rate,* but require no formal definition to be committed to memory at this time.

‡ In the last six problems on this page, find the *one per cent* by using the full-faced figures as a dividend. Note that in problem (f) $275.50 is the base; in problem (g), $27.60 is 6 hundredths of the base; in problem (h), $560 is the base.

Review.

(a) Find the sum of all the numbers that are exactly divisible by 5, from 40 to 95 inclusive.

(b) What are the prime factors of 255? (c) Of 325?

(d) If $\frac{3}{4}$ of a ton of hay is worth \$7.20, how much are $5\frac{1}{4}$ tons worth at the same rate?

(e) If potatoes are worth $\frac{1}{4}$ of a dollar a bushel, how many bushels can I buy for $17\frac{1}{2}$ dollars? $17\frac{1}{2} \div \frac{1}{4} =$

(f)	(g)	(h)
\$3) \$5.^{v}22	\$.03) \$5.22^{v}	\$.3) \$5.2^{v}2
(i)	(j)	(k)
\$.3) \$522.0^{v}	\$.3) \$52.2^{v}	\$.03) \$.52^{v}2

1. Ten centimeters make one ——.
2. Ten ———— make one meter.

(l) How many centimeters long is this page?

(m) Is one foot more or less than 30 centimeters?

3. A piece of board 12 ft. long, 1 in. thick, and 9 in. wide, is —— feet of lumber.

(n) How many feet of lumber in 5, 12-foot, 1-inch boards whose widths are 11 in., 12 in., 15 in., 11 in., and 14 in.?

4. Antecedent is 30; consequent, 10; ratio, ——.
5. Antecedent is 10; consequent, 30; ratio, ——.
6. Antecedent is 2.5; consequent, 7.5; ratio, ——.
7. Antecedent is 7.5; consequent, 2.5; ratio, ——.

(o) At the rate of 5 %, my commission for selling goods was \$27.40. What was the selling price of the goods? *

* (1) Solve by finding 1 per cent of the selling price; then 100 %, or the whole of it. (2) Solve by making use of the fact that 5 % of a sum is $\frac{1}{20}$ of the sum.

Miscellaneous Problems.

1. The 29th day of August, 1896, was Saturday; the 29th day of August, 1897, was ——. The first day of September, 1897, was ——.*

(a) If a bicycle moves forward 16 ft. during one revolution of the pedals, how many times do the pedals revolve in going 1 mile?

(b) How many cubic yards of earth must be removed to make a cellar 36 feet long, 20 feet wide, and 6 feet deep?

(c) Find the number of square feet in the sides and ends of a cellar 20 feet long, 15 feet wide, and 8 feet deep.

(d) Mr. Bell sold his apples at 20¢ a peck; he received for them $24.80. How many bushels did he sell?

(e) How many $\frac{1}{2}$-inch squares equal a 6-inch square?

(f) How many $\frac{1}{2}$-inch cubes equal a 6-inch cube?

(g) A piece of land 4 rods by 8 rods is what part of an acre?

(h) What is the cost of 320 lb. of sugar at $5.30 per cwt.?

(i) What is the cost of 940 lb. of hay at $8 per ton?

(j) What is the cost of 1530 brick at $6 per M?

(k) A man spent in one year $864 which was $\frac{4}{5}$ of his salary. How much was his salary? (l) How much did he save?

(m) Harry has 65¢; this is 25¢ more than his sister has. How much have both?

(n) How many cubic feet of air in a room that is 12 ft. by 13 ft. by 7 ft.?

(o) After paying $\frac{1}{3}$ of my indebtedness, I owe $456. How much did I pay?

* See Book I, pages 124, 134 and 144.

Simple Numbers.*

Lead the pupil to see that *the prime factors of a multiple of a number include all the prime factors of the number itself.*

NUMBERS AND THEIR MULTIPLES.

$12 = 2 \times 2 \times 3$.	$60 = 2 \times 2 \times 3 \times \mathbf{5}$.	12 in 60 ——— times.
$18 = 2 \times 3 \times 3$.	$54 = 2 \times 3 \times 3 \times \mathbf{3}$.	18 in 54 ——— times.
$10 = 2 \times 5$.	$70 = 2 \times 5 \times 7$.	10 in 70 ——— times.
$15 = 3 \times 5$.	$60 = 3 \times 5 \times \mathbf{2} \times \mathbf{2}$.	15 in 60 ——— times
$27 = 3 \times 3 \times 3$.	$81 = 3 \times 3 \times 3 \times \mathbf{3}$.	27 in 81 ——— times.
$21 = 3 \times 7$.	$126 = 3 \times 7 \times \mathbf{2} \times \mathbf{3}$.	21 in 126 ——— times.

$132 = 2 \times 2 \times 3 \times 11$.

1. 12 ($2 \times 2 \times 3$) is contained in 132 ——— times.†
2. 33 (3×11) is contained in 132 ——— times.†
3. 6 (2×3) is contained in 132 ——— times.
4. 22 (2×11) is contained in 132 ——— times.
5. 44 ($2 \times 2 \times 11$) is contained in 132 ——— times.

$280 = 2 \times 2 \times 2 \times 5 \times 7$.

6. 40 ($2 \times 2 \times 2 \times 5$) is contained in 280 ———.†
7. 35 (5×7) is contained in 280 ——— times.†
8. 8 ($2 \times 2 \times 2$) is contained in 280 ——— times.
9. 20 ($2 \times 2 \times 5$) is contained in 280 ——— times.
10. 56 ($2 \times 2 \times 2 \times 7$) is contained in 280 ——— times.

11. Is 132 exactly divisible by 7? by 11? by 5? by 55?†
12. Is 280 exactly divisible by 11? by 7? by 21? by 13?†

Resolve the following numbers into their prime factors and learn by inspection how many times one number (of each two numbers) is contained in the other.

(a) 720 and 144. (b) 320 and 64. (c) 360 and 60.

* Review page 51, statements 1 to 9, and page 151.

† Answer by comparing the factors of which each number is composed; then prove the statement by actual division.

Common Fractions.

1. If a man earns $25 in $2\frac{1}{2}$ days, in $\frac{1}{2}$ of a day he earns ——— dollars; in 1 day he earns ——— dollars.

2. Mr. Holland paid $21 for $3\frac{1}{2}$ cords of wood; for $\frac{1}{2}$ of a cord he paid ——— dollars; for 1 cord he paid ——— dollars.

3. Henry rode his bicycle 36 miles in $4\frac{1}{2}$ hours; this was at the rate of ——— miles in $\frac{1}{2}$ of an hour, or ——— miles an hour.

To THE TEACHER.—By means of the foregoing problems and others similar to these, lead the pupil to see that $25 ÷ $2\frac{1}{2}$ means, *find* $\frac{1}{2}$ *of* $25; $21 ÷ $3\frac{1}{2}$ means, *find* $\frac{1}{2}$ *of* $21; 36 miles ÷ $4\frac{1}{2}$ means, *find* $\frac{1}{2}$ *of* 36 *miles*, etc.

$25 ÷ $2\frac{1}{2}$ This means, *find how many times* $2\frac{1}{2}$ *are contained in* $25. $2\frac{1}{2}$ are contained in $25, ——— times. At $2\frac{1}{2}$ a yard, how many yards of cloth can be bought for $25?

$25 ÷ 2 This means, *find* 1 *half of* $25. One half of $25 is $12\frac{1}{2}$. If 2 tons of hay are worth $25, what is 1 ton worth?

$25 ÷ $2\frac{1}{2}$ This means, *find* 2 *fifths* * *of* $25. Two fifths of $25 are ——— dollars. If $2\frac{1}{2}$ tons of hay are worth $25, what is 1 ton worth?

(1) Tell the meaning of each of the following, (2) solve it, and (3) make a problem that corresponds to it as suggested in the foregoing.

(a) 75 bu. ÷ $2\frac{1}{2}$ bu. =† (b) 75 bu. ÷ 2 =‡ (c) 75 bu. ÷ $2\frac{1}{2}$ =§

(d) 64 mi. ÷ $2\frac{1}{2}$ mi. = ‖ (e) 64 mi. ÷ 2 =¶ (f) 64 mi. ÷ $2\frac{1}{2}$ =**

(g) If $3\frac{1}{2}$ acres of land yield 168 bu. of corn, how much does 1 half acre yield? (h) One acre? 168 bu. ÷ $3\frac{1}{2}$ =

(i) If $5\frac{1}{2}$ tons of coal are worth $35.75, how much is 1 half a ton worth? (j) One ton? $35.75 ÷ $5\frac{1}{2}$ =

* $\frac{1}{2\frac{1}{2}} = \frac{2}{5}$. † Put oats in bags. ‡ 2 acres yield 75 bu. § $2\frac{1}{2}$ acres yield 75 bu.
‖ A horse walks $2\frac{1}{2}$ miles per hour. ¶ A locomotive moves 64 mi. in 2 hours.
** A locomotive moves 64 mi. in $2\frac{1}{2}$ hours.

Decimal Fractions.*

1. If a teacher earns $125 in 2.5 months,† in 1 tenth of a month she earns ——;‡ in 1 month she earns —— dollars.

2. Mr. Holland paid $70 for 3.5 tons of oil meal; for 1 tenth of a ton he paid —— dollars; for 1 ton he paid —— dollars.

3. A railroad train went 90 miles in 4.5 hours; this was at the rate of —— miles in 1 tenth of an hour, or —— miles an hour.

TO THE TEACHER.—By means of the foregoing problems and others similar to these, lead the pupil to see that $125 ÷ 2.5 means, *find* 10 *twenty-fifths of* $125; $70 ÷ by 3.5 means, *find* 10 *thirty-fifths of* $70; 90 miles ÷ 4.5, means, *find* 10 *forty-fifths of* 90 *miles*, etc.

$125 ÷ $2.5 This means, *find how many times* $2.5 *are contained in* $125. $2.5 are contained in $125 —— times. At $2.5 a barrel, how many barrels of apples can be bought for $125?

$125 ÷ 2 This means, *find* 1 *half of* $125. One half of $125 is —— dollars. If 2 acres of land are worth $125, what is 1 acre worth?

$125 ÷ 2.5 This means, *find* 10 *twenty-fifths of* $125. Ten twenty-fifths of $125 are —— dollars. If 2.5 acres of land are worth $125, what is 1 acre worth?

(1) Tell the meaning of each of the following; (2) solve it, and (3) make a problem that corresponds to it as suggested in the foregoing.

(a) 84 bu. ÷ 2.1 bu. = (b) 84 bu. ÷ 2 = (c) 84 bu. ÷ 2.1 =

(d) 81 mi. ÷ 2.7 mi. = (e) 81 mi. ÷ 2 = (f) 81 mi. ÷ 2.7 =

(g) If 3.4 acres of land yield 238 bu. of corn, how much does 1 tenth acre yield? (h) One acre? 238 bu. ÷ 3.4 =

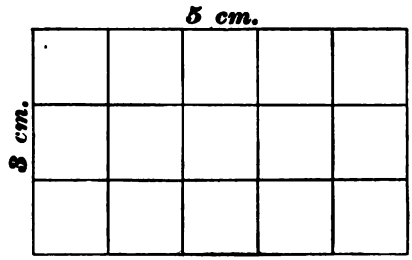
(i) If 4.3 tons of coal are worth $19.35, how much is 1 tenth of a ton worth? (j) One ton? $19.35 ÷ 4.3 =

* Note the relation between the several parts of this page and the corresponding parts of the preceding page.

† In many states the school month is 20 days; 2 days are 1 tenth of such a month; 4 days, 2 tenths, etc.

‡ If the pupil hesitates on this problem, require him to read 2.5 months, 25 *tenths months;* he will then readily see how he must find how much the teacher earns in *one* tenth of a month.

Denominate Numbers — Metric System.



Teach the abbreviation for meter, *m.*; for decimeter, *dm.*; for centimeter, *cm.*; for square centimeter, *scm.* or *qcm.*

1. An oblong 3 cm. by 5 cm. contains —— square centimeters. 4 cm. by 7 cm. =

2. The perimeter of a 3 cm. square is ——. The area of a 3 cm. square is ——.

3. A square decimeter is equal to —— square centimeters.

4. How many square decimeters in a square meter? *

5. A square meter equals how many square centimeters?

6. An oblong 4 dm. by 5 dm. contains —— square decimeters. It is equal to how many square centimeters?

7. An oblong 4 m. by 7 m. contains —— square meters. Is such an oblong larger or smaller than the floor of your school-room? The oblong is equal to how many square decimeters?

Review.

8. Fifty feet are —— rods and —— foot.
9. Fifty inches are —— feet and —— inches.
10. Fifty ounces are —— pounds and —— ounces.
11. Fifty feet are —— yards and —— feet.
12. Fifty quarts are —— gallons and —— quarts.
13. Fifty pecks are —— bushels and —— pecks.
14. Fifty quarts are —— pecks and —— quarts.
15. Fifty days are —— weeks and —— day.
16. Fifty months are —— years and —— months.

* Draw a square meter, a square decimeter, and a square centimeter, upon the blackboard and allow them to remain for several days. Require pupils to speak the names of these figures many times and to estimate in these denominations the areas of surfaces in the room. In this way they may obtain a working familiarity with the metric system.

Measurements — Lumber.

One piece of **2 × 4, 12,*** means 1 piece of lumber 2 inches thick, 4 inches wide, and 12 feet long. The abbreviation for pieces is *pcs.*

1. Think of a piece of 2 × 4, 12, cut into strips each 1 inch square and 12 feet long. There would be —— such strips.

2. One piece of 2 × 4, 12, contains —— feet of lumber.

(a) How many feet of lumber in 46 pcs. 2 × 4, 12?

3. One piece 4 × 4, 12, contains —— feet of lumber.

(b) How many pieces of lumber in 37 pcs. 4 × 4, 12?

4. One piece of 2 × 10, 12, contains —— feet of lumber.

(c) How many feet of lumber in 56 pcs. 2 × 10, 12?

(d) Find the amount of lumber in the following bill:

28 pcs. 2 × 8, 12.
15 pcs. 4 × 6, 12.
30 pcs. 2 × 4, 12.
25 12-foot boards, each 12 inches wide.

In measuring boards† of different widths a tape measure is sometimes used. The tape is so manipulated as to show *the sum of the widths of the boards.* If the first three boards are 15 in., 10 in., and 12 in. respectively, the width of the second board is taken on the tape from the 15 in. point to the 25 in. point; the width of the third board, from the 25 in. point to the 37 in. point. This shows the sum of the widths to be 37 inches, and, if they are 12-foot boards, they contain 37 feet of lumber.

(e) How many feet of lumber in a pile of 12-foot boards the sum of whose widths is 6 ft. 3 in.? (f) 7 ft. 5 in.? (g) 8 ft. 4 in.?

5. Mr. Jones wants 200 feet of 12-foot boards. He must be given boards the sum of whose widths is —— inches, or —— ft. —— inches.

(h) If I should place 1000 feet of 12-foot boards upon the ground, "side by side," they would cover a piece of ground 12 ft. wide and how many inches long? (i) How many feet long?

* To be read—*two by four, twelve.*
† The term *board* designates lumber 1 inch thick *unless the thickness is specified.* We may, however, speak of, *½-in. boards, 1¼ in. boards,* etc.

Ratio and Proportion.

1. The antecedent is 54, the consequent is 12; the ratio is ———.

2. The first term of a ratio is 32; the second term is 48; the ratio is ———.

What is the ratio of—

3. 36 pecks to 54 pecks?
4. 54 pecks to 36 pecks?
5. 1.4 to 2.1? *
6. 2.1 to 1.4?
7. 6 to 4.8? †
8. 4.8 to 6?
9. 6 bushels to 3 pecks? ‡
10. 3 pecks to 6 bushels?
11. .9 to 3 fifths? §
12. 3 fifths to .9?
13. 24 to $\frac{1}{2}$?
14. 6 men to 2 dollars? ‖

Find the ratio of—

(a) 6 bushels to 3 quarts.
(b) 5 yards to 6 inches.
(c) 3 meters to 6 decimeters.
(d) 127.5 to 2.5.
(e) 2.5 to 127.5.
(f) $\frac{5}{8}$ to $\frac{1}{10}$. ¶

15. The ratio of a 3-inch square to a 2-inch square is ———.

(g) What is the ratio of an 8-inch square to an oblong that is 3 in. by 4 in.?

16. The ratio of 1 rod to 1 acre is ———.

(h) What is the ratio of an oblong 6 rods by 4 rods to one acre?

17. The ratio of a 3-inch cube to a 2-inch cube is ———.

(i) What is the ratio of an 8-inch cube to a 2-inch cube?

18. The ratio of the surface of a 2-inch cube to the surface of a 1-inch cube is ———.

(j) What is the ratio of the surface of a 6-inch cube to the surface of a 3-in. cube?

* 14 tenths to 21 tenths.
† Change 6 to tenths. (6.0).
‡ Change 6 bu. to pecks.
§ Change 3 fifths to tenths.
‖ Why is this absurd?
¶ Change to equivalent fractions having a common denominator.

Percentage.

Case I. Find 16 % of $24.35.

Operation		*Explanation.*
$24.35		
.16		One % of $24.35 = $.2435.
$1.4610	(6 % of $24.35.	16 % of $24.35 = 16 times $.2435,
$2.435	(10 % of $24.35.)	or $3.896.
$3.8960	(16 % of $24.35.)	

Case II. 435 is 8 % of what?

Operation.		*Explanation.*
Let x = the number sought.		
8)435.	(8 % of x.)	Since 435 is 8 % of the number sought, 1 % is $\frac{1}{8}$ of 435, or 54.375.
54.375	(1 % of x.)	Since 54.375 is 1 %, 100 % is 100 times 54.375, or 5437.5
100		
5437.5	(x.)	

Case III. 185.76 is what % of 1376?

Operation.	*Explanation.*
1376 ÷ 100 = 13.76	
13.76)185.76(13.5 t's.	13.76 is 1 % of 1376.
13.76	185.76 is as many % of 1376 as 13.76 is contained times in 185.76. 13.76 is contained 13.5 times in 185.76; therefore 185.76 is 13½ % of 1376.
48.16	
41.28	
6.880	
6.880	

(a) Find 24½ % of 748. (748 is the base.)
(b) 72 is 5 % of what?* (72 is $\frac{5}{100}$ of the base.)
(c) 26.25 is what % of 375? (375 is the base.)
(d) Find 84 % of 296. (What is the base?)
(e) 96 is 24 % of what? (96 is what part of the base?)
(f) 39.20 is what % of 560? (What is the base?)

* Solve in two ways. (1) If 72 is 5 %, 1 % is ⅕ of 72, etc. (2) 5 % of a number is $\frac{5}{100}$ of the number, etc.

Percentage — Buying on Commission.

When an agent *buys goods* on commission, his pay for doing this is usually reckoned on the *buying price* as a *base*.

(a) An agent bought for me 10,000 bushels of corn at 28¢ a bushel. How much did he pay for the corn? (b) His commission for buying was ½ of 1 %.* How many dollars was his commission? (c) How much did the ten thousand bushels of corn cost me? (d) If the agent afterwards sold the same corn for me at 30¢ a bushel, how much did he receive for it? (e) If his commission for selling was ½ of 1 %,* how many dollars was his commission? (f) How many dollars did I receive for the corn? (g) What was my profit on the corn?

(h) At ½ of 1 % * an agent's commission for buying oats amounted to exactly $5. How many dollars' worth of oats did he buy? (i) If he paid 20¢ a bushel, how many bushels did he buy? (j) How many dollars did the oats cost the man for whom he bought them?

(k) At ½ of 1 % * an agent's commission for selling wheat was $6. How many dollars' worth of wheat did he sell? (l) If he sold it at 60¢ a bushel, how many bushels did he sell? (m) How many dollars did the man receive, for whom the wheat was sold?

(n) An agent's commission for buying $1200 worth of rye was $9. What was the rate of commission?† (o) If the agent paid 40¢ a bushel for the rye, how many bushels did he purchase? (p) How many dollars did the rye cost the man for whom he bought it?

(q) Buying price, $475.50; rate of commission 2 %. Commission?

(r) Commission, $32; rate of commission 5%. Buying price?

(s) Commission, $17.10; buying price $570. Rate of commission? ‡

* ½ of 1 % of what? ½ of 1 % is sometimes written, ½ %.
† 1 % of $1200 is $12. ‡ First find 1 % of $570.

Review.

$350 = 2 \times 5 \times 5 \times 7.$

1. 35 (5×7) is contained in 350 —— times.

2. 25 (5×5) is contained in 350 —— times.

3. 50 ($2 \times 5 \times 5$) is contained in 350 —— times.

4. 70 ($2 \times 5 \times 7$) is contained in 350 —— times.

5. I paid 45¢ for $2\frac{1}{2}$ lb. cheese; 1 half pound cost —— cents; 1 pound cost —— cents. $45¢ \div 2\frac{1}{2}$, means, find —— ——ths of 45¢; 1 fifth of 45¢ is —— cents; 2 fifths of 45¢ are —— cents.

6. I paid \$8.75 for $3\frac{1}{2}$ barrels of apples; 1 half barrel cost ——; 1 barrel cost ——. $\$8.75 \div 3\frac{1}{2}$, means, find —— ——ths of \$8.75; 1 seventh of \$8.75 is ——; 2 sevenths of \$8.75 are ——.

7. A railroad train went 63 miles in 2.1 hours; this was at the rate of —— miles in 1 tenth of an hour, or —— miles an hour. 63 miles ÷ 2.1, means, find —— —— of 63 miles; one 21st of 63 miles is —— miles; ten 21sts of 63 miles are ——.

8. Mr. A paid \$92 for 2.3 acres of land; for 1 tenth of an acre he paid —— dollars; for one acre he paid ——. \$92 ÷ 2.3, means, find —— ——of \$92; one 23rd of \$92 is ——; ten 23rds of \$92 are ——.

9. In an oblong 3 dm. by 4 dm. there are —— square decimeters.

(a) How many square centimeters in the oblong described in problem 9?

10. 5 pcs. 2×6, 12, contain —— feet of lumber.

(b) How many feet of lumber in 37 pcs. 4×4, 12?

(c) How many feet of lumber in 53 pcs. 2×8, 12?

(d) How many feet of lumber in 42 pcs. 2×10, 12?

(e) How many feet of lumber in a pile of 12-ft. boards, the sum of whose widths is 23 ft. 8 in.?

Miscellaneous Problems.

1. The 5th day of September, 1896, was Saturday; the 5th day of September, 1895, was ——.* The 13th day of September, 1895, was ——.

2. If 5 men can do a piece of work in 4 days, 1 man should be able to do it in —— days; 2 men should be able to do it in —— days.

(a) If 11 men can do a piece of work in 15 days, in how many days should 3 men do it? (b) 5 men?

3. In .12½† of a yard there are —— and —— —— inches.

(c) How many feet in .12½ of a mile?

(d) How many bushels in 2895 lb. of wheat?

(e) Find the cost of 2130 lb. wheat at 70¢ a bushel.

(f) A road 4 rd. wide must be how long to contain 1 acre?

(g) How many acres in 1 mile of 4-rod road?

(h) How many railroad ties will be required per mile if they are put 3 feet apart?

(i) How many days from April 20th to September 5th?

(j) If a train moves 186⅔ rods a minute, how many rods will it move in 1 hour? (k) How many miles?

(l) From $\frac{5}{6}$ of a gross take $\frac{2}{3}$ of a dozen.

(m) How many times as much lumber in a 12-foot board, 16 inches wide, as there is in 1 piece of 2 × 4, 12?

(n) How many square feet in a 4-foot walk surrounding a rectangular grass plot 16 feet by 20 feet?‡

(o) A lot 40 feet square is all in grass except 40 square feet. How many square feet of grass?

(p) How many square yards of pavement in a street that is 60 feet wide and 252 feet long?

* Remember that the year between these two dates includes February 29th, and is therefore 366 days, or 52 weeks and two days.

† 12½ hundredths—the same as 12½ %.

‡ Make a diagram of the grass plot and walk, on a scale of ⅛ inch to the foot.

Simple Numbers.

A common multiple of two or more numbers contains all the prime factors of each of them.

EXAMPLE.

$30 = 2 \times 3 \times 5.$
$40 = 2 \times 2 \times 2 \times 5.$
$50 = 2 \times 5 \times 5.$

A common multiple of 30, 40, and 50 must contain as factors three 2's, two 5's, and one 3.* The *least* common multiple can contain no other factors than these; therefore the l. c. m. of these numbers is $2 \times 2 \times 2 \times 5 \times 5 \times 3 = 600$.

1.

$24 = 2 \times 2 \times 2 \times 3.$
$36 = 2 \times 2 \times 3 \times 3.$
$40 = 2 \times 2 \times 2 \times 5.$

A common multiple of 24, 36, and 40 must contain as factors ——— 2's, ——— 3's, and ——— 5.

(a) What is the l. c. m. of 24, 36, and 40?

2.

$32 = 2 \times 2 \times 2 \times 2 \times 2.$
$42 = 2 \times 3 \times 7.$
$50 = 2 \times 5 \times 5.$

A common multiple of 32, 42, and 50 must contain as factors ——— 2's, ——— 3, ——— 5's, and ——— 7.

(b) What is the l. c. m. of 32, 42, and 50?

3.

$34 = 2 \times 17.$
$44 = 2 \times 2 \times 11.$
$54 = 2 \times 3 \times 3 \times 3.$

A common multiple of 34, 44, and 54 must contain as factors ——— 2's, ——— 3's, ——— 11, and ——— 17.

(c) What is the l. c. m. of 34, 44, and 54?

(d) What is the l. c. m. of 36, 46, and 56?

(e) What is the l. c. m. of 18, 24, and 40?

(f) What is the l. c. m. of 30, 36, and 48?

4.

$18 = 2 \times 3 \times 3.$
$20 = 2 \times 2 \times 5.$
$28 = 2 \times 2 \times 7.$

A common multiple of 18, 20, and 28 must contain as factors ——— 2's, ——— 3's, ——— 5, and ——— 7.

The l. c. m. is $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$.

(g) How many times is 18 contained in 1260? †

(h) How many times is 20 contained in 1260?

(i) How many times is 28 contained in 1260?

* One of the numbers (40) contains three 2's; one of the numbers (50) contains two 5's; one of the numbers (30) contains one 3.

† $2 \times 5 \times 7$ times. See page 161.

Common Fractions.

The **least common denominator** of two or more fractions is the least common multiple of their denominators.

Reduce to equivalent fractions having their l. c. d. Example—

$\frac{7}{30}, \frac{8}{40}$. $30 = 2 \times 3 \times 5$. $40 = 2 \times 2 \times 2 \times 5$. } L. c. m. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$. $\frac{7}{30} = \frac{28}{120}$. $\frac{8}{40} = \frac{24}{120}$.

(a) $\frac{3}{40}$ and $\frac{9}{50}$ (L. c. m. of 40 and 50?) $\frac{3}{40} =$ $\frac{9}{50} =$

(b) $\frac{7}{24}$ and $\frac{1}{36}$ (L. c. m. of 24 and 36?) $\frac{7}{24} =$ $\frac{1}{36} =$

(c) $\frac{7}{18}$ and $\frac{5}{24}$ (L. c. m. of 18 and 24?) $\frac{7}{18} =$ $\frac{5}{24} =$

(d) $\frac{11}{36}$ and $\frac{9}{48}$ (L. c. m. of 36 and 48?) $\frac{11}{36} =$ $\frac{9}{48} =$

(e) Find the sum of $\frac{1}{18}$ and $\frac{1}{24}$. (f) $\frac{1}{24} + \frac{1}{36} =$

(g) Find the sum of $\frac{1}{36}$ and $\frac{1}{48}$. (h) $\frac{1}{40} + \frac{1}{50} =$

(i) Find the difference of $\frac{1}{18}$ and $\frac{1}{30}$. (j) $\frac{1}{18} - \frac{1}{36} =$

(k) Find the difference of $\frac{1}{16}$ and $\frac{1}{24}$. (l) $\frac{1}{30} - \frac{1}{50} =$

1. Divide $\frac{5}{16}$ by $\frac{1}{24}$. This means, *find how many times* $\frac{1}{24}$ *is contained in* $\frac{5}{16}$. (L. c. m. of 16 and 24 is ———.) $\frac{5}{16} = \frac{15}{48}$. $\frac{1}{24} = \frac{2}{48}$. 2 forty-eighths are contained in 15 forty-eighths ——— times.* I divided $\frac{5}{16}$ of an acre of land into lots each containing $\frac{1}{24}$ of an acre; there were ——— lots.

(m) Divide $\frac{5}{32}$ by $\frac{1}{48}$. (n) Divide $\frac{11}{40}$ by $\frac{1}{16}$.

(o) Divide $\frac{9}{16}$ by $\frac{3}{40}$. (p) Divide $\frac{11}{32}$ by $\frac{5}{48}$.

2. If $2\frac{1}{2}$ tons of coal are worth $13.50 how much is $\frac{1}{2}$ of a ton worth? 1 ton? $13.50 ÷ $2\frac{1}{2}$, means what? †

3. If $2\frac{1}{2}$ acres of land yield 99 bu. of oats, $\frac{1}{2}$ of an acre yields ——— bushels. 1 acre yields ———. 99 bu. ÷ $2\frac{1}{2}$ means ————.†

* 2 apples are contained in 15 apples how many times? 2 inches in 15 inches, how many times? 2 forty-eighths in 15 forty-eighths how many times?

† See page 162.

Decimal Fractions.

1. A man paid \$46 for 2.3 acres of land; 1 tenth of an acre cost ——— dollars; 1 acre cost ——— dollars. \$46 ÷ 2.3, means, *find* 10 *twenty-thirds of* \$46; 1 twenty-third of \$46 is ——— dollars; 10 twenty-thirds of \$46 are ——— dollars.

EXAMPLE.

If 2.3 acres of land cost \$149.50, how much does 1 acre cost?

Operation No. 1.	Operation No. 2.
23)\$149.$\overset{v}{5}$0(\$6.5 × 10 = \$65.	2.3)\$149.5$\overset{v}{0}$(\$65.
138	138
11.5	11.5
11.5	11.5

Explanation.

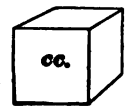
Dividing \$149.50 by 23 gives the cost of $\frac{1}{10}$ of an acre, and multiplying the quotient by 10, gives the cost of 1 acre. Observe that the same result may be obtained by taking *one twenty-third of a dividend* 10 *times as large*, that is, \$1495. See Operation No. 2.

Observe further, that the method of "pointing off" in division of decimals given on page 143 and illustrated on page 153, may be mechanically applied to problems like the foregoing. See rule printed in italics on page 143.

Operation.	*Explanation*
2.4)\$127.2$\overset{v}{0}$(\$53. 120 7.2 7.2	This means, find 10 twenty-fourths of \$127.20. For convenience, I find one twenty-fourth of ten times this dividend. One twenty-fourth of \$1272 is \$53.
(a) 2.6)\$109.2$\overset{v}{0}$	This means, find ten 26ths of \$109.20. I may find one 26th of —

(b)	(c)	(d)
4.3)\$279.50	3.4)\$255.	3.5)\$129.50

Denominate Numbers — Metric System.

A 1-centimeter cube, *or its equivalent*, is called 

a cubic centimeter. Abbreviation, cc.*

1. A 2-centimeter cube is equal to ——cc. †
2. A 3-centimeter cube is equal to ——cc.
3. A 5-centimeter cube is equal to ——cc.

(a) A 6-centimeter cube is equal to how many cc.?

(b) A 7-centimeter cube is equal to how many cc.?

(c) An 8-centimeter cube is equal to how many cc.?

To THE TEACHER.—Give pupils practice in estimating cubic centimeters of water. A small glass graduate showing volume in centimeters, is necessary for testing their estimates.

Review.

4. $5\frac{1}{2}$ yards are —— feet, or —— rod.
5. 100 yards are —— rods and —— feet. ‡
6. 200 yards are —— rods and —— feet.

(d) One mile is how many yards?

(e) 500 yards are how many feet more than one fourth of a mile?

(f) 12 rods are how many yards?

(g) 140 yd. are how many rods?

(h) 40 rods are how many yards?

(i) 20 rods are how many feet?

(j) 275 feet are how many inches?

(k) 540 inches are how many feet?

(l) 320 rods are how many feet?

(m) 65 yards are how many inches?

(n) 1760 yards are how many feet?

* Sometimes written ccm.

† Prove by a drawing; also by cutting cubes from turnip, potato, or apple.

‡ Remember that 100 feet = 6 rd. 1 ft.

Measurements — Lumber.

1. A 16-foot board, contains ——— ———* more lumber than a 12-ft. board of the same width.

2. An 18-ft. board contains ——— ——— more lumber than a 12-ft. board of the same width.

3. A 14-ft. board contains ——— ——— more lumber than a 12-foot board of the same width.

4. A 10-foot board contains ——— ——— less lumber than a 12-ft. board of the same width.

To measure a pile of 16-ft. boards:

Measure them as though they were 12*-ft. boards,*† *then add* 1 *third*‡ *of itself to the sum thus obtained.*

5. Make a rule for measuring 18-ft. boards; 20-ft. boards; 14-ft. boards; 10-ft. boards.

In each of the following columns of figures, the widths of ten boards are given. The length of the boards is indicated at the top of each column. How many feet of lumber in each lot?

(a) 12-ft. bds.	(b) 14-ft. bds.	(c) 16-ft. bds.	(d) 10-ft. bds.
11 in.	12 in.	14 in.	13 in.
13 in.	13 in.	9 in.	11 in.
15 in.	14 in.	12 in.	12 in.
12 in.	12 in.	10 in.	9 in.
16 in.	9 in.	13 in.	12 in.
10 in.	15 in.	12 in.	13 in.
13 in.	11 in.	11 in.	11 in.
15 in.	14 in.	12 in.	13 in.
14 in.	12 in.	10 in.	14 in.
13 in.	10 in.	13 in.	13 in.

* One third more lumber, because 16 is 1 third more than 12.

† See pp. 155 and 165. ‡ Take the number that is nearest to 1 third of the sum; if the sum is 34, add 11; if the sum is 35, add 12.

Ratio and Proportion.

Observe that *the terms of a fraction are the terms of a ratio.* The numerator is the antecedent, the denominator is the consequent, and *the fraction itself is the ratio.*

1. The ratio of 3 to 4 is $\frac{3}{4}$. Here the number 3 is the ——— of the ratio and it becomes the ——— of the fraction. The number 4 is the ——— of the ratio and it becomes the ——— of the fraction. The fraction itself is the ratio of 3 to 4; that is, it is the quotient of 3 divided by 4.

2. The ratio of 4 to 5 is ——— ———. The quotient of 4 divided by 5 is ——— ———.

3. The ratio of 5 to 6 is ——— ———. The quotient of 5 divided by 6 is ———.

4. The ratio of 1 to 2 is ——— ———. The quotient of 1 divided by 2 is ——— ———.

5. The ratio of 3 inches to 2 feet is ——— ———.

6. The ratio of 5 ounces to 2 pounds is ——— ———.

(a) What is the ratio of 15 inches to 5 yards?

(b) What is the ratio of 18 inches to 1 rod?

(c) What is the ratio of 8 sq. feet to 4 sq. yards?

(d) What is the ratio of 12 cu. feet to 4 cu. yards?

(e) What is the ratio of 18 cubic feet to 6 cubic yards?

(f) If 18 cubic feet of masonry cost $5.25, how much will 6 cubic yards of similar masonry cost?

(g) What is the ratio of 15 sq. feet to 5 sq. yards?

(h) If it cost $1.35 to finish 15 square feet of hard wood surface, how much will it cost to finish 5 square yards in a similar manner?

(i) If a 2-centimeter cube weighs 3 ounces, how many ounces will a 6-centimeter cube of the same material weigh?

Percentage.

Case I. What number is 7%* greater than 245?

Operation.

$$\begin{array}{r} 245 \\ .07 \\ \hline 17.15 \quad (7\% \text{ of } 245.) \\ 245 \\ \hline 262.15 \quad (107\% \text{ of } 245.) \end{array}$$

Explanation.

1% of 245 is 2.45.
7% of 245 is 7 times 2.45 or 17.15
245 + 17.15 = 262.15

Case II. 654 is 9%† greater than what number?

Let x = the number sought.
654 = 109% of x
109)654(6 = 1% of x
654 600 = x

Since 654 is 109% of x, 1% of x is $\frac{1}{109}$ of 654, or 6. Since 6 is 1% of x, 100 times 6, or 600, equals the whole of x.

Case III. 364 is what %‡ greater than 350?

Operation.

364
350
3.5)14.0(4 times
14.0

Explanation.

364 is 14 more than 350.
1% of 350 is 3.5.
14 is as many % of 350 as 3.5 is contained times in 14. It is contained 4 times: so 364 is 4% (of 350) more than 350.

(a) What number is 12 % greater than 340?§
(b) 756 is 8 % more than what number?
(c) 567 is what % greater than 540?
(d) What number is 19 % greater than 265?
(e) 750 is 25% more than what number?
(f) 270 is what % greater than 250?
(g) 824 is 3 % more than what number?
(h) 765 is what % greater than 750?
(i) 329.6 is what % greater than 320?

*7 % of what? †9 % of what? ‡ Per cent of what?

§ In problem (a), the *base* is 340. In problem (b), the *base* is the number sought. In problem (c), the *base* is 540. Tell the *base* in each of the problems from (d) to (i) inclusive.

Miscellaneous Applications of Percentage.

(a) In 1895, the average yield of corn per acre in a certain state, was 35 bushels; in 1896 it was 7 % greater. What was the average yield per acre in 1896?

(b) Mr. A raised 400 bushels of corn on 10 acres; Mr. B raised 500 bushels on 10 acres. How many per cent greater was Mr. B's yield than Mr. A's? * (c) How many per cent less was Mr. A's than Mr. B's? *

(d) A man sold a farm for $7020; this was 8 % † more than he paid for it. How much did the farm cost him? ‡

(e) Mr. Harris's income for the year 1896 was $1500; his expenses were $1680. His expenses were how many % more than his income? *

(f) The employes of a certain manufacturing establishment are to have their wages increased 15 %. The following figures are taken from the old schedule. What will each receive after the increase?

A, $80 per month. B, $18 per week. C, $1.80 per day.
D, $120 per month. E, $13 per week. F, $2.20 per day.

(g) The following figures are taken from the new schedule of wages of the employes of the manufactory referred to in problem (f). How much did each receive before the increase?

G, $69 per month. § H, $17.25 per week. I, $1.84 per day.
K, $115 per month. L, $13.80 per week. M, $2.76 per day.

(h) Mr. N, one of the employes of the manufactory referred to in the preceding problems, had been receiving $150 per month. As a reward for unusual faithfulness, he was given $180 per month. What was Mr. N's per cent of increase?

* What is the base? † 8 % of what? ‡ $7020 is how many % of what the farm cost him? § $69 is $\frac{23}{25}$ = $\frac{23}{25}$ of the wages named in the old schedule.

Review.

(a) What is the l. c. m. of 14, 21, and 28?

(b) What is the l. c. m. of 24, 40, and 48?

$14 = 2 \times 7.$ } Least common multiple,
$21 = 3 \times 7.$ } $2 \times 3 \times 7 = 42.$

(c) Find the sum of $\frac{5}{14}$ and $\frac{1}{21}$. (d) $\frac{11}{14} + \frac{8}{21} =$

(e) Find the difference of $\frac{5}{14}$ and $\frac{1}{21}$. (f) $\frac{11}{14} - \frac{8}{21} =$

(g) Divide $\frac{5}{14}$ by $\frac{1}{21}$. (h) $\frac{11}{14} \div \frac{8}{21} =$

(i) If $3\frac{1}{2}$ bbl. of flour are worth \$17.50, how much is $\frac{1}{2}$ of a barrel worth? (j) 1 barrel? \$17.50 ÷ $3\frac{1}{2}$, means what?

(k) 2.4)\$151.2^{v}0 *

(l) 2.5)\$130.0^{v}

(m) 2.3)\$106.9^{v}5

(n) If a man earns \$11.70 in 2.6 days, how much does he earn in one day? \$11.70 ÷ 2.6, means what?

1. A rectangular solid 7 cm. long, 5 cm. wide, and 2 cm. thick contains ——cc.

2. A rectangular solid 8 centimeters long, 2 centimeters wide, and 1 centimeter thick contains ——cc.

(o) How many feet of lumber in five 16-ft. boards whose widths are 13 in., 12 in., 15 in., 14 in., and 12 in.?

(p) How many feet of lumber in a pile of 18-ft. boards, the sum of whose widths is 12 feet 6 inches?

(q) What is the ratio of 1 bushel to 6 quarts? (r) A man paid 15¢ for 6 quarts of oats. At that rate how much would he pay for 1 bushel?

3. The ratio of 12 to 24 is —— ——; of 18 to 36,—— ——; of 24 to 48,—— ——; of $\frac{1}{6}$ to $\frac{1}{3}$,—— ——; of .3 to .6,—— ——.

* This means, find ten 24ths of \$151.20. Find *one* twenty-fourth of 10 times \$151.20. See pp. 163 and 173.

Miscellaneous Problems.

1. The first day of February, 1895, was Friday; the first day of February, 1896, was ——; the first day of March, 1895, was ——; the first day of March, 1896, was ——.

2. The area of a certain rectangular piece of land is 48 square rods; it is 3 rods wide and —— rods long.

(a) A certain rectangular field contains 6 acres and it is 20 rods wide. How long is the field?

3. A piece of lumber 6 inches square and 12 feet long contains —— feet.

(b) How many feet of lumber in a timber that is 12 inches square and 30 feet long?

(c) Find the value of a pile of wood 4 feet wide, 6 feet high, and 24 feet long, at $4.50 per cord.

(d) Find the cost of linoleum sufficient to cover a floor that is 24 ft. by 15 ft., at 65¢ a square yard.

(e) Telegraph poles are usually 165 feet * apart. If a train passes 16 poles every minute, what is its rate of speed per hour?

(f) How many garments, each containing $\frac{3}{4}$ of a yard, can be made from 15 yards of cloth? 15 yd. ÷ $\frac{3}{4}$ yd. =

(g) Peter buys chestnuts at $2.00 a bushel and sells them at 5¢ a pint. How much does he gain on 2 bushels?

4. What number is that to which if you add 1 half of itself the sum will be 12?

(h) What number is that to which if you add 1 half of itself the sum will be 435.6?

(i) How many cords of wood at $4.50 per cord must I give in exchange for 6 tons of coal at $7.50 a ton?

(j) Two thirds of 75 is what part of 450?

* 165 feet equals how many rods?

Simple Numbers.

1.

$16 = 2 \times 2 \times 2 \times 2.$
$30 = 2 \times 3 \times 5.$

The l. c. m. of 16 and 30 contains as factors how many 2's? 3's? 5's?

(a) What is the l. c. m. of 16 and 30?

(b) 16 is contained how many times in the l. c. m. of 16 and 30?

(c) 30 is contained how many times in the l. c. m. of 16 and 30?

2. The continued product of three numbers is 120. Two of the numbers are 4 and 5; the other number is ——.

(d) The continued product of three numbers in 2400; two of the numbers are 8 and 12. What is the other number?

3. $4 \times 5 \times ? = 40.$ (e) $6 \times 9 \times ? = 378.$

4. $3 \times 6 \times ? = 54.$ (f) $8 \times 5 \times ? = 480.$

5. $8 \times 2 \times ? = 64.$ (g) $12 \times 10 \times ? = 1800.$

6. 18 ($2 \times 3 \times 3$) is contained how many times in 216 ($2 \times 2 \times 2 \times 3 \times 3 \times 3$)? (h) Divide 216 by 18.

7. 35 (5×7) is contained how many times in 770 ($2 \times 5 \times 7 \times 11$)? (i) Divide 770 by 35.

(j) The salary of the president of the United States is $50,000 a year. What is his salary per month?

8. Henry and William together have 24 marbles, and Henry has twice as many as William. How many has each?

(k) A and B together have 7248 sheep, and A has twice as many sheep as B. How many has B? (l) How many has A?

(m) A grocer bought 4 tubs of butter, each containing 56 lb., at 22¢ a pound. In retailing it there was a shrinkage of 2 lb. on each tub. He sold it at 28¢ a lb. How much did he gain?

Common Fractions.

ADDITION OF FRACTIONS—EXAMPLE.

$\frac{7}{18} + \frac{18}{24}$. $18 = 2 \times 3 \times 3$. $24 = 2 \times 2 \times 2 \times 3$. } L. c. m. $= 2 \times 2 \times 2 \times 3 \times 3 = 72$.

$\frac{7}{18} = \frac{28}{72}$. $\frac{18}{24} = \frac{54}{72}$.

$\frac{28}{72} + \frac{54}{72} = \frac{82}{72}$.*

(a) Find the sum of $\frac{11}{16}$ and $\frac{7}{24}$. (b) $\frac{5}{12} + \frac{17}{36} =$

(c) Find the sum of $\frac{5}{36}$ and $\frac{7}{48}$. (d) $\frac{7}{16} + \frac{5}{24} =$

(e) Find the sum of $\frac{5}{16}$ and $\frac{11}{18}$. (f) $\frac{13}{16} + \frac{5}{36} =$

SUBTRACTION OF FRACTIONS—EXAMPLE.

$\frac{18}{24} - \frac{5}{36}$. $24 = 2 \times 2 \times 2 \times 3$. $36 = 2 \times 2 \times 3 \times 3$. } L. c. m. $= 2 \times 2 \times 2 \times 3 \times 3 = 72$.

$\frac{18}{24} = \frac{54}{72}$. $\frac{5}{36} = \frac{10}{72}$.

$\frac{54}{72} - \frac{10}{72} = \frac{44}{72}$.

(g) Find the difference of $\frac{9}{30}$ and $\frac{9}{50}$. (h) $\frac{7}{16} - \frac{3}{32} =$

(i) Find the difference of $\frac{11}{40}$ and $\frac{11}{60}$. (j) $\frac{13}{24} - \frac{7}{30} =$

(k) Find the difference of $\frac{17}{32}$ and $\frac{7}{30}$. (l) $\frac{7}{16} - \frac{5}{24} =$

DIVISION OF FRACTIONS—EXAMPLE.

$\frac{9}{16} \div \frac{5}{24}$. $16 = 2 \times 2 \times 2 \times 2$. $24 = 2 \times 2 \times 2 \times 3$. } L. c. m. $= 2 \times 2 \times 2 \times 2 \times 3 = 48$.

$\frac{9}{16} = \frac{27}{48}$. $\frac{5}{24} = \frac{10}{48}$.

$\frac{27}{48} \div \frac{10}{48} = 2\frac{7}{10}$.†

(m) Divide $\frac{13}{16}$ by $\frac{3}{32}$. (n) Divide $\frac{3}{40}$ by $\frac{3}{50}$.

(o) Divide $\frac{11}{32}$ by $\frac{5}{36}$. (p) Divide $\frac{13}{16}$ by $\frac{5}{24}$.

(q) Divide $\frac{11}{36}$ by $\frac{5}{48}$. (r) Divide $\frac{13}{20}$ by $\frac{7}{30}$.

1. If $2\frac{1}{8}$ yd. of ribbon are worth \$1.75, how much is $\frac{1}{8}$ of a yard worth? 1 yard? \$1.75 ÷ $2\frac{1}{8}$ means what?‡

* Lead the pupil to see that this work can be abridged if he can find the l. c. m. of the denominators by inspection, and that he can solve the problem correctly by using any multiple of the denominators.

† This means, find how many times 10 forty-eighths are contained in 27 forty-eighths.

‡ $\frac{1}{8} \div 2\frac{1}{8} = \frac{1}{17}$. Find $\frac{1}{17}$ of \$1.75.

Decimal Fractions.

1. A man paid \$75 for .3 of an acre of land; 1 tenth of an acre cost —— dollars; 1 acre at that rate would cost —— dollars. \$75 ÷ .3 means, *find* 10 *thirds of* \$75; * 1 third of \$75 is —— dollars; 10 thirds of \$75 = ——.

EXAMPLE.

If .3 of an acre of land cost \$74.25, how much would one acre cost at the same rate?

Operation No. 1.	Operation No. 2.
3)\$74.25	.3)\$74.25
\$24.75 × 10 = \$247.50	\$247.50

Dividing \$74.25 by 3 gives the cost of 1 tenth of an acre, and multiplying the quotient by 10 gives the cost of 1 acre.

Observe that the same result may be obtained by taking 1 third of *a dividend* 10 *times as large;* that is, \$742.5. See Operation No. 2.

Observe further that the method of "pointing off" in division of decimals given on page 143, and illustrated on page 153, may be mechanically applied to problems like the foregoing. See rule printed in italics on page 143.

(a) Divide \$106.75 by .7 (b) \$213.50 ÷ \$.7 =

(c) Divide \$47.25 by .9 (d) \$94.50 ÷ \$.9 =

(e) Divide \$157.50 by .5 (f) \$315. ÷ \$.5 =

(g) Divide \$91.20 by .8 (h) \$182.40 ÷ \$.2 =

(i) Divide \$91.80 by .6 (j) \$183.60 ÷ \$.6 =

(k) Divide \$34.02 by 2.7 (l) \$68.04 ÷ \$2.7 =

(m) Divide \$34.88 by 3.2 (n) \$69.76 ÷ \$3.2 =

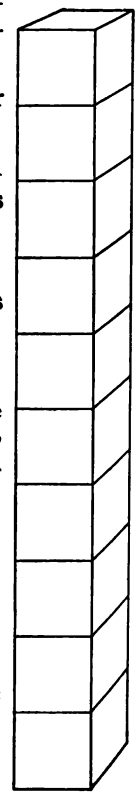
(o) Divide \$31.68 by 2.4 (p) \$63.36 ÷ \$2.4 =

(q) Divide \$17.25 by 2.5 (r) \$34.50 ÷ \$2.5 =

(s) If 2.4 tons of coal cost \$15.60, how much does 1 ton cost? (t) 5 tons? (u) 4.8 tons?

* Tell the pupil that the expression "10 thirds of \$75" means 10 times 1 third of \$75. See Book I., page 192.

Denominate Numbers.



1. A rectangular solid 1 decimeter long, 1 centimeter wide, and 1 centimeter thick contains ——cc.

2. A figure 1 decimeter long and 1 decimeter wide, contains —— square centimeters.

3. A rectangular solid 1 decimeter long, 1 decimeter wide, and 1 centimeter thick, contains ——cc.

4. A rectangular solid 1 decimeter long, 1 decimeter wide, and 1 decimeter thick, contains ——cc.

5. A **cubic decimeter** is equal to ——cc.

TO THE TEACHER.—This work cannot be well done unless pupils see a decimeter cube. If, in addition to this, each pupil can draw and *make* one, the work will certainly be efficient.

Review.

6. Two square feet are —— square inches.

(a) A 2-foot square contains how many sq. inches?

(b) 864 square inches are how many square feet?

(c) An 18-inch square contains how many square feet? *

(d) Eight square feet are how many square inches?

(e) Five square yards are how many square feet?

(f) 918 square feet are how many square yards?

(g) 1120 square rods are how many acres?

(h) Nine acres are how many square rods?

* Draw upon the blackboard an 18-inch (1½-ft.) square and show that it contains —— square feet. Draw upon paper a 3½-inch square and show that it contains —— square inches.

Measurements—Lumber.

Boards *less than* 1 *inch in thickness* are measured as though they were 1 inch thick; thus, a board 12 ft. long, 6 inches wide, and ½ inch thick, is said to contain 6 feet of lumber.*

In measuring boards that are *more than* 1 *inch in thickness*, the amount of material in the board must be regarded; thus, a 1¼-inch board is said to contain ¼ more lumber than a 1-inch board of the same length and width.

1. A 12-ft. 1¼-inch board, 9 inches wide, contains —— feet of lumber. †

2. A 12-ft. ⅝-inch board, 10 inches wide, contains —— feet of lumber.

3. A 12-ft. 1½-inch board, 14 inches wide, contains —— feet of lumber.

4. A 12-ft. ⅞-inch board, 11 inches wide, contains —— feet of lumber.

To measure a pile of 16-ft. 1¼-inch boards.

Measure them as though they were 12*-ft. boards; then, because they are* 16 *ft. long, add* 1 *third of itself to the sum thus obtained; then, because they are* 1¼ *inches thick, add* 1 *fourth of itself to the sum last obtained.* ‡

5. Make a rule for measuring 18 ft. 1¼-inch boards; 16 ft. 1½-inch boards; 14 ft. 1¼-inch boards; 16 ft. ⅝-inch boards.

In each of the following columns the widths of six boards are given. How many feet of lumber in each lot?

(a) 18-ft. 1¼-in. b'ds.	(b) 14-ft. ⅝-in. b'ds.	(c) 16-ft. 1½-in. b'ds.	(d) 12-ft. 1¼-in. b'ds.
12 in.	13 in.	15 in.	14 in.
13 in.	12 in.	12 in.	15 in.
9 in.	15 in.	9 in.	12 in.
10 in.	10 in.	10 in.	10 in.
12 in.	9 in.	13 in.	12 in.
14 in.	12 in.	12 in.	13 in.

* Of course a half-inch board contains only ½ as much material as an inch board of the same length and width; but custom in the lumber yards determines that no distinction shall be made on account of this difference when the thickness is less than 1 inch.

† One fourth of 9 is 2¼; omit the fraction. If, in such a division, the fraction is greater than ½, omit the fraction and add 1 to the integral part of the quotient; thus, 2¾ becomes 3. If the fraction is ½, add 1 to the integral part of the quotient, or not, as you please.

‡ Discard fractions as indicated in the foot-note immediately preceding this.

Ratio and Proportion.

The terms of a ratio (the antecedent and the consequent) are together called **a couplet.** Two couplets whose ratios are equal are called **a proportion.** The two couplets of a ratio are often written thus: 4 : 2 = 20 : 10, and should be read, *The ratio of* 4 *to* 2 *equals the ratio of* 20 *to* 10.*

1. The ratio of 8 to 2 equals the ratio of 20 to ——.†
2. The ratio of 9 to 3 equals the ratio of —— to 10.
3. The ratio of 2 to 10 equals the ratio of 10 to ——.
4. The ratio of $\frac{1}{4}$ to $\frac{1}{8}$ equals the ratio of 9 to ——.
5. The ratio of .8 to .4 equals the ratio of 15 to ——.
6. The ratio of 7 to $\frac{1}{2}$ equals the ratio of —— to 2.
7. The ratio of 1.5 to .5 equals the ratio of 36 to ——.
8. If 10 books are worth 7 dollars, 30 books are worth how many dollars?

10 books : 30 books = 7 dollars : —— dollars. ‡

9. If 1 bu. 4 qts. of oats are sufficient for 6 horses, 18 horses would require how many oats?

6 horses : 18 horses = 1 bu. 4 qt. oats : —— oats.

(a) If 2 tons of coal cost $11.60, how much will 8 tons cost at the same rate?

2 tons : 8 tons = $11.60 : $——.

(b) If 6 horses consume 23640 lb. of hay in a year, how much will 9 horses consume?

6 horses : 9 horses = 23640 lb. : —— lb.

(c) If $2\frac{1}{2}$ tons of hay cost $18.50, how much will 10 tons cost at the same rate?

(d) If a man can dig $27\frac{1}{2}$ rods of ditch in 3 days, how many rods can he dig in 2 weeks?

(e) If a woman can earn $9.45 in 5 days, how much can she earn in the month of January, provided there are 5 Sundays and 1 holiday in the month?

* A proportion is often written thus: 4 : 2 : : 20 : 10, and read, 4 *is to* 2 *as* 20 *is to* 10. Think of the ratio sign (:) as a division sign (÷) with the horizontal line omitted.

† Observe that 8 is 4 times 2; so 20 is 4 times the number sought.

‡ To be read: *The ratio of* 10 *books to* 30 *books equals the ratio of* 7 *dollars to* —— *dollars.*

Percentage.

1. Two hundred per cent of 36 is ——.*
2. Thirty-six is 200 % of ——.
3. Three hundred per cent of 23 is ——.
4. Forty-eight is 300 % of ——.
5. One hundred fifty per cent of 24 is ——.†
6. Twenty-four is 150 % of what?
7. One hundred twenty-five per cent of 48 is ——.‡
8. Thirty-five is 125 % of ——.
9. Two hundred fifty per cent of 40 is ——.§
10. Sixty is 250 % of ——.

(a) Find 200 % of $475.34 (b) 300 % of 97½ =

(c) $345 is 200 % of what? (d) 76 is 300 % of ——.

(e) Find 150 % of $324.60 (f) 150 % of 84½ =

(g) $465 is 150 % of what? (h) 87 is 150 % of ——.

(i) Find 125 % of $346.20 (j) 125 % of 86 =

(k) $245 is 125 % of what? (l) 75 is 125 % of ——.

(m) A physician's charges during the month of May amounted to $436; of this sum he succeeded in collecting $370.60. What per cent of his charges did he collect?‖

(n) A teacher earned $1500 a year and his expenses were $1140. What per cent of his salary did he save?¶

(o) A beef animal alive weighed 1450 lb.; dressed, it weighed only 783 lb. What per cent did it "dress away"?

(p) A farmer raised 480 bushels of potatoes; of these 120 bu. were small and therefore not marketable. What per cent of his crop was marketable?

* 200 % = ²⁰⁰/₁₀₀ = 2, or 2; that is, 200 % of a number means, 200 times 1 hundredth of the number, or *twice the number*.

† 150 % = ¹⁵⁰/₁₀₀ = 1½. ‡ 125 % = ¹²⁵/₁₀₀ = —. § 250 % = ²⁵⁰/₁₀₀ = 2½.

‖ If he had collected only 1 % of his charges, he would have collected $4.36.

¶ If he had saved $15, he would have saved 1 % of his salary.

Percentage—Loss and Gain.

The per cent of *loss or gain* by the purchase and sale of an article is reckoned *on the cost* as a *base*.

1. Bought for 20¢; sold for 25¢; gained ——%.*
2. Bought for 25¢; sold for 20¢; lost ——%.†
3. Bought for 15¢; sold for 20¢; gained ——%.
4. Bought for 20¢; sold for 15¢; lost ——%.
5. Bought for 25¢; sold for 30¢; gained ——%.
6. Bought for 30¢; sold for 25¢; lost ——%.

(a) Bought for $250; sold for $265. What was the gain per cent?

(b) Bought for $350; sold for $322. What was the loss per cent?

7. Cost, $60; gain, 10 %; selling price, ——.
8. Cost, $60; loss, 10 %; selling price, ——.
9. Cost, $50; gain, 12½ %; selling price, ——.
10. Cost, $50; loss, 12½ %; selling price, ——.
11. Cost, $640; gain, 5 %; selling price, ——.
12. Cost, $640; loss 5 %; selling price, ——.

(c) Cost $846; gain 18½ %. What was the selling price?

(d) Cost $534; loss 15 %. What was the selling price?

13. Selling price, $60; gain, 20 %; cost, ——.‡
14. Selling price, $60; loss, 20 %; cost, ——.§

(e) Selling price, $120; gain, 25 %. Find the cost.

(f) Selling price, $120; loss, 25 %. Find the cost.

(g) Selling price, $284; gain, 33⅓ %. Find the cost.

(h) Selling price, $364; loss, 33⅓ %. Find the cost.

(i) Selling price, $371; gain, $21. Find % of gain.

(j) Selling price, $230; loss, $20. Find % of loss.

* 20¢ is the base. † 25¢ is the base. ‡ $60 is $\frac{4}{5}$ = $\frac{1}{5}$ of the cost. § $60 is $\frac{4}{10}$ = $\frac{1}{5}$ of the cost.

Review.

(a) Find the l. c. m. of 24 and 40.

(b) Find the sum of $\frac{5}{24}$ and $\frac{3}{40}$. (c) $\frac{5}{24} - \frac{3}{40} =$

(d) Find the quotient of $\frac{5}{24}$ divided by $\frac{3}{40}$.

(e) If .4 of a ton of coal cost \$3.20 how much will 1 ton cost? \$3.20 ÷ .4, means ———.

(f) I paid \$3.20 for coffee at \$.4 a pound, how many pounds did I buy? \$3.20 ÷ \$.4, means ———.

(g) A 2-foot cube contains how many cubic inches?

(h) A 2-decimeter cube contains how many cc.?

(i) How many feet of lumber in 5, 16-foot, $\frac{3}{4}$-inch boards, whose widths are 11 in., 13 in., 14 in., 12 in., and 15 in.?

(j) How many feet of lumber in 8, 18-ft. boards* each 10 inches wide?

(k) If 5 acres of land are worth \$136.50, $12\frac{1}{2}$ acres are worth how much? †

5 acres : $12\frac{1}{2}$ acres = \$136.50 : \$———.

(l) If the assessment for a street pavement, on a lot with 40 feet of frontage, is \$216.20, what should the assessment be on a lot with 50 feet of frontage?

40 feet : 50 feet = \$216.20 : \$ ———.

(m) Bought for \$450; sold for \$540. Per cent of gain?

(n) Cost, \$242; gain, 15 %. Selling price?

(o) Cost, \$364; loss, 15 %. Selling price?

(p) Selling price, \$240; gain, 25 %. Find the cost. ‡

(q) Selling price, \$240; loss, 25 %. Find the cost. ‡

(r) Buying price, \$140; gain, 25 %. Selling price?

* See foot-note, page 165.

† (1) Solve by finding the cost of 1 acre; then the cost of $12\frac{1}{2}$ acres. (2) Solve by finding the ratio of $12\frac{1}{2}$ acres to 5 acres.

‡ \$240 is how many fourths of the cost?

Miscellaneous Problems.

(a) How many cows at $35 each should be given in exchange for 5 horses at $105 each?

(b) Mr. Wheeler sold his farm of 64 acres for $5440. What was the price per acre?

1. The sum of two numbers is $46\frac{1}{2}$; one of the numbers is 22; the other number is ——.

(c) The sum of two fractions is $\frac{11}{16}$; one of the fractions is $\frac{5}{8}$. What is the other fraction?

(d) $16\frac{3}{4}$ lb. of butter at 24¢ a pound will pay for how much sugar at 5¢ a pound?

(e) The circumference of the wheel of a wagon is 12 feet. How many times does it revolve in going 1 mile?

(f) If 15 tons of coal cost $31.50, what is the price per ton? $31.50 ÷ 15, means ————.

(g) If $\frac{5}{8}$ of a ton of coal cost $4.75, what is the price per ton? $4.75 ÷ $\frac{5}{8}$, means ————.

(h) A rectangular field is 15 rods 5 ft. long, and 12 rd. $6\frac{1}{4}$ ft. wide. What is the distance around it?

(i) Two boys start from the same place and ride on their bicycles in opposite directions. One rides at the rate of 7 miles an hour and the other at the rate of $8\frac{1}{2}$ miles an hour. How far apart will they be in 4 hours?

(j) Two boys start from the same place and ride on their bicycles in the same direction. One rides at the rate of $7\frac{1}{4}$ miles an hour and the other at the rate of $8\frac{3}{4}$ miles an hour. How far apart will they be in 4 hours?

(k) B spent $\frac{3}{4}$ of his money and had $5.65 remaining. How much did he spend?

Simple Numbers.

Two integral numbers are said to be *prime to each other* when they have no prime factor in common; thus, 12 (2 × 2 × 3) and 25 (5 × 5) are prime to each other. When two numbers are prime to each other, their least common multiple is their product.

1. The l. c. m. of 7 and 12 is ——.
2. The l. c. m. of 5 and 13 is ——.

(a) What is the l. c. m. of 12 and 25?

(b) What is the l. c. m. of 16 and 35?

Find the l. c. m. of the following:

(c) 17 and 21.	(d) 36 and 60.	(e) 35, 45, and 15.
(f) 24 and 50.	(g) 18 and 13.	(h) 32, 48, and 64.
(i) 13 and 40	(j) 25 and 40.	(k) 24, 26, and 60.

(l) B paid $7250 for a farm; he built upon it a barn that cost $1340 and a corn crib for $156; he laid tile at a cost of $375 and paid $246 for repairing the fences; he then sold the farm for $10,000. How much did he gain?

(m) Mr. A bought 650 acres of land at $15 per acre; he made a cash payment of $4500. How much remains unpaid?

(n) A dealer bought 25 cows at $35 each and 28 sheep at $4 each; he sold the cows for $1000 and the sheep for $125. How much did he gain?

(o) A street car line carried 2140 passengers Monday; 1896, Tuesday; 2324, Wednesday; 1954, Thursday; 2642, Friday; 2427, Saturday, and 3123, Sunday. What was the average number carried daily? (p) If each fare was 5¢, how much did the company earn per day on an average?

(q) Willis averages 8 miles an hour on his bicycle. If he rides 7 hours a day, how long will it take him to go from Chicago to Denver, a distance of 1000 miles?

Common Fractions.

(a) Find the l. c. m. of 11 and 15.*

(b) Find the sum of $\frac{7}{11}$ and $\frac{3}{15}$. (c) $\frac{7}{11} - \frac{3}{15}$ =

(d) Find the quotient of $\frac{7}{11}$ divided by $\frac{3}{15}$.

(e) Find the sum of $\frac{5}{7}$ and $\frac{3}{16}$. (f) $\frac{5}{7} - \frac{3}{16}$ =

(g) Find the quotient of $\frac{5}{7}$ divided by $\frac{3}{16}$.

(h) Find the sum of $\frac{4}{9}$ and $\frac{3}{11}$. (i) $\frac{4}{9} - \frac{3}{11}$ =

(j) Find the quotient of $\frac{4}{9}$ divided by $\frac{3}{11}$.

(k) At $\frac{3}{4}$ of a dollar a basket, how many baskets of peaches can be bought for $\$9\frac{3}{4}$?

$\$9\frac{3}{4} \div \$\frac{3}{4}$, means, ———.

(l) If $\frac{3}{4}$ of a ton of hay costs $\$9\frac{3}{4}$, how much does one ton cost at the same rate?

$\$9\frac{3}{4} \div \frac{3}{4}$, means, ———. †

1. How much will $3\frac{1}{2}$ yd. cost at $2\frac{1}{8}$ dollars a yard? ($\$2\frac{1}{8} \times 3\frac{1}{2}$, means, 3 *times* $\$2\frac{1}{8}$ *plus* $\frac{1}{2}$ *of* $\$2\frac{1}{8}$.)

(m) How much will $6\frac{1}{2}$ dozen pairs of shoes cost at $\$25\frac{1}{2}$ a dozen? $\$25\frac{1}{2} \times 6\frac{1}{2}$, means ———.

(n) Mr. C's farm contains 148 acres and is divided into three fields; in field No. 1, there are $56\frac{7}{8}$ acres; in field No. 2, there are $45\frac{3}{4}$ acres. How many acres in field No. 3?

2. If a man can do a piece of work in $4\frac{1}{2}$ weeks, what part of the work can he do in 1 half of a week? In 1 week?

3. $\$90 \div 4$ = $\$90 \div 4\frac{1}{2}$ = $\$90 \div 2\frac{1}{2}$ =

* These numbers have no prime factor in common; therefore their least common multiple is their product.

† To THE TEACHER.—This means, find $\frac{3}{4}$ of $\$9\frac{3}{4}$.—One third of $\$9\frac{3}{4}$ = the cost of 1 fourth of a ton; four thirds of $\$9\frac{3}{4}$ = the cost of a whole ton. Later the pupil may learn to solve problem (l) by a thought process similar to that employed in the solution of problem (k); thus, he may think—*If the coal had been* 1 *dollar per ton,* $\frac{3}{4}$ *of a ton would have cost* $\frac{3}{4}$ *of a dollar. But* $\frac{3}{4}$ *of a ton cost* $\$9\frac{3}{4}$*; therefore the price per ton must have been as many dollars as* $\$\frac{3}{4}$ *is contained times in* $\$9\frac{3}{4}$. *It is contained* 13 *times; so the price was* $13 *per ton.*

Decimal Fractions.

1. C paid $40 for .08 of an acre of land; 1 hundredth of an acre cost ——— dollars; 1 acre at the same rate would cost ——— dollars. $40 ÷ .08 means, *find* 100 *eighths of* $40; * 1 eighth of $40 is ——— dollars; 100 eighths of $40 = ———.

EXAMPLE.

If .08 of an acre of land cost $36.80, how much would one acre of land cost at the same rate?

Operation No. 1.	Operation No. 2.
8)$36.80	.08)$36.80
$4.60 × 100 = $460.	$460.

Dividing $36.80 by 8 gives the cost of 1 hundredth of an acre, and multiplying the quotient by 100 gives the cost of 1 acre.

Observe that the same result may be obtained by taking 1 eighth of a dividend 100 times as large; that is, $3680. See Operation No. 2.

Observe further that the method of "pointing off" in division of decimals given on page 143, and illustrated on page 153, may be mechanically applied to problems like the foregoing. See rule printed in italics on page 143.

(a) Divide $104.34 by .02 † (b) $104.34 ÷ $.02 = †

(c) Divide $104 by .02 (d) $208 ÷ $.02 =

(e) Divide $75.60 by .08 (f) $75.60 ÷ $.08 =

(g) Divide $24.50 by .05 (h) $49.15 ÷ $.05 =

(i) Divide $37.50 by .25 (j) $75. ÷ $.25 =

(k) Divide $12.25 by .35 (l) $24.50 ÷ $.35 =

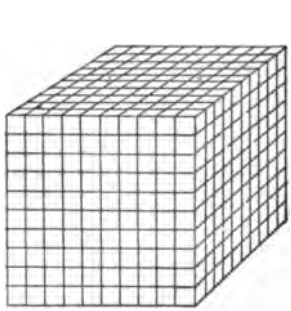
(m) Divide $67.50 by 1.25 (n) $67.50 ÷ $1.25 =

(o) Divide $83.70 by 1.35 (p) $83.70 ÷ $1.35 =

(q) If 1.35 tons of flour cost $48.60, how much does 1 ton cost? (r) 8 tons?

* See foot-note, page 183. † Tell the meaning; tell a suggested number story. Require pupils to "point off" (1) by thinking the meaning of the problem, and (2) by applying the rule given on page 143. Lead the pupil to see that in problems (a) and (c) he is to find $\frac{1}{100}$ of the dividend; in problem (e), $\frac{1}{100}$; in problem (g), $\frac{1}{100}$, etc.

Denominate Numbers—Metric System.



In the measurements of liquids, as molasses, oil, vinegar, milk, a cubic decimeter is called a *liter*.*

1. One liter is equal to ——— cubic centimeters.
2. One hundred cc. are ——— ———th of a liter.
3. Five hundred cc. are ——— ——— of a liter.
4. Ten cubic centimeters are ——— ———th of a liter.
5. A cubic meter is equal to how many liters?
6. A rectangular tank 5 decimeters long, 4 decimeters wide, and 3 decimeters high, will contain how many liters of water?

(a) In a tank 6.5 decimeters long and 4 decimeters wide, there is water 2.5 decimeters deep. How many liters of water in the tank? (b) How many cc. of water in the tank?

Review.

(c) Two cubic feet are how many cubic inches?

(d) A 2-foot cube contains how many cubic inches?

(e) 5184 cubic inches are how many cubic feet?

(f) 1296 square inches are how many square feet?

7. Two square yards are ——— square feet.
8. A 2-yard square contains ——— square feet.

(g) Two cubic yards are how many cubic feet?

* A liter is a little larger than a "liquid quart." 1 liter = 61 cubic inches.
1 "liquid quart" = 57¾ cubic inches. 1 "dry quart" = 67¼ cubic inches

Measurements—Lumber.

The term **stock boards**, unless otherwise specified, has reference to boards that are 1 foot wide. The term **fencing**, or **fence boards**, unless otherwise specified, has reference to boards 6 in. wide.

1. A 12-ft. stock board contains —— feet of lumber.
2. A 14-ft. stock board contains —— feet of lumber.
3. A 16-ft. stock board contains —— feet of lumber.
4. A 12-ft. fence board contains —— feet of lumber.
5. A 14-ft. fence board contains —— feet of lumber.
6. An 18-ft. fence board contains —— feet of lumber.

(a) How much lumber in 38, 14-ft. stock boards?

(b) How much lumber in 65, 16-ft. fence boards?

(c) Mr. Randall ordered 1000 feet of 16-ft. fencing. How many boards should he receive?

(d) Mr. Hardy ordered 2970 feet of 18-ft. stock boards. How many boards should he receive?

7. At \$1 per M.,* 2000 feet of lumber cost —— dollars.

3000 ft. cost ——; 4000 ft. cost ——; 2500 ft. cost ——;
2560 ft. cost ——; 2565 ft. cost ——; 375 ft. cost ——;
1460 ft. cost ——; 75 ft. cost ——; 8 ft. cost ——.

Observe that at \$12 per M., lumber will cost 12 times as much as at \$1 per M.; at \$15 per M., 15 times as much; at \$14.50 per M., 14½ times as much, etc.

Find the cost of the following lots of lumber:

(e) 2500 ft. @ \$12 per M. (f) 3400 ft. @ \$15 per M.
(g) 3240 ft. @ \$13 per M. (h) 575 ft. @ \$16 per M.
(i) 140 ft. @ \$15.50 per M. (j) 1250 ft. @ \$24 per M.
(k) Find the cost of 35, 14-ft. stock boards at \$15 per M.
(l) Find the cost of 36, 16-ft. fence boards at \$12 per M.

* M. is the abbreviation for the Latin word *mille*, which means *thousand*.

Ratio and Proportion.

1. The ratio of 2 to 5 equals the ratio of 8 to ——.
2. The ratio of 7 to $3\frac{1}{2}$ equals the ratio of 15 to ——.
3. 12 yards : 30 yards = 11 dollars : —— dollars.*
4. 8 horses : 10 horses = 20 bushels : —— bushels.†
5. 12 hours : 8 hours = 30 miles : —— miles.
6. 15 men : 12 men = —— dollars : 40 dollars.
7. 8 quarts : 2 bushels = 2 horses : —— horses.‡
8. 3 inches : 1 foot = —— rods : 28 rods.
9. 50 cents : 3 dollars = 9 pounds : —— pounds.§

(a) If a man earns $35 in 6 days, how much does he earn in 9 days?

6 days : 9 days = 35 dollars : —— dollars.

(b) If it costs $2.50 to paint a floor that is 10 feet by 12 feet, how much should it cost to paint a floor that is 12 feet by 15 feet?

120 sq. ft. : 180 sq. ft. = $2.50 : ——.

(c) If a man agrees to pay $3.50 a week for his board, how much should he pay for the month of April?

7 days : 30 days = $3.50 : ——.

(d) If a man agrees to pay $27.50 a month for board for himself and wife, how much should he pay for board from the morning of April 1st to the morning of April 16th?

(e) What is the ratio of a 4-in. square to a 6-in. square?

(f) Of a 4-in. cube to a 6-in. cube?

* To be read as follows: *The ratio of* 12 *yards to* 30 *yards equals the ratio of* 11 *dollars to* —— *dollars.*

† 8 is $\frac{4}{5}$ of 10; 20 is $\frac{4}{5}$ of ——.

‡ Observe that the numbers of each couplet must be of the same kind or they must be such as can be reduced to the same kind.

§ Change each of the proportions from 3 to 9 inclusive, into a number story thus: Problem 3. *If* 12 *yards of cloth cost* $11, 30 *yards will cost* —— *dollars.*

Percentage.

Case I. What number is 7 % * less than 324?

Operation.	*Explanation.*
3)24	1 % of 324 is 3.24.
.07	7 % of 324 is 7 times 3.24 or 22.68.
22.68	324 less 22.68 = Ans.

324 − 22.68 = 301.32.

Case II. $744 is 7 % * less than what number?

Operation No. 1.

Let x = the number of dollars sought.

Then $744 = 93 % of x.

93)$744(8 (1 % of x)
744
100
$800 ($x$)

Operation No. 2.

.93)$744.00(800.
744
00

Dividing $744 by 93 gives 1 hundredth of x, and multiplying the quotient by 100 gives 100 hundredths of x.

Observe that the same result may be obtained by taking 1 ninety-third of a dividend 100 times as large; that is, $74400. See operation No. 2.

Observe further, that dividing $744 by .93, means, finding 100 ninety-thirds of $744, which is equal to 1 ninety-third of $74400.

Case III. $595.20 is what % less than $640?

Operation.	*Explanation.*
$640	$595.20 is $44.80 less than $640.
$595.20	1 % of $640 is $6.40.
$6.40)$44.80(7 times.	$44.80 is as many % of $640 as $6.40 is contained times in $44.80.
44.80	It is contained 7 times; so $595.20 is 7 % less than $640.

(a) What number is 13 % less than 750?

(b) What number is 13 % more than 750?

(c) 478.40 is 8 % less than what number? †

(d) 561.60 is 8 % more than what number? ‡

(e) $770.40 is what % more than $720?

* 7 % of what?

† 478.40 is 92 hundredths of what number? .92)478.40

‡ 561.60 is 108 hundredths of what number? 1.08)561.60

Percentage—Loss and Gain.*

The following figures give cost prices. Tell how the goods must be marked if they are to be sold at a profit of 20 %.

1. Shoes @ $36 per dozen pairs.	$24 per dozen.
2. Buttons @ 60¢ a gross.	$1.20 per gross.
3. Gloves @ $9 per dozen pairs.	$6 per dozen.
4. Nails @ $2.50 per keg.	$1.75 per keg.

(a) A dealer bought hats at $1.75 each; for how much must he sell them to gain 20 per cent?

(b) If shoes cost $30 per dozen pairs, for how much must they be sold per pair in order to gain 25 %? (c) 30 %?

(d) If I buy apples at $2.75 per barrel and sell them at $3.25, will I gain more or less than 20%?

(e) Mr. Harvey bought a horse for $116, and sold it for $130.50. What was his per cent of gain?

(f) Mr. Gordon sold a farm for $6480 and by so doing made a gain of 8 % on what the farm cost him. What was the cost of the farm? †

(g) Mr. Judd sold a farm for $5580 and by so doing lost 7 % on the cost of the farm. How much did the farm cost him?

(h) If 1 sixth of the selling price is profit, what is the per cent of profit? ‡

(i) If 1 fifth of the selling price is profit, what is the per per cent of profit?

(j) If my profit is 33⅓ per cent, what part of the selling price is profit? §

* Review page 188. † 6480 is 108 % of what?

‡ If 1 sixth of the selling price is profit, 5 sixths of the selling price equals the cost. Then for every 5 dollars of cost there is 1 dollar of profit.

§ Problems (h), (i), (j) should be solved without the aid of a pencil. Give many more problems similar to these.

Review.

1. The l. c. m. of 5 and 17 is ——. The l. c. m. of 10 and 16 is ——.

(a) Find the l. c. m. of 15 and 17. (b) Of 15 and 25.

(c) Find the sum of $\frac{11}{15}$ and $\frac{8}{17}$. (d) $\frac{11}{15} + \frac{8}{17} =$.

2. If .06 of a ton of coal costs \$.30, 1 ton costs ——.

\$.30 ÷ .06, means ——.* \$.30 ÷ .06 =

\$.30 ÷ \$.06, means ——. \$.30 ÷ \$.06 =

(e) Divide \$45.20 by .08.† (f) \$45.20 ÷ \$.08 = ‡

3. A rectangular tank 2 decimeters by 3 decimeters by 4 decimeters contains —— liters.

(g) A rectangular tank 1 meter long, 1 meter wide, and 2 decimeters deep contains how many liters?

(h) Does the tank described in problem (g) contain more or less than 200 "liquid quarts?" §

4. The ratio of 4 to 9 equals the ratio of 1 to ——.

(i) If a man can earn \$134 in 4 weeks, how much can he earn in 9 weeks at the same rate?

(j) If a man can earn \$256.50 in 9 weeks, how much can he earn in 4 weeks?

5. \$329 is how many dollars less than \$350?

(k) \$329 is how many per cent less than \$350?

6. \$486 is how many dollars more than \$450?

(l) \$486 is how many per cent more than \$450?

(m) Bought goods for \$320 and sold them for \$368. I gained what per cent?

(n) Bought goods for \$320 and sold them for \$304. I lost what per cent?

* See problem 1, page 193.

† If .08 of an acre of land cost \$45.20, how much will 1 acre cost at the same rate?

‡ If 1 pound costs \$.08, how many pounds can be bought for \$45.20?

§ See foot-note, page 194.

Miscellaneous Problems.

(a) Bricks are usually 8 in. long, 4 in. wide, and 2 in. thick. How many bricks in a cubic foot of bricks? (b) In a cubic yard? (c) In a pile 8 ft. by 4 ft. by 4 ft.?

(d) If bricks are laid on their largest face, how many bricks will be required for a square foot of brick walk? (e) For a square yard? (f) For a walk 20 ft. long and 5 ft. wide?

(g) A man sold 54 lb. of grapes in ¾-lb. packages. How many packages did he sell?

(h) A man sold 350 lb. sugar at the rate of 16 lb. for one dollar. If 4 % of the sugar was lost to the seller in "down weights," * waste, and shrinkage by drying, how much did he receive for the sugar?

(i) Abraham Lincoln was born Feb. 12th, 1809; at the time of his death he was 56 years 2 months 3 days old. What was the date of his death?

(j) 45⅛ yards of carpet is to be cut into 7 equal pieces What will be the length of each piece?

(k) Find the cost:

27½ lb. butter at 17¢ a pound.
100 eggs at 15¢ a dozen.
1½ bu. hickory nuts at 6¢ a quart.
5 gal. kerosene at 13¢ a gallon.

(l) If a man can row a boat at the rate of 1 mile in 14 minutes 30 seconds, in how long a time can he row 12 miles? (m) 12½ miles?

(n) 48 minutes is what part of an 8-hour day?

* "Down weight" means the over-weight that is often given to the purchaser, which makes the scale containing the article weighed, go *down*.

Simple Numbers.

1. $5 \times 3 \times ? = 30$.
2. $8 \times 5 \times ? = 120$.
3. $4 \times 5 \times ? = 240$.

(a) $16 \times 4 \times ? = 256$.

(b) $15 \times 5 \times ? = 525$.

(c) $36 \times 2 \times ? = 1728$.

(d) The product of three factors is 512; two of the factors are 4 and 2. What is the third factor?

4. The product of two numbers is 240; one of the numbers is 12; the other number is ——.

(e) The product of two numbers is 1024; one of the numbers is 8. What is the other number?

5. The sum of three numbers is 64; two of the numbers are 12 and 25; the third number is ——.

(f) The sum of three numbers is 4572; two of the numbers are 436 and 975. What is the third number?

6. The product of three numbers is 90; two of the numbers are 3 and 5; the third number is ——.

(g) The product of three numbers is 1728; two of the numbers are 8 and 12. What is the third number?

(h) A pile of wood is 4 ft. wide and 4 ft. high. How long must it be to contain 3 cords?*

(i) A stone is 24 inches wide and 2 inches thick. How long must it be to contain 1 cu. ft.?

(j) A bin is $8\frac{1}{3}$ ft. wide and 6 feet deep. How long must it be to contain 1000 bushels?†

(k) A room is 15 ft. wide and 21 ft. long. How high must the room be to contain 105 cu. yd. of air?‡

(l) A tank is 24 inches wide and 30 inches long. How deep must it be to contain 5 cubic feet?

* 4 cu. ft. $\times 4 \times ? = 384$ cu. ft.

† A bushel of oats occupies about $1\frac{1}{4}$ cu. ft.

‡ 105 cu. yd. equals how many cu. ft.?

Common Fractions.

(a) Find the l. c. m. of 12 and 18.

(b) Find the sum of $\frac{11}{12}$ and $\frac{5}{18}$. (c) $\frac{11}{12} - \frac{5}{18} =$

(d) Find the quotient of $\frac{11}{12}$ divided by $\frac{5}{18}$.

(e) Find the l. c. m. of 14 and 20.

(f) Find the sum of $\frac{9}{14}$ and $\frac{3}{20}$. (g) $\frac{9}{14} - \frac{3}{20} =$

(h) Find the quotient of $\frac{9}{14}$ divided by $\frac{3}{20}$.

(i) At $\$\frac{1}{8}$ a bushel, how many bushels of apples can be bought for $\$34\frac{1}{8}$?

$\$34\frac{1}{8} \div \$\frac{1}{8}$, means, ————.

(j) If $\frac{1}{8}$ of a ton of hay costs $\$11\frac{1}{8}$, how much does 1 ton cost at the same rate?

$\$11\frac{1}{8} \div \frac{1}{8}$, means, ————.*

(k) How much will $3\frac{1}{2}$ acres of land cost at $\$25\frac{1}{2}$ an acre?†

$\$25\frac{1}{2} \times 3\frac{1}{2}$, means, ————.

(l) How much will $\frac{1}{2}$ of an acre of land cost at $\$364$ an acre?

$\$364 \times \frac{1}{2}$, means, ————.

(m) Mr. Miller earns $\$19.25$ in $5\frac{1}{2}$ days. How much does he earn in 1 day?

$\$19.25 \div 5\frac{1}{2}$, means, ————.

(n) Mr. Finley earns $\$2\frac{3}{4}$ a day. In the month of May he earned $\$46\frac{3}{4}$. How many days did he work?

$\$46\frac{3}{4} \div \$2\frac{3}{4}$, means, ————.

(o) From a piece of cloth containing $47\frac{3}{4}$ yards, there were sold $14\frac{1}{2}$ yd., $9\frac{3}{4}$ yd.; $5\frac{1}{4}$ yd., and $6\frac{1}{2}$ yd. How many yards were left?

* See problem (1), page 192, and foot-note.

† See problem 1, page 192. Do not change these numbers to improper fractions.

Decimal Fractions.

(a) If a teacher earns $262.50 in 3.5 months, how much does she earn in 1 month?

$262.50 ÷ 3.5, means, ————.*

(b) At $3.5 each, how many sheep can be bought for $262.50?

$262.50 ÷ $3.5, means, ————.†

(c) A railroad train moved 103.5 miles in 2.3 hours. At what rate per hour did it move?

$103.5 miles ÷ 2.3, means, ————.

(d) A canal boat moves at the rate of 2.3 miles an hour. In how many hours does it move 103.5 miles?

$103.5 miles ÷ 2.3 miles, means, ————.

(e) A teacher earns $75 per month. How much does she earn in 3.5 months?

$75 × 3.5, means, ————.

(f) At $3.5 each, how much will 75 sheep cost?

$3.5 × 75, means, ————.

(g) If a railroad train moves at the rate 45 miles an hour, how far will it move in 2.3 hours?

45 miles × 2.3, means, ————.

(h) If a canal boat moves at the rate of 2.3 miles an hour, how far will it move in 45 hours?

2.3 miles × 45, means, ————.

(i) Divide $153.6 by 2.4. (j) $153.6 ÷ $2.4 =

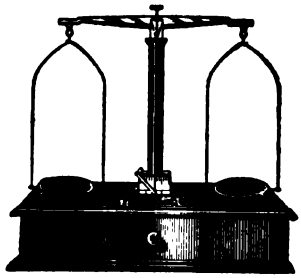
(k) Divide $116.1 by 2.7. (l) $116.1 ÷ $2.7 =

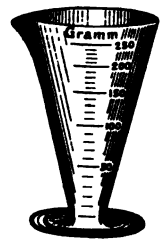
* Find 10 thirty-fifths of $262.50, or 1 thirty-fifth of a number 10 times as large.

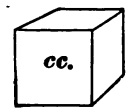
† Find how many times $3.5 are contained in $262.50.

TO THE TEACHER.—Compare problems (a) and (b). The pupil might solve problem (a) as follows: At $1 a month, in 3½ months, the teacher would earn $3½. If she earned $262.50 in 3½ months, she must have earned as many dollars per month as $3½ are contained times in $262.50. This makes the problem identical with problem (b); but in this case the pupil does not divide $262.50 by 3.5 but by 3.5 *dollars*. See page 163.

Denominate Numbers — Metric System.*







1. The weight of a cubic centimeter of water is called a **gram.**

2. One thousand grams equal 1 **kilogram.** †

3. Two cc. of water weigh —— grams.

4. Water equivalent to a 2-centimeter cube of water weighs —— grams.

5. Water equivalent to a 3-centimeter cube of water weighs —— grams.

6. A cubic decimeter of water weighs —— grams, or —— kilogram.

7. A liter of water weighs ————.

8. A rectangular vessel 3 cm. long, 2 cm. wide, and 2 cm. high will contain —— grams of water.

9. A rectangular vessel 3 dm. long, 2 dm. wide, and 2 dm. high will contain —— kilograms of water.

10. One quart of water weighs a little ——‡ than 1 kilogram.

11. One gallon of water weighs nearly —— kilograms.

12. One cubic meter of water weighs —— kilograms.

* A balance, a set of metric weights, and a small metric graduate are necessary appliances to be used in preparation for the work of this page. Measure water; then weigh it. Do this again and again until the pupil is *familiar with the fact* that 1 cc. of water weighs 1 gram.

† A kilogram is equal to 2.2 + lb.

‡ More or less? See foot-note, page 194.

Measurements—Lumber.

Pieces of lumber more than 1 inch thick, and uniform in all their dimensions, are sometimes called "**dimension stuff**"; thus, a pile of 2 by 4, 12's,* or a pile of 4 by 4, 16's, is *dimension stuff*. The smaller sizes of dimension stuff, as 2 by 4's, 4 by 6's, 2 by 6's, etc., are called **scantlings**. The larger sizes, as 4 by 6's, 6 by 8's, 10 by 12's, etc., are called **timbers**.

In measuring dimension stuff, the number of feet of lumber in each piece is regarded as integral; thus, if the actual measurement is 9⅓ feet, it is regarded as 9 feet; if it is 9⅔ feet, it is regarded as 10 feet, etc.

1. 1 piece 2 × 4, 12,† contains —— feet of lumber.
(a) 25 pcs. 2 × 4, 12, contain how many feet?
2. 1 piece 2 × 4, 14, contains ——‡ feet of lumber.
(b) 25 pcs. 2 × 4, 14, contain how many feet?
3. 1 piece 2 × 4, 18, contains —— feet of lumber.
(c) 25 pcs. 2 × 4, 18, contain how many feet?
4. 1 piece 4 × 4, 12, contains —— feet of lumber.
(d) 25 pcs. 4 × 4, 12, contain how many feet?
5. 1 piece 4 × 4, 16, contains —— feet of lumber.
(e) 25 pcs. 4 × 4, 16, contain how many feet?
6. 1 piece 4 × 4, 18, contains —— feet of lumber.
(f) 25 pcs. 4 × 4, 18, contain how many feet?

Find the amount of lumber in each of the following:

(g) 36 pcs. 2 × 4, 18.
(h) 24 pcs. 2 × 6, 14.
(i) 32 pcs. 2 × 8, 12.
(j) 15 pcs. 2 × 10, 12.
(k) 23 pcs. 4 × 6, 16.
(l) 18 pcs. 4 × 4, 16.
(m) 16 pcs. 2 × 4, 16.
(n) 26 pcs. 2 × 10, 12.
(o) 28 16-ft. fence boards
(p) 35 14-ft. stock boards.

* Lumber 2 in. thick, 4 inches wide, and 12 ft. long.

† To be read, *two by four, twelve.*

‡ One piece of 2 × 4, 14, actually contains 9⅓ feet of lumber; but the custom in the lumber yards is to regard it as 9 ft.; 1000 such pieces are said to contain 1000 times 9 ft., *not* 1000 *times* 9⅓ *ft.*

Ratio and Proportion.

1. The ratio of $\frac{6}{8}$ to $\frac{1}{4}$ equals the ratio of 10 to ———.*
2. The ratio of 2.7 to .9 equals the ratio of 45 to ———.†
3. The ratio of $\frac{1}{2}$ to 3.5 equals the ratio of 12 to ———.
4. The ratio of .4 to $2\frac{1}{2}$ equals the ratio of 12 to ———.‡
5. 1 sq. centimeter : 1 sq. decimeter = 1 : ———.§
6. 1 sq. decimeter : 1 sq. meter = 1 : ———.
7. 1 cu. centimeter : 1 cu. decimeter = 1 : ———.
8. 1 cu. decimeter : 1 cu. meter = 1 : ———.
9. 1 pc. 1 × 1, 12 : 1 pc. 2 × 4, 12 = 1 : ———.
10. 1 pc. 2 × 4, 12 : 1 pc. 4 × 4, 12 = 1 : ———.
11. 1 pc. 4 × 4, 12 : 1 pc. 2 × 8, 12 = 1 : ———.
12. 1 pc. 2 × 6, 12 : 1 pc. 2 × 6, 18 = 1 : ———.
13. 1 pc. 2 × 4, 12 : 1 pc. 2 × 4, 14 = 1 : ———.

14. If 1 M. feet of lumber is worth \$15, 1200 feet is worth ———.‖

1000 ft. : 1200 ft. = \$15 : ———.

(a) If 1 M. feet of lumber is worth \$14.50, how much is 1200 feet worth?¶

(b) If 1 M. feet of lumber is worth \$13.50, how much is 1600 feet worth?

(c) If 500 feet of lumber is worth \$8.50, how much is 1700 feet worth?

(d) What is the ratio of a 7-cm. square to a 2-cm. square?

(e) Of a 9-cm. square to a 2-cm. square?

(f) What is the ratio of an 8-cm. cube to a 2-cm. cube?

(g) Of a 10-cm. cube (a cubic decimeter) to a 5-cm. cube?

(h) What is the ratio of a 10-inch cube to a 5-inch cube?

(i) Of a 10-foot cube to a 5-foot cube?

* Change $\frac{6}{8}$ to eighths. † The ratio of 27 tenths to 9 tenths.
‡ Change $2\frac{1}{2}$ to tenths. § Change 1 qdm. to qcm.'s.
¶ The pupil is expected to solve this problem by making use of the fact that the ratio of 1000 to 1200 is $\frac{5}{6}$. The cost of 1200 ft. is $\frac{6}{5}$ of the cost of 1000 ft.
‖ For use of the word *is* in this and in other problems on this page, see page 236, foot-note (†).

Percentage.

10 20 30 40 50*

1.

10 is ——% of 20. 20 is ——% of 10.
10 is ——% less than 20. 20 is ——% more than 10.

2.

20 is ——% of 30. 30 is ——% of 20.
20 is ——% less than 30. 30 is ——% more than 20.

3.

30 is ——% of 40. 40 is ——% of 30.
30 is ——% less than 40. 40 is ——% more than 30.

4.

40 is ——% of 50. 50 is ——% of 40.
40 is ——% less than 50. 50 is ——% more than 40.

5.

10 is ——% of 30. 30 is ——% of 10.
10 is ——% less than 30. 30 is ——% more than 10.

6.

10 is ——% of 40. 40 is ——% of 10.
10 is ——% less than 40. 40 is ——% more than 10.

7.

10 is ——% of 50. 50 is ——% of 10.
10 is ——% less than 50. 50 is ——% more than 10.

8.

20 is ——% of 40. 40 is ——% of 20.
20 is ——% less than 40. 40 is ——% more than 20.

9.

20 is ——% of 50. 50 is ——% of 20.
20 is ——% less than 50. 50 is ——% more than 20.

* To The Teacher.—This page is for drill. Review it until every pupil has mastered it. In a similar manner compare other numbers; as, 30 and 50, 40 and 60, 50 and 60, etc.

Percentage — Commercial Discount.

Dealers in hardware, rubber boots and shoes, belting, rubber hose, and many other kinds of goods, sell from a *list price*. This price may remain unchanged for many years. The actual price is usually less than the list price, and is found by making a certain discount from the list price. "10 % off" or "10 off," means that the list price is to be discounted 10 per cent. "20 and 10 off," means, that the list price is to be discounted 20 % and what remains is to be discounted 10 %. Sometimes as many as four successive discounts from the list price are allowed. Observe that in computing these, the base changes with each discount.

(a) The list price of men's rubber boots is $3.00 per pair. What is the actual price if they are sold at "20 and 10 off"?* (b) What is the actual price if they are sold at "30 off"? Compare the answers to (a) and (b). Why are they not alike?

(c) The list price of 1-inch wire brads is 30¢ per pound. How much will 100 lb. cost if they are sold at "50 and 10 off"? (d) How much will 100 lb. cost if they are sold at "60 off"?

(d) The list price of carriage bolts, 10 × $\frac{5}{8}$,‡ is $9.20 per hundred. How much will 300 such bolts cost if they are sold at "50 and 10 off"? (e) How much will they cost if they are sold at "60 off"?

(f) Find the cost of 12 pairs men's rubber boots, list price $3.00 per pair, discount "40 and 10 off" and "2 off for cash"? §

(g) Find the cost of 24 pairs men's arctics, list $1.50 per pair, discount "30 and 10 off and 2 off for cash"?

(h) Bought for "20 off" from list price and sold at list price. What was my per cent of gain?

* Take $\frac{1}{4}$ of $3.00 from $3.00; then take $\frac{1}{10}$ of what remains from what remains.

‡ 10 inches long and $\frac{5}{8}$ of an inch in diameter.

§ First, find the cost at $3.00 per pair; then discount this 40 %; then discount what remains 10 %; then discount what still remains 2 %.

Review.

1. The product of two numbers is 360; one of the numbers is 12; the other number is ——.

(a) 45 sq. ft. × ? = 1080 square feet. *Story.*

2. The product of three numbers is 270; two of the numbers are 5 and 6; the third number is ——.

(b) 12 cu ft. × 7 × ? = 1260 cu. feet. *Story.*

(c) A real estate agent offered $2\frac{3}{4}$ acres of land for $1199. This was equal to what rate per acre?

$1199 ÷ $2\frac{3}{4}$, means ————.

(d) A real estate agent offered 2.7 acres of land for $1215. This was equal to what rate per acre?

$1215 ÷ 2.7, means ————.

1. 12 cubic centimeters of water weigh —— grams.

2. 35 cubic decimeters of water weigh —— kilograms.

(e) How many feet of lumber in 25 pcs. 4 × 6, 16? (f) How much is the lumber worth at $15 per M. ?

(g) How much lumber in 32 16-ft. fence boards? (h) How much is it worth at $14 per M. ?

3. What is the ratio of 1 piece of 4 × 6, 12, to 1 piece of 4 × 6, 18?

4. 50 is ——% of 60. 60 is ——% of 50.

5. 50 is ——% less than 60. 60 is ——% more than 50.

6. Bought for 50 and sold for 60. What was the per cent of gain?

7. Bought for 60 and sold for 50. What was the per cent of loss?

8. The buying price was 2 thirds of the selling price. What was the per cent of gain?

Miscellaneous Problems.

(a) On a bill of goods amounting to $154.50, there is a discount of 4 % for cash. How much money will be required to pay the bill?

(b) How many square yards of carpet in a piece that is 56 yards long and $\frac{3}{4}$ of a yard wide?

(c) What is the value at $85 an acre, of a square field whose side is 80 rods?

(d) A gallon of water weighs about $8\frac{1}{3}$ lb. Vinegar is a little heavier than water. Does a barrel of vinegar (48 to 51 gal.) weigh more or less than 375 lb.?

(e) Paid the freight on 24600 lb. coal at 5¢ per hundred weight. How much did I pay?

(f) Minnie took a teaspoonful of medicine every hour, beginning at 5 o'clock Sunday afternoon and taking the last dose at 11 o'clock Monday morning. How many teaspoonfuls did she take?

(g) A box (inside measure) is 10 inches long, 5 inches wide, and 4 inches high. It is filled with 1-inch wooden cubes. How many of the cubes touch the bottom of the box? (h) How many touch each end of the box? (i) How many touch each side of the box? (j) How many do not touch either end of the box? (k) How many do not touch either side of the box? (l) If a cover is put upon the box, how many of the cubes cannot touch the top, the bottom, either side, or either end of the box? *

* TO THE TEACHER.—It is desirable that the pupil should learn to solve such problems as these, (g) to (l) inclusive, without cubes and without drawing a figure of the box. The learner is supposed to have handled 1-inch cubes earlier in the course, and to be able at this time to make a mental picture of them and to arrange them in this picture as suggested in the problem.

Simple Numbers.*

Teach orally the reading and the writing of numbers as high as millions.

Write in figures:

1. Eight. Seventy-five. Six hundred seventy-four.

2. Five thousand six hundred seventy-two.

3. Forty-five thousand six hundred twenty-eight.

4. Three hundred forty-five thousand six hundred fifty-seven.

5. Two million three hundred thirty-two thousand six hundred ninety-one.

6. Twelve million one hundred thirty-four thousand two hundred seventy-five.

(a) Find the sum of all the numbers given above.

Read the following, first by column, then by line; then add each column.

	(b)	(c)	(d)
7.	25 346 251	8 156 347	175 342 154
8.	25 846 851	7 106 207	6 130 245
9.	25 006 001	5 600 500	34 112 306
10.	16 600 400	3 407 208	51 175 318
11.	16 650 430	9 101 101	124 124 124
12.	16 653 432	6 006 006	100 000 000
13.	1 001 001	4 444 444	7 200 000
14.	1 010 010	2 000 000	9 350 000

(e) Find the sum of six million two hundred four thousand one hundred five, eight million six thousand twenty-five, and three million forty-five thousand two hundred thirty-seven.

Common Fractions.

(a) Find the l. c. m. of 16 and 20.

(b) Find the sum of $\frac{9}{16}$ and $\frac{3}{20}$. (c) $\frac{9}{16} - \frac{3}{20} =$

(d) Find the quotient of $\frac{9}{16}$ divided by $\frac{3}{20}$.

1. Bought ribbon as follows:*

5 yd. for 60¢; 1 yd. cost ——.	60¢ + 5, means ——.
4 yd. for 60¢; 1 yd. cost ——.	60¢ + 4, means ——.
3. yd. for 60¢; 1 yd. cost ——.	60¢ + 3, means ——.
2 yd. for 60¢; 1 yd. cost ——.	60¢ + 2, means ——.
$2\frac{1}{2}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $2\frac{1}{2}$, means ——.
$\frac{1}{2}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $\frac{1}{2}$, means ——.
$\frac{2}{3}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $\frac{2}{3}$, means ——.
$\frac{3}{4}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $\frac{3}{4}$, means ——.
$\frac{4}{5}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $\frac{4}{5}$, means ——.
$1\frac{1}{2}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $1\frac{1}{2}$, means ——.
$1\frac{1}{3}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $1\frac{1}{3}$, means ——.
$1\frac{2}{3}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $1\frac{2}{3}$, means ——.
$1\frac{1}{4}$ yd. for 60¢; 1 yd. cost ——.	60¢ + $1\frac{1}{4}$, means ——.

(e) Bought $2\frac{2}{4}$ tons of hay for $33. How much would 1 ton cost at the same rate?

(f) Bought $3\frac{3}{5}$ acres of land for $816. How much was paid per acre?

(g) Bought $9\frac{1}{2}$ cords of wood for $42.75. What was the price per cord?

* To the Teacher.—Observe that there are two ways of thinking the solution of such problems as those that follow: (1) The pupil may think as indicated by the figures given after each problem, thus: If 5 yd. cost 60¢, 1 yd. cost $\frac{1}{5}$ of 60¢; if $2\frac{1}{2}$ yd. cost 60¢, $\frac{1}{2}$ a yd. cost $\frac{1}{5}$, and 1 yd. $\frac{2}{5}$ of 60¢; if $1\frac{1}{2}$ yd. cost 60¢, $\frac{1}{2}$ yd. cost $\frac{1}{5}$, and 1 yd. $\frac{3}{5}$ of 60¢, etc. (2) The pupil may think—At 1¢ a yd. 5 yd. would cost 5¢, and if 60¢ were paid for 5 yd. as many cents were paid per yd. as 5¢ are contained times in 60¢. This method of thinking may be applied in the solution of all similar *problems.*

Decimal Fractions.

Teach orally the reading and writing of decimals to *millionths*. Lead the pupil to see—(1) That the denominator of a decimal is 1 with as many zeros annexed to it as there are decimal places; thus, the denominator of .7 is 10; of .47, 100; of .375, 1000, etc. (2) That in millionths there are six decimal places; in hundred-thousandths, five; in ten-thousandths, four; in thousandths, three; in hundredths, two, and in tenths, one.

1. The numerator of .07 is —; the denominator is —.*
2. The numerator of .056 is —; the denominator is —.
3. The numerator of .0405 is —; the denominator is —.
4. The numerator of .00308 is —; the denominator is —.
5. The numerator of .002346 is —; the denominator is —.

(a) Find the sum of the decimals given in statements 1 to 5, inclusive.

Write in figures:

6. Two *and* seven hundredths. †
7. Forty-five *and* sixty-four thousandths.
8. Seventy-four *and* three hundred seventy-five thousandths.
9. Eight *and* four hundred thirty-six ten-thousandths.
10. Two hundred thirty-five *and* seven ten-thousandths.

(b) Find the sum of numbers given in lines 6 to 10, inclusive.

Read the following, first by column, then by line; then add each column:

	(c)	(d)	(e)
11.	245.3234 ‡	.275324	27.00275
12.	137.0245	.046256	36.54024
13.	164.0036	.007246	52.06281
14.	243.0007	.000325	71.00005
15.	327.2006	.000074	46.00023
16.	175.0202	.000009	34.80875

* One hundred, not hundredths. The numerator is 7, the denominator is 100; the fraction is seven one-hundredths or seven hundredths.

† Observe that the decimal point takes the place of the word *and* when these numbers are written in figures.

‡ In reading these numbers, use the word *and* only in the place of the decimal point; say, two hundred forty-five, not two hundred *and* forty-five; three thousand two hundred thirty-four, not three thousand two hundred and thirty-four.

Denominate Numbers—Specific Gravity.

Teach the meaning and use of the term specific gravity.*

1. *A cubic foot of water weighs* 62½ *lb.*

(a) How many ounces in a cu. ft. of water?

2. The specific gravity of dry white pine is about ½, or .5.† A cubic foot of dry white pine weighs about —— pounds.

(b) If the specific gravity of a piece of pine is .6, how much does a cubic foot of it weigh? (c) How much does a piece 1 foot long, 1 foot wide, and 1 inch thick, weigh?

3. The specific gravity of cork is about ¼, or .25. A cubic foot of cork weighs about how many pounds?

(d) If the specific gravity of a piece of cork is .24, how much does a cubic foot of it weigh?

4. The specific gravity of dry sand is about 2. A cubic foot of dry sand weighs about —— pounds.

(e) If the specific gravity of dry sand is 2.1, how much does a cubic foot of it weigh?

5. The specific gravity of limestone is about 2½. A cubic foot of limestone weighs about —— pounds.

(f) If the specific gravity of a piece of limestone is 2.6, how much does a cubic foot of it weigh?

6. The specific gravity of iron is about 7. A cubic foot of iron weighs about —— pounds.

(g) If the specific gravity of a piece of iron is 7.3, how much does a cubic foot of it weigh?

* When we say that the *specific gravity* of a substance is 7, we mean that it is seven times as heavy as water, the same bulk, or mass, of each being considered. If the specific gravity of a substance is 4, it is 4 times as heavy as water; if it is ½. it is ½ as heavy as water; if it is 2½, it is 2½ times as heavy as water.

† Put a piece of wood into water. If ½ of the wood is below the surface of the water and ½ above the surface, the specific gravity of the wood is ½; if ¾ of the wood *is below and* ¼ *above* the surface, the specific gravity is ¾.

Measurements—Lumber.

Make a diagram like the following and fill the spaces with figures that will indicate the number of feet of lumber in one piece of each kind described; thus, instead of (a) write the number of feet in one piece of 2 × 4, 12; instead of (b) write the number of feet in one piece of 2 × 4, 14, etc. Review carefully page 205.

LUMBER TABLE—"DIMENSION STUFF."

	12	14	16	18	24
2 × 4	a	b*	c	d	e
4 × 4	f	g	h	i	j
2 × 6	k	l	m	n	o
2 × 8	p	q	r	s	t
8 × 8	u	v	w	x	y

Find the cost of the following:

(z) 15 pcs. 2 × 4, 12, at $16 per M.

(aa) 23 pcs. 4 × 4, 16, at $15 per M.

(bb) 26 pcs. 2 × 6, 18, at $14 per M.

(cc) 12 pcs. 2 × 8, 16, at $15 per M.

(dd) 2 pcs. 8 × 8, 24, at $18 per M.

(ee) 35 14-foot stock boards at $19 per M.

(ff) 28 16-foot fence boards at $13 per M.

(gg) 650 feet of common boards at $15 per M.

(hh) 1 16-foot 1½-inch board, 15 inches wide, at $25 per M.

* 8 ft. + ⅛ of 8 ft. = 9⅛ feet, to be regarded as 9 ft. Custom determines that in the measurement of any piece of "dimension stuff", a fraction of a foot shall be omitted if it is less than ½ a foot and shall be regarded as 1 foot if it is more than ½ a foot. One piece of 2 × 4, 14, is said to contain 9 feet; 7 pieces 2 × 4, 14, are said to contain 63 feet.

Ratio and Proportion.

Cubic foot and gallon compared.

1. Is the ratio of 37 to 5 more or less than $7\frac{1}{2}$?

2. Is the ratio of 39 to 7 more or less than $5\frac{1}{2}$?

(a) Is the ratio of 295 to 12 more or less than $24\frac{1}{2}$?

(b) Is the ratio of 435 to 13 more or less than $33\frac{1}{2}$?

(c) Is the ratio of 715 to 160 more or less than $4\frac{1}{2}$?

(d) One cubic foot contains 1728 cubic inches. *One gallon contains* 231 *cubic inches.** Is the ratio of one cubic foot to 1 gallon more or less than $7\frac{1}{2}$?

(e) Regarding the ratio of one cubic foot to one gallon as $7\frac{1}{2}$, find the approximate capacity, in gallons, of a milk tank 6 feet long, $2\frac{1}{2}$ feet wide, and 2 feet deep.

(f) Is the capacity of a tank 8 feet long, 5 feet wide, and 2 feet deep, more or less than 600 gallons?

(g) What is the ratio, expressed in its lowest terms, of an oblong 3 feet by 6 feet, to an oblong twice as wide and twice as long?

(h) What is the ratio, expressed in its lowest terms, of an oblong 2 feet by 8 feet, to an oblong twice as wide and twice as long?

3. Compare the answers to problems (g) and (h). What is the ratio, expressed in its lowest terms, of any oblong to another twice as wide and twice as long?

4. What is the ratio, expressed in its lowest terms, of a 3-ft. square to a 6-ft. square? Of a 4-ft. square to an 8-ft. square? Of a 5-ft. square to a 10-ft. square?

(i) A locomotive goes 4 miles in 5 minutes; how far would it go in 1 hour at the same rate?

5 minutes : 1 hour = 4 miles : ———.

* *Pupils should memorize the fact stated in italics.*

Percentage.*

1. 390 is what % of 400? 388 of 400?
(a) 390 is what % of 450? (b) 388 of 450?

2. 390 is what per cent less than 400?
(c) 390 is what per cent less than 450?

3. 410 is what % of 400? 412 of 400?
(d) 410 is what % of 350? (e) 412 of 350?

4. 410 is what per cent more than 400?
(f) 410 is what per cent more than 350?

5. A lawyer collected $750. Of this sum he "paid over" to the man for whom he collected the money $705 and retained the remainder as his commission for collecting. What was his rate of commission?

Am't collected.	Am't "paid over."	Rate of commission.
(g) $724.50	$666.54	?
(h) $635.50	$597.37	?

6. A bill before it was discounted amounted to $450; the amount paid was $441. What was the rate of discount?

Before discounting.	After discounting.	Rate of discount.
(i) $365.20	$346.94	?
(j) $436.50	$427.77	?

7. A merchant bought goods for $300 and sold them for $337.50. What was the per cent of profit?

Buying price.	Selling price.	Per cent of gain.
(k) $365.20	$419.98	?
(l) $275.60	$344.50	?

* To the Teacher.—Impress upon the pupil the fact that to find what % one number is of another, he must first determine which is the base; then *divide the other number by one per cent of the base.* Such is the nature of the language employed in giving such problems as these that the base always follows the word *of* or the word *than.*

Percentage — Interest.

Teach orally the meaning and use of the terms, *interest*, *principal*, and *amount*. Tell the pupil that when a man borrows money, and agrees to pay 6 % interest, that he must pay a sum equal to 6% of the principal for every year he has the use of the money, and that the interest and the principal together make the *amount* that he must "pay back."

1. What part of the principal equals the interest if the time is 1 year and the rate 6 %? Ans. .06 or 6 %.

2. The interest on $500 for 1 year at 6 % is ——; * the amount is ——.

3. The interest on $700 for 1 year at 7 % is ——; * the amount is ——.

4. The interest on $60 for 1 year at 4 % is ——; * the amount is ——.

5. The interest on $400 for 6 months at 6 % is ——; * the amount is ——.

6. The interest on $600 for 1 year 6 months at 6 % is ——; * the amount is ——.

7. The interest on $500 for 1 year 6 months at 8 % is ——; * the amount is ——.

8. The interest on $300 for 1 year 4 months at 6 % is ——; * the amount is ——.

(a) What is the interest on $325 for 1 year at 6 %? (b) The amount?

(c) What is the interest on $276 for 1 year at 4 %? (d) The amount?

(e) What is the interest on $340 for 1 year 6 months at 6 %? (f) The amount?

(g) What is the interest on $260 for 1 year 6 months at 5 %? (h) The amount?

* What % of the principal equals the interest? How much is one per cent of the *principal*?

Review.

(a) Find the sum of five million two hundred four thousand three hundred forty-seven, and seven hundred twenty-five thousand six.

(b) A million is how many more than eight hundred twenty-four thousand one hundred seven?

(c) A dealer offered $2\frac{5}{8}$ cords of wood for $11.55. This was equal to what rate per cord?

(d) A dealer sold a lot of wood for $8.55, the price per cord being $3.60. How many cords did he sell?

(e) From two hundred seventy-five thousand, subtract two hundred seventy-five thousandths.

(f) Multiply seven and forty-five ten thousandths by six.

(g) If the specific gravity of seasoned white oak is .8, how much does a cubic foot of it weigh?

(h) A piece of wood was observed floating in the water; $\frac{3}{4}$ of the wood was below the surface of the water. How much would a cubic foot of such wood weigh? *

(i) How many feet of lumber in 1 piece 12 × 12, 20? (j) How much is the lumber worth at $15 per M.? (k) How many cubic feet in the piece of lumber described in problem (i)? (l) If the specific gravity of this piece of timber was .7, how much would it weigh?

(m) If the specific gravity of a certain kind of stone is $2\frac{1}{2}$, how much will a piece weigh, that is 6 feet long, 3 feet wide and 4 inches thick?

* See foot-note (†) page 214.

Miscellaneous Problems.

1. Three per cent of a number is 12; the number is ——.

(a) Seven per cent of a number is 238. What is the number?

2. What per cent is gained on paper bought at 8¢ a quire and sold at the rate of 2 sheets for a cent?

(b) What per cent is gained on paper bought at $2 a ream and sold at the rate of 3 sheets for 2 cents?

(c) A certain wheel makes 250 revolutions in a minute. How many revolutions does a wheel revolving $2\frac{1}{2}$ times as fast make in $\frac{2}{3}$ of an hour?

(d) If a brick 8 inches long, 4 inches wide, and 2 inches thick, weighs exactly 4 lb., is its specific gravity more or less than 2? *

(e) If bricks weigh $4\frac{1}{4}$ lb. each, how much more than 2 tons do 1000 bricks weigh?

(f) One thousand bricks of the size of those described in problem (d) are how many cubic feet of bricks?

(g) If 22 bricks, including the mortar that is between them, will make 1 cubic foot of wall, how many cubic feet will 1000 bricks make?

(h) At $5.50 per M., how much will 28360 bricks cost?

(i) When the sun rises at 4:58 and sets at 6:57, what is the length of time from sunrise to sunset?

3. What time did the sun rise this morning? What time will it set to-night?

(j) How long from sunrise to sunset to-day?

* One brick is what part of a cubic foot of bricks?

Simple Numbers—Ledger Columns.

(a)	(b)	(c)
24.75	356.40	154.20
6.42	74.30	2.86
.74	6.42	.75
325.60	124.57	371.56
8.73	5.65	27.31
2.96	24.32	8.27
.47	.56	2.55
32.56	35.50	46.24
12.53	8.74	.86
7.29	271.33	8.96
25.34	7.15	5.13
6.15	5.22	2.52
2.25	63.40	34.60
36.40	25.42	52.35
35.24	.55	5.50
5.50	352.57	253.75
253.75	60.75	57.60
75.60	8.43	4.28
3.48	1.52	5.21
2.51	6.41	4.16
1.46	8.20	2.08

An experienced accountant can find the sum of either of the above ledger columns, *adding twice*, in one minute. Can you do it in four minutes? Do not, however, sacrifice accuracy to speed. In such work as this nothing short of absolute accuracy has any value whatever.

To THE TEACHER.—Much drill similar to that presented on this page should be given. Allow no pupil to give his "answer" in such drill as this until he has added upward, and proved his work by adding downward. Every pupil should be made to feel that unless he can learn to add accurately, he will be worthless as an accountant.

Common Fractions—Multiplication.

1. At $\frac{1}{4}$ of a dollar a yard, how much will $\frac{1}{3}$ of a yard of ribbon cost? It will cost $\frac{1}{3}$ of $\$\frac{1}{4}$.*

1 third of $\$\frac{1}{4}$ =
1 third of $\$\frac{1}{4}$ =
2 thirds of $\$\frac{1}{4}$ =

2. $\frac{1}{4}$ multiplied by $\frac{1}{3}$, means, take $\frac{1}{3}$ of $\frac{1}{4}$. $\frac{1}{3}$ of $\frac{1}{4}$ =
3. $\frac{1}{5}$ multiplied by $\frac{1}{3}$, means, take $\frac{1}{3}$ of $\frac{1}{5}$. $\frac{1}{3}$ of $\frac{1}{5}$ =
4. $\frac{1}{7}$ multiplied by $\frac{1}{3}$, means, take $\frac{1}{3}$ of $\frac{1}{7}$. $\frac{1}{3}$ of $\frac{1}{7}$ =

5. $\frac{1}{3} \times \frac{1}{4}$, means, take $\frac{1}{4}$ of $\frac{1}{3}$ or $\frac{1}{3}$ of $\frac{1}{4}$. $\frac{1}{3} \times \frac{1}{4}$ = †
6. $\frac{1}{5} \times \frac{1}{4}$, means, take $\frac{1}{4}$ of $\frac{1}{5}$ or $\frac{1}{5}$ of $\frac{1}{4}$. $\frac{1}{5} \times \frac{1}{4}$ =
7. $\frac{1}{7} \times \frac{1}{3}$, means, take $\frac{1}{3}$ of $\frac{1}{7}$ or $\frac{1}{7}$ of $\frac{1}{3}$. $\frac{1}{7} \times \frac{1}{3}$ =

(a) Multiply $\frac{1}{3}\frac{1}{4}$ by $\frac{1}{3}$. (b) Multiply $\frac{1}{3}\frac{1}{7}$ by $\frac{1}{5}$.
(c) Multiply $\frac{1}{3}\frac{1}{4}$ by $\frac{1}{5}$. (d) Multiply $\frac{1}{3}\frac{1}{7}$ by $\frac{1}{7}$.

8. At $\$4\frac{1}{2}$ a cord, how much will $2\frac{1}{2}$ cords of wood cost?

First Method.	Second Method.
It will cost 2 times $\$4\frac{1}{2}$ plus $\frac{1}{2}$ of $\$4\frac{1}{2}$.	It will cost $\$4\frac{1}{2} \times \frac{5}{2}$, or 5 times $\frac{1}{2}$ of $\$4\frac{1}{2}$.
2 times $\$4\frac{1}{2}$ plus $\frac{1}{2}$ of $\$4\frac{1}{2}$ = ‡	$\$4\frac{1}{2} \times \frac{5}{2}$ = ‡

(e) Multiply $6\frac{1}{3}$ by $3\frac{1}{4}$. (f) Multiply $12\frac{1}{2}$ by $3\frac{1}{4}$.
(g) Multiply $8\frac{1}{3}$ by $7\frac{1}{4}$. (h) Multiply $15\frac{1}{4}$ by $5\frac{1}{3}$.

* By means of problems 1 to 7 and other similar exercises, lead the pupil to see that one fraction can be multiplied by another fraction by taking the product of the numerators for the required numerator, and the product of the denominators for the required denominator; that is, by "*multiplying the numerators together for the new numerator and the denominators together for the new denominator.*"

† The pupil should now understand that the sign of multiplication may mean *times* or *multiplied by;* that it indicates that the product of the numbers between which it stands, is to be taken, and that in abstract problems either of the numbers may be regarded as the multiplicand.

‡ The pupil should now learn that to multiply a mixed number by a mixed number, he may reduce them to improper fractions or not as he chooses and as seems most convenient.

Decimal Fractions.

Group 1.

$\frac{12}{4}$ $\frac{16}{2}$ $\frac{15}{5}$ $\frac{24}{8}$ $\frac{40}{8}$

Group 2.

$\frac{13}{4}$ $\frac{7}{2}$ $\frac{11}{5}$ $\frac{21}{8}$ $\frac{35}{10}$

Group 3.

$\frac{1}{2}$ $\frac{2}{5}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{8}$

Group 4.

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{3}{7}$ $\frac{2}{9}$

Observe that any fraction in either of the above groups may be changed in form by dividing its numerator by its denominator.*

Each fraction in Group 1 thus becomes an integer.

Each fraction in Group 2 thus becomes a mixed number.

Each fraction in Group 3 thus becomes a *finite decimal.*†

Each fraction in Group 4 thus becomes an *infinite decimal.*†

Observe that every fraction is an expression of division; the numerator is the dividend, the denominator is the divisor, and the fraction itself is the quotient. ‡

Observe that by changing the numerator of each fraction in Group 2, to tenths, hundredths, or thousandths, and dividing it thus changed, by its denominator, the fraction becomes a mixed decimal.

1. To reduce a common fraction to a decimal, *change its numerator to tenths, hundredths, thousandths, etc., and divide it, thus changed, by its denominator.*

$\frac{3}{4} = 3 \div 4 = 4)\underline{3.00}$ $\qquad$ $\frac{3}{5} = 3 \div 5 = 5)\underline{3.0}$

Reduce the following to decimals:

(a) $\frac{3}{8}$. (b) $\frac{5}{8}$. (c) $\frac{7}{8}$. (d) $\frac{1}{125}$. (e) $\frac{5}{6}$. §

(f) $\frac{9}{40}$. (g) $\frac{11}{40}$. (h) $\frac{11}{16}$. (i) $\frac{2}{9}$. § (j) $\frac{1}{7}$. §

* To make this division in Groups 3 and 4, change the numerators to tenths, hundredths, etc.

† Lead the pupil to associate in mind the word *finis* (end) and the word *finite*. *Infinite* means *without end*. Let the pupil attempt to change $\frac{1}{3}$ or $\frac{1}{7}$ to a finite decimal.

‡ Compare this statement with the first part of page 176.

§ This cannot be reduced to a finite decimal. Lead the pupil to "carry out" his reduction to two or three decimal places, and to write the plus sign after the last figure, thus: $\frac{1}{3} = .333+$

Denominate Numbers—Specific Gravity.

A cubic yard of sand, gravel, or soil is called a load.

(a) If the specific gravity of sand is 2, how much does a load of sand weigh?

(b) I wish to cover a roadway that is 657 ft. long and 12 ft. wide with gravel 6 inches deep. To cover 1 linear foot of the road (that is, a piece 1 foot by 12 feet) will require how many cubic feet? (c) One load of gravel will cover how many linear feet of the roadway? (d) How many loads will be required for the entire roadway?

(e) If the specific gravity of granite is 2.7, how much does a cubic yard of it weigh?

(f) One thousand feet of 1-inch boards are how many cubic feet? *

(g) If the specific gravity of a certain grade of lumber is .7, what is the weight of 1000 feet of it? †

(h) How much do 1000 feet of $\frac{1}{8}$-inch boards weigh, if the specific gravity of the lumber is .5? ‡

(i) One thousand bricks, each 8 in. by 4 in. by 2 in., are how many cubic feet of bricks?

(j) Regarding a thousand bricks as exactly 37 cubic feet, and the specific gravity as exactly 2, how much do 1000 bricks weigh?

(k) If a brick, 8 in. by 4 in. by 2 in., weighs $4\frac{1}{2}$ pounds is its specific gravity more or less than 2? §

* How many feet of 1-inch lumber are equal to 1 cubic foot?

† Regard this lumber as 1 inch or more in thickness. Each foot of it is 1 ft. by 1 ft. by 1 in. or its equivalent.

‡ In this case a foot of lumber is a piece 1 ft. by 1 ft. by $\frac{1}{8}$ in. or its equivalent. How many feet of such lumber make a cubic foot?

§ That is, does 1 cubic foot of bricks weigh more or less than exactly twice as much as 1 cubic foot of water?

Measurements—Miscellaneous.

1. In a surface 10 feet by $12\frac{1}{2}$ feet there are —— square feet. How many square yards?

(a) How many square yards in a surface 35 feet by 40 ft.?

2. In a piece of oil-cloth 24 feet long and $\frac{1}{2}$ a yard wide there are —— square yards.

(b) How many square yards in a walk 224 feet long and $1\frac{1}{2}$ yards wide?

(c) How many square yards of plastering will be required for a ceiling that is 25 feet by 21 feet?

3. How many feet of boards will be required for a close board fence 200 feet long and 6 feet high?

(d) How many feet of boards will be required for a close board fence $5\frac{1}{3}$ feet high, inclosing a lot that is 90 feet by 228 feet?

4. At $15 a thousand, how much will the boards cost for the fence described in problem 3?

(e) At $15 per M., how much will the boards cost for the fence described in problem (d)?

5. How many boards, each 12 feet long and 1 foot wide, must be purchased for the fence described in problem 3?

(f) How many boards, each 16 feet long and 1 foot wide, must be purchased for the fence described in problem (d)?

6. If floor tiles are 4 inches square, every square foot of floor will require —— tiles. (Make a diagram to prove the correctness of your statement.)

(g) How many 4-inch floor tiles will be required to cover a floor that is 38 ft. by 23 ft.? (h) How much will the tile cost at $20 per M.?

Ratio and Proportion.

Cubic foot and bushel compared.

1. Is the ratio of 36 to 49 more or less than $\frac{3}{4}$?

2. Is the ratio of 31 to 45 more or less than $\frac{2}{3}$?

(a) Is the ratio of 340 to 855 more or less than $\frac{2}{5}$?

(b) Is the ratio of 305 to 450 more or less than $\frac{2}{3}$?

(c) One cubic foot contains 1728 cubic inches. *One bushel contains* 2150.4 *cubic inches.** Is the ratio of 1 cubic foot to 1 bushel more or less than $\frac{4}{5}$?

(d) Regarding the ratio of 1 cubic foot to 1 bushel as $\frac{4}{5}$, find the approximate capacity, in bushels, of a bin 8 feet long, 6 feet wide, and 5 feet deep.

(e) Is the capacity of a bin 8 feet long, 5 feet wide, and 7 feet deep, more or less than 224 bushels?

(f) What is the ratio, expressed in its lowest terms, of an oblong 2 feet by 3 feet, to an oblong three times as wide and three times as long?

(g) What is the ratio, expressed in its lowest terms, of an oblong 3 feet by 4 feet, to an oblong three times as wide and three times as long?

3. Compare the answers to problems (f) and (g). What is the ratio, expressed in its lowest terms, of any oblong to another three times as wide and three times as long?

4. What is the ratio of a 2-ft. square to a 6-ft. square? Of a 5-foot square to a 15-foot square? Of a 6-foot square to a 12-foot square?

(h) If 24 sheep cost $86 how much will 36 sheep cost at the same rate?

24 sheep : 36 sheep = $86 : $———.

* Require pupils to memorize the fact printed in italics.

Percentage.

Explain to pupils the meaning of the expression *per cent of attendance.*

1. The number of pupils belonging to a certain school was 30. The school was in session 20 days during the month of February. If every pupil had been present every day, the number of "days attendance" would have been ——.* But one pupil was absent 4 days, another 3 days, and another 2 days. The number of days of absence was ——. The number of days attendance was ——. (a) What was the per cent of attendance? †

SCHOOL REPORTS.

Room 1.	Room 2.
Number belonging, 45.	Number belonging, 36.
In session 20 days.	In session 20 days.
Days of absence, 30.	Days of absence, 35.
Days of attendance, (b)	Days of attendance, (d)
Attendance, %, ‡ (c)	Attendance, % § (e)
Room 3.	**Room 4.**
Number belonging, 37.	Number belonging, 46.
In session 20 days.	In session 19 days.
Days of absence, 54.	Days of absence, $13\frac{1}{2}$.
Days of attendance, (f)	Days of attendance, (h)
Attendance, %, (g)	Attendance, %, (i)

* 20 times 30 = 600. † Observe that every 6 days of attendance makes 1% of attendance.

‡ Observe that perfect attendance would have given (20 times 45) 900 days of attendance; therefore every 9 days of attendance makes 1 % of attendance.

§ Every 7.2 days of attendance makes 1 % of attendance.

TO THE TEACHER.—In practice, if there are permanent withdrawals from school during the month, the base upon which the percentage is to be computed is correspondingly less than the number obtained by taking the possible attendance for 1 day as many times as the number of days the school was in session.

Percentage — Interest.*

1. What per cent of the principal equals the interest if the rate is 6 % and the time is ——

1 month? †	1 year 1 month?	2 years 1 month?
2 months?	1 year 2 months?	2 years 2 months?
3 months?	1 year 3 months?	2 years 3 months?
4 months?	1 year 4 months?	2 years 4 months?
5 months?	1 year 5 months?	2 years 5 months?
6 months?	1 year 6 months?	3 years 6 months?
7 months?	2 years 7 months?	3 years 7 months?
8 months?	2 years 8 months?	3 years 8 months?
9 months?	2 years 9 months?	3 years 9 months?
10 months?	2 years 10 months?	3 years 10 months?
11 months?	2 years 11 months?	3 years 11 months?

(a) Find the interest of $500 for 1 year 1 month at 6 %.

(b) Find the interest of $250 for 2 years 5 months at 6 %.

(c) Find the interest of $375 for 1 year 8 months at 6 %.

(d) Find the interest of $344 for 3 years 7 months at 6 %.

2. From Jan. 1, 1896 to June 1, 1897, it is —— yr. —— mo.

3. From Apr. 10, 1896, to Dec. 10, 1897, it is —— yr. —— mo.

4. From June 4, 1895, to Nov. 4, 1897, it is —— yr. —— mo.

5. From Aug. 17, 1895, to Oct. 17, 1896, it is —— yr. —— mo.

(e) On the first day of January, 1895, Mr. Peterson borrowed of Mr. Dix, for 1 year 6 months, at 6 %, $248. When must Mr. Peterson pay Mr. Dix? (f) How much must he pay Mr. Dix?

* Review page 218. † $\frac{1}{2}$ of 1 %, or $\frac{1}{2}$ %.

Review.

1. At $\$\frac{3}{5}$ of a dollar a yard, how much will $\frac{4}{7}$ of a yard of cloth cost? $\frac{3}{5} \times \frac{4}{7} =$ $\frac{4}{7} \times \frac{3}{5} =$

(a) Multiply $\frac{3}{5}\frac{4}{7}$ by $\frac{4}{7}$. (b) $\frac{4}{7}\frac{3}{5} \times \frac{3}{5} =$

(c) At $27\frac{1}{3}$ dollars an acre, how much will $3\frac{3}{4}$ acres of land cost? $27\frac{1}{3} \times 3\frac{3}{4} =$

(d) Reduce $\frac{11}{16}$ to a decimal. (e) $\frac{11}{20}$. (f) $\frac{1}{3}$.

(g) Reduce .325 to a common fraction in its lowest terms.

2. Is the specific gravity of ice more or less than 1?*

(h) Does a block of ice 2 feet by 2 feet by $1\frac{1}{2}$ feet, weigh more or less than 375 lb.?

(i) What is the weight of 8 cubic feet of water?

(j) Regarding the ratio of 1 cubic foot to 1 gallon as $7\frac{1}{2}$, find the approximate capacity, in cubic feet, of a tank that holds 1500 gallons.

(k) The water in a cistern 8 feet square is 2 feet deep; find, approximately, the number of gallons of water in the cistern.

(l) Regarding the ratio of 1 cubic foot to 1 bushel as $\frac{4}{5}$, find the approximate capacity, in cubic feet, of a bin that will hold 500 bushels of oats.

(m) The wheat in a bin 8 feet square is 5 feet deep; find, approximately, the number of bushels of wheat in the bin.

3. What is the ratio of a $2\frac{1}{2}$-foot square to a 5-ft. square? Of a $2\frac{1}{2}$-ft. square to a $7\frac{1}{2}$-ft. square?

(n) If the total days of attendance for a certain school was 740 and the days of absence 60, what was the per cent of attendance?†

(o) Days of attendance 894; days of absence 56. Find per cent of attendance.

* Does the ice sink or float in water?
† The *base* is 740 + 60.

Miscellaneous Problems.

1. The first day of January, 1896, was Wednesday. February 1, 1896, was ——.* Jan. 1, 1897, was ——.

2. The first day of January this year was ——.

February 1, ——.	March 1, ——.
April 1, ——	May 1, ——.
June 1, ——.	July 1, ——.
July 4, ——.	August 1, ——.
September 1, ——.	October 1, ——.
November 1, ——.	December 1, ——.
December 25, ——.	Next New Year's day——.

(a) On Monday morning Mr. Phillips had $231.46 in the bank. His deposits during the week were as follows: $21.75, $36.54, $181.50, $75, $230, and $143.25. He drew from the bank during the week the following: $61.50, $85, $73.20, $11.40, $6.35, $120.31, and $3.60. How much had he on deposit at the end of the week?

(b) Mrs. Allen has loaned to Mr. James $5500 upon which Mr. James agrees to pay interest annually at the rate of 6 %. How much "interest money" will he pay her each year?

(c) Find the ratio of a rectangle 16 ft. by 32 ft. to a 64-foot square.†

3. What is the value at $75 per acre, of a rectangular field 40 rods by 12 rods?‡

(d) What is the value at $75 per acre of a rectangular field 40 rods by 23 rods?

(e) What is the value at $75 per acre of a square field whose side is 32 rods?

* See Book I, pp. 124, 134, and 144.

† Solve in two ways: (1) Draw a diagram of the square on a scale of 8 feet to the inch. Within the square draw a diagram of the rectangle on the same scale. See how many times the rectangle is contained in the square. (2) Find the number of square feet in each and compare them by division.

‡ Land 40 rods by 4 rods contains 1 acre.

Simple Numbers — Short Methods.

1. $36 \times 50 =$ *	**2.** $84 \times 50 =$	**3.** $48 \times 50 =$
4. $35 \times 50 =$	**5.** $63 \times 50 =$	**6.** $55 \times 50 =$
7. $36 \times 51 =$ †	**8.** $48 \times 51 =$	**9.** $24 \times 51 =$
10. $84 \times 51 =$	**11.** $76 \times 51 =$	**12.** $38 \times 51 =$
13. $36 \times 49 =$ ‡	**14.** $24 \times 49 =$	**15.** $26 \times 49 =$
16. $52 \times 49 =$	**17.** $76 \times 49 =$	**18.** $32 \times 49 =$
19. $36 \times 25 =$ §	**20.** $37 \times 25 =$	**21.** $38 \times 25 =$
22. $44 \times 25 =$	**23.** $28 \times 25 =$	**24.** $41 \times 25 =$
25. $36 \times 26 =$ ‖	**26.** $48 \times 26 =$	**27.** $44 \times 26 =$
28. $32 \times 26 =$	**29.** $40 \times 26 =$	**30.** $28 \times 26 =$
31. $36 \times 24 =$ ¶	**32.** $48 \times 24 =$	**33.** $44 \times 24 =$
34. $40 \times 24 =$	**35.** $32 \times 24 =$	**36.** $28 \times 24 =$

Find the cost:

37. 24 bushels apples at 50¢ a bushel.

38. 26 yards cloth at 51¢ a yard. **

39. 28 pairs shoes at 49¢ a pair. ††

40. 32 pounds coffee at 25¢ a pound.

41. 36 yards ribbon at 26¢ a yard. ‡‡

42. 44 dozen lemons at 24¢ a dozen. §§

* $36 \times 50 = (36 \times 100) \div 2 = 18$ hundred, or 1800.
† $36 \times 51 = (36 \times 50) + 36 = 18$ hundred 36, or 1836.
‡ $36 \times 49 = (36 \times 50) - 36 = 17$ hundred 64, or 1764.
§ $36 \times 25 = (36 \times 100) \div 4 = 9$ hundred, or 900.
‖ $36 \times 26 = (36 \times 25) + 36 = 9$ hundred 36, or 936.
¶ $36 \times 24 = (36 \times 25) - 36 = 8$ hundred 64, or 864.
** At 50¢ a yard, 26 yards would cost $13.
†† At 50¢ a pair, 28 pairs would cost $14.
‡‡ At 25¢ a yard, 36 yards would cost $9.
§§ At 25¢ a dozen, 44 dozen would cost $11.

Common Fractions—Division.

1. At $\frac{4}{5}$ of a dollar a yard, how many yards of ribbon can be purchased for $\frac{3}{4}$ of a dollar?

$$\frac{3}{4} \div \frac{4}{5} = \frac{15}{20} \div \frac{16}{20} = \frac{15}{16} = 1\frac{7}{16}.$$

(1) Observe that in dividing $\frac{3}{4}$ by $\frac{4}{5}$ we reduce them to equivalent fractions having a common denominator, and then divide the numerator of the dividend, 15, by the numerator of the divisor, 8.

(2) *Observe that we make no use whatever of the common denominator*, 20.

(3) Observe that the numerator, 15, becomes the numerator of the quotient; that the numerator, 8, becomes the denominator of the quotient.

(4) Observe that the number 15 was obtained by multiplying the numerator, 3, by the denominator, 5; that the number 8 was obtained by multiplying the denominator, 4, by the numerator, 2.

Hence, $\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{15}{16} = 1\frac{7}{16}$.*

(5) Since any two fractions may be divided, the one by the other, in a similar manner and with similar results we have the following rule for division of fractions:

Invert the divisor; then multiply the numerators together for the numerator of the quotient, and multiply the denominators together for the denominator of the quotient.†

(a) Divide $\frac{3}{5}$ by $\frac{5}{6}$. (b) Divide $\frac{7}{8}$ by $\frac{4}{5}$.

(c) Divide $\frac{3}{11}$ by $\frac{5}{7}$. (d) Divide $\frac{9}{13}$ by $\frac{5}{8}$.

(e) Divide $2\frac{3}{4}$ by $\frac{5}{7}$. (f) Divide $\frac{3}{5}$ by $2\frac{3}{4}$.

(g) James can hoe a row of corn in $\frac{3}{5}$ of an hour. How many rows can he hoe in $5\frac{3}{4}$ hours?

* Require pupils to solve several problems similar to this in two ways: (1) by reducing the fractions to equivalent fractions having a common denominator, and (2) by inverting the divisor, etc.

† Teachers who prefer to do so can use the following explanation of the rule for inverting the divisor:

$\frac{3}{4} \div \frac{4}{5}$ { 1 divided by $\frac{4}{5} = \frac{5}{4}$. / $\frac{3}{4}$ divided by $\frac{4}{5} = \frac{3}{4}$ of $\frac{5}{4}$, or $\frac{15}{16}$.

Decimal Fractions.

1. The sum of $24\frac{1}{4}$ and 12.6 is ——.*

(a) Add $386\frac{3}{8}$ and 125.026. (b) $45\frac{5}{8} + 34.25 =$

2. The difference of $29\frac{3}{4}$ and 12.6 is ——.

(c) From $275\frac{7}{8}$ subtract 148.37. (d) $493\frac{7}{10} - 146.256 =$

3. The product of $2\frac{1}{2}$ multiplied by .3 is ——.

(e) Multiply $138\frac{3}{8}$ by 2.6. (f) $271\frac{7}{8} \times 3.5 =$

4. The quotient of $6\frac{1}{2}$ divided by 1.3 is ——.

(g) Divide $275\frac{1}{4}$ by .25. (h) $59\frac{3}{8} \div 1.1 =$

Find the cost:

(i) 54,600 bricks at $5.25 per M.

(j) 850 pounds flour at $2.75 per cwt.

(k) 4852 feet lumber at $15.50 per M.

(l) 2450 pounds beef at $7.25 per cwt.

(m) 48 oranges at 75¢ per hundred.

(n) If hay is $12.50 a ton, how many tons can be purchased for $28.50? (o) How many pounds?

(p) If bricks are $6.25 per M., how many thousand can be purchased for $11.25? (q) How many bricks?

(r) If beef is $6.40 per cwt., how many cwt. can be purchased for $53.44? (s) How many pounds?

(t) If a locomotive moves at the rate of 26.4 miles an hour, in how many hours will it move 84.48 miles?

(u) If a rectangular piece of land is 25 rods long, how wide must it be to contain one acre?

(v) If a rectangular piece of land is 24 rods long, how wide must it be to contain 1 acre?

(w) If a rectangular piece of land is 50 rods long, how wide must it be to contain 1 acre?

* Reduce $\frac{1}{4}$ to a decimal.

Denominate Numbers—Addition, Etc.

1. 11 ft. 8 in. + 13 ft. 10 in. + 2 ft. 3 in. =

(a) 18 ft. 9 in. + 25 ft. 8 in. + 36 ft. 3 in. + 13 ft. 7 in.

2. 5 min. 20 sec. − 1 min. 50 sec. =

(b) 27 min. 12 sec. − 8 min. 52 sec.

3. 8 bu. 3 pk. × 3 =

(c) 45 bu. 1 pk. × 12.

4. 12 gal. 1 qt. × 6 =

(d) 74 gal. 3 qt. × 8.

5. 10 wk. 2 da. ÷ 3 =

(e) 52 wk. 1 da. ÷ 5.

6. 9 yr. 8 mo. ÷ 4 =

(f) 26 yr. 6 mo. ÷ 6.

7. 5 lb. 10 oz. ÷ 1 lb. 4 oz. =

(g) 45 lb. 10 oz. ÷ 1 lb. 9 oz.

8. 2 bu. 3 qt. = ——— quarts.

(h) Change 47 bu. 13 qt. to quarts.

9. 2 tons 300 lb. = ——— pounds.

(i) Change 7 tons 1340 lb. to pounds.

(j) A liveryman had 50 bu. of oats; he fed from them 2 bu. 3 pk. each day for 2 weeks. How many oats remained?

(k) Six miles are how many feet?

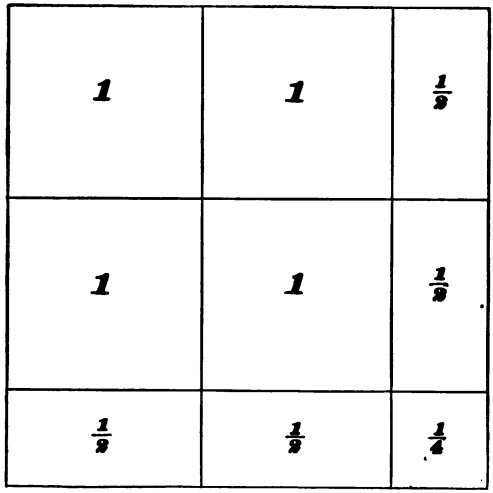
(l) If a bicycle moves forward 18 feet at each revolution of the pedal shaft, how many revolutions of this shaft will be made in going one mile?

(m) At 18¢ a gallon, how much will the January milk bill be for a hotel that uses 14 quarts each Sunday and 10 quarts a day on other days, the first day of January being Saturday?

(n) At 48 cents a bushel, what is the value of 1 ton of oats?

(o) At 90¢ a bushel, what is the value of 1 ton of wheat?

Measurements.



1. The above is a $2\frac{1}{2}$-inch square. It contains —— and —— —— square inches. $2\frac{1}{2} \times 2\frac{1}{2}$, means, 2 times $2\frac{1}{2}$ plus 1 half of $2\frac{1}{2}$. $2\frac{1}{2} \times 2\frac{1}{2}$ =

2. Draw a $3\frac{1}{2}$-inch square and divide it in such a manner that you can count the square inches in its area. $3\frac{1}{2} \times 3\frac{1}{2}$ =

3. Draw an oblong $4\frac{1}{2}$ inches by $3\frac{1}{4}$ inches and divide it in such a manner that you can count the square inches in its area. $4\frac{1}{2} \times 3\frac{1}{4}$, means, 3 times $4\frac{1}{2}$ plus 1 fourth of $4\frac{1}{2}$. $4\frac{1}{2} \times 3\frac{1}{4}$ =

(a) What is the area of a $7\frac{1}{2}$-in. square? (b) Of an $8\frac{1}{2}$-in. square?

(c) What is the area of an oblong $8\frac{1}{2}$ inches by $3\frac{1}{4}$ inches? (d) $12\frac{1}{2}$ inches by $7\frac{1}{4}$ inches?

(e) How many square rods in a piece of land $15\frac{1}{2}$ rods long and $8\frac{1}{4}$ rods wide?

Ratio and Proportion.

1. $5\frac{1}{2}$ inches : 1 foot = $5\frac{1}{2}$: ——; this = 11 : ——.
   $5\frac{1}{2}$ inches is† —— —— of a foot.*

2. 8 inches : 1 yard = 8 : —— ; this = 2 : ——.
   8 inches is —— —— of a yard.

3. 1 ft. 4 in. : 1 yard = 16 : ——; this = 4 : ——.
   1 ft. 4 in. is —— —— of a yard.

4. 7 feet : 1 rod = 7 : ——; this = 14 to ——.
   7 feet is —— —— of a rod.

5. 12 ounces : 1 pound = 12 : ——; this = 3 : ——.
   12 ounces is —— —— of a pound.

6. 1 pk. 6 qt. : 1 bushel = 14 : ——; this = 7 : ——.
   1 pk. 6 qt. is —— —— of a bushel.

(a) Change $7\frac{1}{4}$ inches to the fraction of a foot.

(b) Change $4\frac{1}{2}$ inches to the fraction of a yard.

(c) Change 1 ft. 8 in. to the fraction of a yard.

(d) Change $5\frac{1}{2}$ feet to the fraction of a rod.

(e) Change $11\frac{1}{2}$ ounces to the fraction of a pound.

(f) Change 2 pk. 4 qt. to the fraction of a bushel.

(g) Change 240 rods to the fraction of a mile.

(h) Change 120 rods to the fraction of an acre.

* To be read—The ratio of $5\frac{1}{2}$ inches to 1 foot equals the ratio of $5\frac{1}{2}$ to 12; this equals the ratio of 11 to 24. $5\frac{1}{2}$ inches is $\frac{11}{24}$ of a foot.

† The word *is* with $5\frac{1}{2}$ *inches*, 8 *inches*, etc., as its subject is used advisedly on this page. In the early work in number and always when emphasis is to be placed upon the minor units of which a major unit is composed, it is quite proper to use the plural form of a verb with such a subject. But when the necessity for such emphasis does not exist, we naturally and correctly adopt the singular form. Thus, in the primary room with objects divided into groups before the pupil, we say two thirds of 9 blocks are 6 blocks. But in the eighth grade we correctly say two thirds of $75 is $50. So one may say, 8 inches *is* or 8 inches *are*, the choice of the verb to be determined by the desire of the speaker to make emphatic or not the multiplicity in this 8-inch unity.

Percentage—Miscellaneous.

1. To insure a house for $3000 at 1½ % will cost ——.

(a) How much will it cost to insure property to the amount of $9370 at 2½ % ?

2. The number of pupils belonging to a certain school was 41. The school was in session 20 days. If there had been no absence the number of days attendance would have been ——. There were 20½ days of absence. (b) What was the per cent of absence? (c) What was the number of days attendance? (d) What was the per cent of attendance?

3. I loaned $800 at 6 % interest. For the use of this money for one year, I should receive —— dollars. (e) How much should I receive for the use of the same sum at the same rate for 7½ months?

(f) A Chicago Commission House sold for Mr. H. B. Gurler, 1350 lb. butter @ 22¢ a pound. What was the sum received at the Commission House for the butter? (g) The rate of commission was 5 %; the commission amounts to what sum? (h) If the Commission House had paid $12.75 freight on the butter, and $1.25 cartage, how much money should be remitted to Mr. Gurler?

(i) A dealer imported goods valued at $586.40. The rate of duty was 30 %. How much must the dealer pay to the government?

(j) A house and lot was assessed (apprised by the assessor) at $7360. The taxes upon this house were as follows:

State tax,	3½ mills "on a dollar."
County tax,	6 mills "on a dollar."
Road and Bridge tax,	3 mills "on a dollar."
Town tax,	½ mill "on a dollar."
School tax,	1 per cent.

Find the amount of the taxes upon this property.

Percentage — Interest.*

In computing interest, 30 days is usually regarded as a month; 15 days, ½ of a month; 6 days, ⅕ of a month, etc.

1. What per cent of the principal equals the interest if the rate is 6 % and the time 1 day? Ans. $\frac{1}{30}$ of ⅙ %, or $\frac{1}{180}$ %.†

2. The interest of any sum for 1 day at 6 % equals $\frac{1}{180}$ of 1 % of the principal; for 2 days, $\frac{2}{180}$, or $\frac{1}{90}$ of 1 % of the principal; for 3 days, $\frac{3}{180}$, or $\frac{1}{60}$ of 1 % of the principal; for 4 days ————.

3. What per cent of the principal equals the interest if the rate is 6 % and the time is —

5 days?	2 months 5 days? ‡	1 yr. 2 mo. 5 da.?	
6 days?	2 months 6 days?	1 yr. 6 mo. 2 da.?	
7 days?	2 months 7 days?	1 yr. 2 mo. 7 da.?	
8 days?	4 months 8 days?	1 yr. 4 mo. 8 da.?	
9 days?	4 months 9 days?	1 yr. 4 mo. 9 da.?	
10 days?	6 months 10 days?	1 yr. 6 mo. 10 da.?	
15 days?	8 months 15 days?	1 yr. 8 mo. 15 da.?	
20 days?	10 months 20 days?	1 yr. 10 mo. 20 da.?	
29 days?	10 months 29 days?	1 yr. 10 mo. 29 da.?	

(a) Find the interest of $800 for 1 yr. 2 mo. 15 da. @ 6 %.§

(b) Find the interest of $650 for 1 yr. 4 mo. 12 da. @ 6 %.

(c) Find the interest of $850 for 1 yr. 6 mo. 10 da. @ 6 %.

(d) Find the interest of $320 for 1 yr. 8 mo. 20 da. @ 6 %.

(e) Find the interest of $530 for 1 yr. 10 mo. 24 da. @ 6 %.

(f) Find the interest of $360 for 2 yr. 4 mo. 6 da. @ 6 %.

* Review page 218.

† This means $\frac{1}{30}$ of 1 per cent.

‡ $1\frac{1}{2}$ per cent.

§ 7½ % of $800, or 7½ times $8.

Review.

1. 32 lb. butter at 25¢ a pound cost ——.
2. 32 lb. butter at 26¢ a pound cost ——.*
3. 32 lb. butter at 24¢ a pound cost ——.
4. 33 lb. butter at 25¢ a pound cost ——.
5. 35 lb. butter at 25¢ a pound cost ——.

(a) At $\frac{3}{4}$ of a dollar a bushel, how many bushels of apples can be purchased for $18\frac{1}{2}$ dollars? †

(b) At $\frac{5}{8}$ of a dollar a pound, how many pounds of tea can be purchased for $18\frac{1}{2}$ dollars?

(c) If a man in walking, steps $2\frac{1}{2}$ feet each time, how many steps will he take in walking 1 mile?

(d) If a man takes 3-foot steps, 1000 steps will equal what part of a mile?

(e) How many square feet in a blackboard $4\frac{1}{2}$ feet by $2\frac{1}{4}$ feet? ‡

(f) How many square rods in a piece of land $10\frac{1}{2}$ rods by $2\frac{1}{4}$ rods?

6. $1\frac{1}{4}$ feet are —— —— of a yard.
7. $1\frac{1}{4}$ pecks are —— —— of a bushel.
8. $1\frac{1}{4}$ inches are —— —— of a foot.
9. $1\frac{1}{4}$ quarts are —— —— of a gallon.

(g) Change $12\frac{1}{4}$ ounces to the fraction of a pound.

(h) Change $12\frac{1}{4}$ feet to the fraction of a rod.

(i) Change $12\frac{1}{4}$ inches to the fraction of a yard.

* Observe that at 26¢ a pound it costs 32¢ more than it costs at 25¢ a pound.

† Solve in three ways: (1) Change the number to be divided to fourths; (2) Change $18\frac{1}{2}$ to halves and invert the divisor, etc.; (3) Change both fractions to decimals.

‡ Draw an oblong $4\frac{1}{2}$ feet by $2\frac{1}{4}$ feet and divide it in such a manner that you can count the square feet in its area. See page 235.

Miscellaneous.

1. Dec. 25, 1896, was Friday. Jan. 1, 1897, was ——.
   January 1, 1898 ——.* January 1, 1899 ——.
   January 1, 1900 ——. January 1, 1901 ——.†
   January 1, 1902 ——. December 25, 1902 ——.
   January 1, 1903 ——. December 25, 1903 ——.

(a) If bricks are 8 in. by 4 in. by 2 in., how many bricks laid on their 2-in.-by-8-in. face will be required to pave 1 square foot?

(b) Paving bricks are usually somewhat larger than those described in problem (a). If the size is such that $7\frac{1}{2}$ bricks are required to the square foot, how many bricks must be purchased to pave a street 10 rods long and 64 feet wide?

(c) How many paving tiles 6 in. by 6 in. would cover a surface 6 ft. by 24 ft.?

(d) A piece of land is 18 rods long; how wide must it be to contain 1 acre?

(e) How many miles an hour does a man walk who takes 2 steps a second, each step being $2\frac{1}{2}$ feet?

(f) How many bushels of oats will weigh as much as 40 bushels of wheat?

(g) A dealer bought 5 reams of paper at \$2.50 per ream and sold it at 18¢ a quire. How much was his gain?

(h) How many hours in 2 weeks 3 days?

(i) When oats cost 32¢ a bushel, what is the price per ton?

(j) Find the cost of excavating a cellar 18 feet square and 4 feet deep at \$1.20 per cubic yard.

* Remember that there are 52 weeks and 1 day in a common year.
† Observe that 1900 is not a leap year.

Simple Numbers—Short Methods.

1. $36 \times 33\frac{1}{3} =$ *	2. $27 \times 33\frac{1}{3} =$	3. $48 \times 33\frac{1}{3} =$
4. $33 \times 33\frac{1}{3} =$	5. $45 \times 33\frac{1}{3} =$	6. $30 \times 33\frac{1}{3} =$
7. $36 \times 34\frac{1}{3} =$ †	8. $24 \times 34\frac{1}{3} =$	9. $48 \times 34\frac{1}{3} =$
10. $45 \times 34\frac{1}{3} =$	11. $18 \times 34\frac{1}{3} =$	12. $39 \times 34\frac{1}{3} =$
13. $36 \times 32\frac{1}{3} =$ ‡	14. $30 \times 32\frac{1}{3} =$	15. $48 \times 32\frac{1}{3} =$
16. $45 \times 32\frac{1}{3} =$	17. $21 \times 32\frac{1}{3} =$	18. $39 \times 32\frac{1}{3} =$
19. $32 \times 12\frac{1}{2} =$ §	20. $34 \times 12\frac{1}{2} =$	21. $36 \times 12\frac{1}{2} =$
22. $40 \times 12\frac{1}{2} =$	23. $64 \times 12\frac{1}{2} =$	24. $56 \times 12\frac{1}{2} =$
25. $32 \times 13\frac{1}{2} =$ ‖	26. $40 \times 13\frac{1}{2} =$	27. $64 \times 13\frac{1}{2} =$
28. $56 \times 13\frac{1}{2} =$	29. $24 \times 13\frac{1}{2} =$	30. $48 \times 13\frac{1}{2} =$
31. $32 \times 11\frac{1}{2} =$ ¶	32. $64 \times 11\frac{1}{2} =$	33. $40 \times 11\frac{1}{2} =$
34. $56 \times 11\frac{1}{2} =$	35. $24 \times 11\frac{1}{2} =$	36. $72 \times 11\frac{1}{2} =$

Find the cost:

37. 36 bushels of oats at $33\frac{1}{3}¢$ a bushel.
38. 75 bushels oats at $34\frac{1}{3}¢$ a bushel. **
39. 45 pounds coffee at $32\frac{1}{2}$c a pound. ††
40. 56 dozen eggs at $12\frac{1}{2}¢$ a dozen.
41. 48 gal. kerosene at $13\frac{1}{2}¢$ a gallon. ‡‡
42. 64 pounds fish at $11\frac{1}{2}¢$ a pound. §§

* $36 \times 33\frac{1}{3} = (36 \times 100) \div 3 = 12$ hundred, or 1200.
† $36 \times 34\frac{1}{3} = (36 \times 33\frac{1}{3}) + 36 = 12$ hundred 36, or 1236.
‡ $36 \times 32\frac{1}{3} = (36 \times 33\frac{1}{3}) - 36 = 11$ hundred 64, or 1164.
§ $32 \times 12\frac{1}{2} = (32 \times 100) \div 8 = 4$ hundred, or 400.
‖ $32 \times 13\frac{1}{2} = (32 \times 12\frac{1}{2}) + 32 = 8$ hundred 32, or 832.
¶ $32 \times 11\frac{1}{2} = (32 \times 12\frac{1}{2}) - 32 = 7$ hundred 68, or 768.
** At $33\frac{1}{3}¢$ a bushel, 75 bushels would cost $25.
†† At $33\frac{1}{3}¢$ a pound, 45 pounds would cost $15.
‡‡ At $12\frac{1}{2}¢$ a gallon, 48 gallons would cost $6.
§§ At $12\frac{1}{2}¢$ a pound, 64 pounds would cost $8.

Common Fractions.

Teach orally the meaning and use of the expressions, simple fraction, compound fraction, complex fraction.

1. $\frac{3}{4}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{12}$, ——, ——, and ——, are simple fractions.

2. $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{2}{3}$ of $\frac{3}{4}$, ——, and ——, are compound fractions.

3. $\frac{4}{5\frac{1}{2}}$,* $\frac{3\frac{1}{2}}{4}$,† $\frac{3\frac{1}{2}}{2\frac{1}{2}}$, $\frac{\frac{1}{2}}{2\frac{1}{2}}$, $\frac{\frac{3}{4}}{\frac{1}{2}}$, are complex fractions.

Reduce the following compound fractions to simple fractions in their lowest terms:

4. $\frac{1}{2}$ of $\frac{1}{4}$ = $\frac{1}{2}$ of $\frac{3}{4}$ = $\frac{1}{2}$ of $\frac{1}{8}$ = $\frac{1}{2}$ of $\frac{6}{8}$ =

5. $\frac{1}{3}$ of $\frac{1}{4}$ = $\frac{2}{3}$ of $\frac{1}{4}$ = $\frac{2}{3}$ of $\frac{3}{4}$ = $\frac{2}{3}$ of $\frac{4}{6}$ =

From the foregoing, make a rule and reduce the following:

(a) $\frac{5}{8}$ of $\frac{17}{24}$ = (b) $\frac{7}{8}$ of $\frac{15}{16}$ = (c) $\frac{3}{5}$ of $\frac{15}{21}$ =

(d) $\frac{3}{8}$ of $\frac{18}{15}$ = (e) $\frac{4}{7}$ of $\frac{14}{32}$ = (f) $\frac{8}{11}$ of $\frac{8}{11}$ =

Observe that reducing a complex fraction to a simple fraction is simply changing the numerator and the denominator into two integral numbers whose ratio is equal to the ratio of the given numerator and denominator.

$$\frac{2}{5\frac{1}{2}} = \frac{4}{11}. \qquad \frac{2\frac{1}{2}}{6} = \frac{5}{12}. \qquad \frac{\frac{3}{4}}{\frac{1}{2}}‡ = \frac{8}{9}.$$

Reduce the following complex fractions to simple fractions:

6. $\frac{3}{5\frac{1}{2}}$ $\frac{3\frac{1}{2}}{8}$ $\frac{2\frac{1}{2}}{5\frac{1}{2}}$ $\frac{\frac{1}{2}}{\frac{3}{4}}$ $\frac{2\frac{1}{2}}{6}$

(g) $\frac{7\frac{1}{4}}{12\frac{1}{4}}$ = (h) $\frac{9\frac{1}{2}}{13\frac{3}{8}}$ = (i) $\frac{8\frac{1}{2}}{7\frac{1}{4}}$ = (j) $\frac{12\frac{1}{4}}{13\frac{1}{8}}$ =§

* This may be read in two ways: (1) 4 over $5\frac{1}{2}$; (2) 4 divided by $5\frac{1}{2}$.

† This may be read in three ways: (1) $3\frac{1}{2}$ over 4; (2) $3\frac{1}{2}$ divided by 4; (3) $3\frac{1}{2}$ fourths.

‡ Change $\frac{3}{4}$ and $\frac{1}{2}$ to 12ths.

§ Change $12\frac{1}{4}$ and $13\frac{1}{8}$ to 80ths.

Decimal Fractions.*

1. Change $1\frac{1}{2}$ feet to the decimal of a yard. $1\frac{1}{2}$ feet = $\frac{1}{2}$ of a yard; $\frac{1}{2}$ of a yd. = ——— tenths of a yard.

2. Change 9 in. to the decimal of a yard. 9 in. = $\frac{1}{4}$ of a yard; $\frac{1}{4}$ of a yard = ——— hundredths of a yard.

Reduce:

3. 27 inches to the decimal of a yard.
4. 4 inches to the decimal of a foot.
5. 2 feet to the decimal of a yard.
6. 9 inches to the decimal of a foot.
7. 4 ounces to the decimal of a pound.
8. 80 rods to the decimal of a mile.
9. 120 sq. rd. to the decimal of an acre.
10. 9 cu. ft. to the decimal of a cu. yard.

(a) 12 quarts to the decimal of a bushel.

(b) 10 ounces to the decimal of a pound.

(c) 20 sq. rd. to the decimal of an acre.

(d) 100 rods to the decimal of a mile.

(e) If a rectangular piece of land is 25 rods long, how wide must it be to contain two acres?

(f) If a piece of carpet is 2 feet wide, how long must it be to contain 3 square yards?

(g) If a piece of gold foil is 2.5 inches wide, how long must it be to contain a square foot?

(h) If a rectangular piece of land is .5 of a rod wide, how long must it be to contain an acre?

(i) A rectangular piece of land 1 mile long and $2\frac{1}{2}$ rods wide contains how many acres?

* Review pp. 223 and 236.

Denominate Numbers — Reduction.

Reduce:

1. 5 bu. 3 pk. to pecks.	(a) 27 bu. 1 pk. =*
2. 2 lb. 9 oz. to ounces.	(b) 23 lb. 5 oz. =
3. 2 bu. 7 qt. to quarts.	(c) 31 bu. 9 qt. =
4. 2 rd. 1 yd. to yards.	(d) 26 rd. 4 yd. =
5. 6 gal. 1 qt. to pints.	(e) 35 gal. 3 qt. =
6. 9 yd. 1 ft. to feet.	(f) 43 yd. 2 ft. =
7. 2 hr. 20 min. to minutes.	(g) 18 hr. 45 min. =
8. 7 wk. 4 da. to days.	(h) 29 wk. 5 da. =
9. 5 yr. 7 mo. to months.	(i) 17 yr. 8 mo. =
10. 1 mile to feet.†	(j) 5 miles =

11. If a watch loses 20 seconds a day, how many minutes and seconds will it lose in a week? (k) In the month of May?

12. If a canal-boat moves at the rate of 1 mile in 17 min. 30 sec., how long will it take it to move 3 miles? (l) 34 miles?

13. Four rods equal —— feet. (m) One ninth of 4 rods equals how many feet and inches?

14. If a bullet weighs $\frac{1}{2}$ an ounce, how many pounds and ounces will 100 bullets weigh? (n) 1000 bullets?

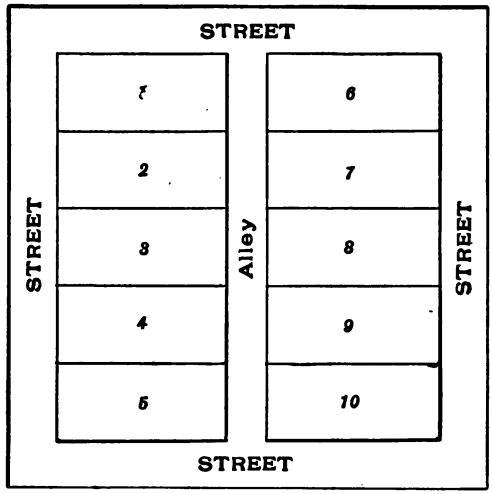
15. A bushel of clover seed weighs 60 lb. How much will 1 peck of clover seed cost at 12¢ a pound? (o) 5 bu. 3 pk.?

16. How many sheets of paper, each 7 in. by $8\frac{1}{2}$ in., can be cut from a piece of paper 22 in. by 28 in.?‡ (p) How many square inches of paper will be wasted?

17. If a horse eats 10 quarts of oats a day, how many bushels and quarts will he eat in a week? (q) In the month of January?

* Change as indicated in the corresponding problem in the first column.
† The answer to this problem should be memorized.
‡ Make an exact diagram of this upon the blackboard or upon a sheet of wrapping paper.

Measurements.



1. Mr. A. owned a block of land 20 rods square. It contained —— acres. He gave for streets a strip 50 ft. wide on each side of the block. He made an alley 30 feet wide through the center of the block. The remainder of the land he divided into 10 equal lots as indicated in the diagram. (a) How many feet wide was each lot? (b) How many feet long? (c) He paid $280 an acre for the land. He sold the corner lots for $250 each and the others for $200 each. If his expenses in laying out the lots, building sidewalks, setting out trees, etc., were $975.40, how much was his gain?

2. A rod is —— yards. (d) A $5\frac{1}{2}$-yd. square contains how many square yards? *

3. A rod is —— feet. (e) A $16\frac{1}{2}$-ft. square contains how many square feet? †

4. A strip of land must be how long to contain 1 acre, if it is 4 rods wide? (f) If it is $4\frac{1}{2}$ rods wide? (g) If it is $12\frac{1}{2}$ rods wide?

* Show by a diagram the number of square yd. in a square rod.

† How many square feet in a square rod?

Ratio and Proportion.

Barrel and cubic foot compared.

(a) $31\frac{1}{2}$ *gallons make* 1 *barrel.** There are 231 cubic inches in a gallon. How many cubic inches in a barrel?

(b) Is the ratio of a barrel to a cubic foot more or less than $4\frac{1}{2}$?

(c) Regarding the ratio of 1 barrel to 1 cubic foot as $4\frac{1}{2}$, find the approximate capacity in barrels, of a tank 16 feet long, 4 feet wide, and 2 feet deep.

(d) Is the capacity of a tank $10\frac{1}{2}$ feet long, 4 feet wide, and 2 feet deep, more or less than 20 barrels?

(e) What is the ratio, expressed in its lowest terms, of an oblong 2 feet by 3 feet, to an oblong 4 times as wide and 4 times as long?

(f) What is the ratio, expressed in its lowest terms, of an oblong 3 feet by 4 feet, to an oblong 4 times as wide and 4 times as long?

1. Compare the answers to problems (e) and (f). What is the ratio, expressed in its lowest terms, of any oblong to another 4 times as wide and 4 times as long?

2. What is the ratio of a 2-ft. square to an 8-ft. square? Of a 3-ft. square to a 12-ft. square? Of a $\frac{1}{2}$-inch square to a 2-inch square? †

(g) If it costs $24.30 to carpet a floor that is 12 ft. square, how much will it cost to carpet a floor that is 24 feet square?

A 12-ft. square : a 24-ft. square = $24.30 : ———.

(h) What is the ratio of a 12-foot square to a 36-foot square?

* Explain to pupils that this is the barrel *used as a measure.* A kerosene barrel or a vinegar barrel contains about 50 gallons. Require pupils to memorize the fact printed in italics.

† *Prove* the answer by a drawing.

Percentage — Miscellaneous.

1. If 1 spoonful of carbolic acid is mixed with 19 spoonfuls of water, what per cent of the mixture is acid? What per cent is water?

2. If 3 bushels of shelled corn are mixed and ground with 1 bushel of oats, *considering bulk only*, what part of the mixture is oats? What per cent? (a) *Considering weight only*, what part of the mixture is oats? (b) What per cent?

3. If goods are bought for $375 and sold for $465, what is the gain? (c) What is the per cent of gain?

4. If goods are bought for $375 and sold for $300, what is the loss? (d) What is the per cent of loss?

5. A merchant plans to make a profit of 25 %. How shall he mark shoes that cost $3 a pair? Rubber boots that cost $2.40 per pair? Slippers that cost 60¢ a pair? (e) What per cent must he discount his marked price to sell at cost?

6. A store rents for $680, which is 8 % of its value. How much is the store worth? (f) For how much per month must the store rent to give 12 % on its value?

7. A bill was presented for wood supposed to be 4 feet long. On investigation, the wood was found to be only 44 inches long. What % should the bill be discounted? (g) If the bill was for 4 cords at $4.50 per cord, how much should be paid?

8. Twenty-five per cent of $160 is what per cent of $50?

Percentage — Interest.

Interest at 7 % = $\frac{1}{6}$ of the interest at 6 %.

1. Interest at 5% equals ——— ——— of the interest at 6%.
2. Interest at 4% equals ——— ——— of the interest at 6%.
3. Interest at 8% equals ——— ——— of the interest at 6%.

(a) Find the interest of $600 for 1 yr. 2 mo. 15 da. at 7%.*
(b) The amount.

(c) Find the interest of $450 for 1 yr. 6 mo. 12 da. at 7%.
(d) The amount.

(e) Find the interest of $360 for 1 yr. 8 mo. 10 da. at 4%.
(f) The amount.

(g) Find the interest of $650 for 2 yr. 2 mo. 24 da. at 5%.
(h) The amount.

Find the interest at 6 %:

(i) Of $254.20 for 1 yr. 4 mo. 5 da. †
(j) Of $356.40 for 1 yr. 6 mo. 4 da.
(k) Of $438.60 for 1 yr. 2 mo. 3 da.
(l) Of $525.50 for 1 yr. 4 mo. 2 da.
(m) Of $146.30 for 1 yr. 6 mo. 1 da.
(n) Of $240 for 1 yr. 4 mo. 7 da. ‡
(o) Of $450 for 1 yr. 2 mo. 8 da.
(p) Of $620 for 1 yr. 6 mo. 9 da.
(q) Of $360 for 2 yr. 10 mo. 10 da.
(r) Of $480 for 1 yr. 2 mo. 11 da.

(s) Find the amount of $270 for 1 yr. 10 mo. 12 da. at 4%.
(t) Find the amount of $360 for 2 yr. 6 mo. 15 da. at 5%.
(u) Find the amount of $420 for 1 yr. 8 mo. 18 da. at 8%.

* To the interest of $600 for 1 yr. 2 mo. 15 da. at 6 %, add $\frac{1}{6}$ of itself.
† $8\frac{1}{3}$ per cent of the principal equals the interest.
‡ $8\frac{1}{3}$ % of $240, or $8\frac{1}{3}$ times $2.40.

Review.

1. 32 lb. cheese at 12½¢ a pound cost ——.
2. 32 lb. cheese at 13½¢ a pound cost ——.*
3. 32 lb. cheese at 11½¢ a pound cost ——.
4. 34 lb. cheese at 12½¢ a pound cost ——.
5. 36 lb. cheese at 12½¢ a pound cost ——.
6. 3½ tenths equals how many twentieths? $\frac{3\frac{1}{2}}{10} =$
7. The ratio of 3½ to 10 = the ratio of 7 to ——.
8. 3½ over 10½, equals 7 over what? $\frac{3\frac{1}{2}}{10\frac{1}{2}} =$
9. The ratio of 3½ to 10½ = the ratio of 7 to ——.
10. 3½ tenths equals how many fortieths? $\frac{3\frac{1}{2}}{10} =$
11. 3½ over 10½, equals 13 over what?
12. 5½ twelfths equals how many twenty-fourths?
13. 2¾ twelfths equals how many forty-eighths?

A owns 24 square rods of ground.
B owns 15 hundredths (.15) of an acre.
C owns a lot 10 rods by 2.4 rods.
D owns 3 twentieths ($\frac{3}{20}$) of an acre.

(a) How many hundredths of an acre does A own? C? D?

(b) How many twentieths of an acre does A own? B? C?

(c) If a watch factory makes on the average a watch in 6 minutes and 40 seconds, how many watches will be made in the month of May if the first day of May is Sunday and the factory is operated 8 hours a day?

(d) If a wagon moves forward 12 feet 6 inches at every revolution of its hind wheel, how many times will the wheel revolve in going 5 miles?

* 32 lb. at 13½¢ cost how much more than 33 lb. at 12½¢?

Miscellaneous.

(a) How many square feet in the walls and ceiling of a room 14 feet wide, 16 feet long, and 10 feet high?

(b) How many cubic feet of air in the room described in problem (a)?

1. $\frac{3}{4}$ is what part of $\frac{7}{8}$? (c) If $\frac{7}{8}$ of an acre of land is worth $141.12, how much is $\frac{3}{4}$ of an acre worth?

2. How many feet of lumber in a joist 4 in. by 10 in. by 18 ft.? (d) Find the cost at $15 per M. of 30 pcs. 4 × 10, 18.

(e) Find the cost of 35 bales of straw averaging 80 lb. each, at $6.50 per ton?

(f) A man owning $\frac{1}{2}$ of $\frac{3}{5}$ of a farm sold his interest in the farm for $1345. What was the value of the whole farm at the same rate?

(g) A cistern 8 feet square and 6 feet deep will hold (approximately) how many barrels? *

3. The sum of three numbers is 46; two of the numbers are 12 and 8; the third number is ——.

(h) The sum of three fractions is $\frac{5}{6}$; two of the fractions are $\frac{1}{3}$ and $\frac{1}{2}$. What is the other fraction?

4. In a mixture of grain there are 10 bushels of corn and 15 bushels of oats. What per cent of the mixture is corn? What per cent is oats?

(i) In a mixture of grain there are 375 bushels of corn and 125 bushels of oats. What per cent of the mixture is corn?

(j) A farmer sold 329 bushels of potatoes; these were 35 % of his whole crop. How many bushels had he left?

* See page 246.

Simple Numbers — "Square."

Teach orally the meaning and use of the term, **square**, as applied to numbers.

1. *To square a number, multiply the number by itself.*
   Square 6; 6^2 = Square 4; 4^2 = Square 8; 8^2 =
   Square 7; 7^2 = Square 3; 3^2 = Square 9; 9^2 =

(a) Find the square of 45. (b) Of 34. (c) Of 73.
(d) 24^2 = (e) 35^2 = (f) 56^2 = (g) 75^2 =

2. What is the square of 10? Of 12? Of 15?
   20^2 = 30^2 = 40^2 = 50^2 = 60^2 =

(h) Find the square of 125. (i) Of 800. (j) Of 342.

3. What is the square of $\frac{1}{2}$? Of $\frac{1}{4}$? Of $\frac{2}{5}$?
   $(\frac{1}{2})^2$ = $(\frac{2}{7})^2$ = $(\frac{5}{8})^2$ = $(\frac{1}{6})^2$ = $(\frac{3}{4})^2$ =

(k) Find the square of $\frac{5}{16}$. (l) Of $\frac{7}{12}$. (m) Of $\frac{8}{25}$.

4. What is the square of .5? Of .3? Of .7?
   $.4^2$ = $.9^2$ = $.6^2$ = $.2^2$ = $.1^2$ =

(n) Find the square of 1.2. (o) Of 2.5. (p) Of 6.4.

5. The area of a 6-foot square is ——— square feet.*

6. The area of a $\frac{1}{2}$-foot square is ——— ——— of a square foot.

7. The area of a $\frac{3}{4}$-foot square is ——— ——— of a square foot.†

8. The area of a .4-foot square is ——— ——— of a square foot.

(q) What is the area of a 3.4-inch square?

* By means of problems 5 to 8 and other similar exercises illustrated by diagrams, lead the pupil to a clear conception of the fact that *squaring the number denoting the length of the side of a square, gives the number denoting its area; that this is true, whether the number denoting the length is integral, fractional, or mixed.*

† Show by a diagram carefully drawn that the area of a $\frac{1}{2}$-foot square is ——— ——— of a square foot.

Common Fractions—Test.*

1. Add $\frac{1}{4}$ and $\frac{5}{8}$.
2. $\frac{3}{4} + \frac{1}{8} =$
3. From $\frac{5}{8}$ subtract $\frac{3}{4}$.
4. $\frac{5}{4} - \frac{3}{8} =$
5. Multiply $\frac{2}{3}$ by 45.
6. $64 \times \frac{3}{4} =$
7. Multiply $\frac{7}{8}$ by $\frac{4}{5}$.
8. $7\frac{1}{2} \times 8 =$
9. Multiply $7\frac{1}{2}$ by $\frac{1}{3}$.
10. $5\frac{1}{4} \times 3\frac{1}{2} =$
11. Divide 8 by $\frac{4}{5}$.
12. $\frac{3}{5} \div 8 =$
13. Divide $\frac{7}{8}$ by $\frac{4}{5}$.
14. $5\frac{1}{4} \div \frac{3}{5} =$
15. Divide $\frac{5}{8}$ by $2\frac{1}{2}$.
16. $4\frac{2}{3} \div 1\frac{1}{2} =$
17. Reduce $\frac{75}{100}$ to its lowest terms.
18. Reduce $9\frac{5}{8}$ to an improper fraction.
19. Reduce $\frac{144}{7}$ to a mixed number.
20. Reduce $\frac{4}{6\frac{2}{3}}$ to a simple fraction.
21. Reduce $\frac{4\frac{1}{2}}{5}$ to a simple fraction.
22. Reduce $\frac{21\frac{1}{2}}{3\frac{2}{3}}$ to a simple fraction.
23. Reduce $\frac{\frac{3}{5}}{7}$ to a simple fraction.
24. Reduce $\frac{\frac{3}{4}}{\frac{2}{3}}$ to a simple fraction.
25. Reduce $\frac{5}{7}$ of $\frac{2}{3}$ to a simple fraction.
26. Reduce $\frac{3}{5}$ and $\frac{2}{3}$ to equivalent fractions having a common denominator.
27. One half of $\frac{3}{4}$ is how many times $\frac{1}{16}$?
28. Three fourths of $\frac{1}{3}$ is what part of $\frac{9}{16}$?

* To the Teacher.—Observe that this page provides for an oral review of the work in common fractions. The pupil who has mastered the preceding pages should be able, without a pencil, to solve any problem here given with little hesitation. If serious weakness is discovered, review pages 12, 22, 32, 42, 52, etc., or such parts of these pages as may be necessary.

Decimal Fractions.*

1.

27.5'4 — 27.54 multiplied by 10 =
60 — 275.4 multiplied by 6 =
1652.4 — 27.54 multiplied by 60 =

2.

3.72'5 — 3.725 multiplied by 100 =
700 — 372.5 multiplied by 7 =
2607.5 — 3.725 multiplied by 700 =

3.

856.2 divided by 10 =
60)85'6.2 — 85.62 divided by 6 =
14.27 — 856.2 divided by 60 =

4.

734.4 divided by 100 =
1200)7'34.4 — 7.344 divided by 12 =
.612 — 734.4 divided by 1200 =

5.

6.4 divided by 1000 =
8000)'006.4 — .0064 divided by 8 =
.0008 — 6.4 divided by 8000 =

(a) 64.29 ÷ 30 =
(b) 85.36 × 60 =
(c) 753.9 ÷ 70 =
(d) 382.6 × 80 =
(e) 3.728 ÷ 40 =
(f) 7.234 × 50 =
(g) 265.2 ÷ 600 =
(h) .3625 × 400 =
(i) 52.63 ÷ 200 =
(j) 253.8 × 300 =
(k) 274.6 ÷ 500 =
(l) 7.231 × 600 =

* Observe that in all cases of multiplication and division of decimals, the separatrix correctly located indicates with certainty the place of the decimal point in the answer

Denominate Numbers — Reduction Ascending.

Reduce:

1. 120 in. to yd. and ft.*	(a) 1560 inches = †
2. 120 qt. to bu. and pk.	(b) 1328 quarts =
3. 120 oz. to lb. and oz.	(c) 1258 ounces =
4. 140 pt. to gal. and qt.	(d) 1420 pints =
5. 150 sec. to min. and sec.	(e) 1275 seconds =
6. 645 rd. to mi. and rd.	(f) 1736 rods =
7. 104 ft. to rd. and ft.	(g) 1264 feet =
8. 130 min. to hr. and min.	(h) 1624 minutes =
9. 240 hr. to wk. and da.	(i) 1872 hours =
10. 250 cm. to m. and dm.	(j) 1490 centimeters =

1 fathom (fth.) **is 6 feet.**
1 hand is 4 inches.

11. If the water in a lake is 15 fathoms deep, how many feet deep is it?

(k) Change 245 fathoms to feet.

(l) Change 1479 feet to fathoms.

12. If a horse is $15\frac{1}{2}$ hands high, ‡ how many inches high is he? How many feet high?

13. Are you as tall as a horse that is $16\frac{1}{4}$ hands high?

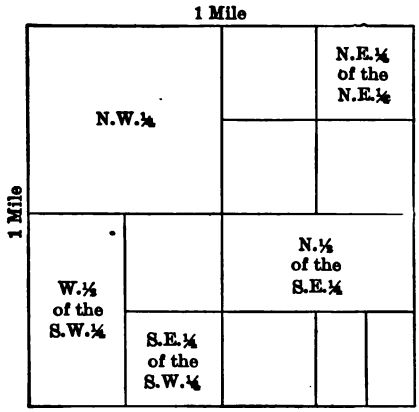
14. At 35¢ an hour, in one day of 8 hours a man earns _____.

(m) At 35¢ an hour, how much does a man earn in the month of May, if he works 9 hours a day every day except the Sundays of the month, the first day of the month being Sunday?

* 120 inches equal 10 feet; 10 feet equal —— yd. —— ft.
† Change as indicated in the corresponding problem in the first column.
‡ Horses are measured over the front feet.

Measurements.

A section (of land) = 1 sq. mile = 640 acres.



1. Mr. A owns the N.W.¼ * of a section of land. How many acres does he own?

2. How many acres has Mr. B, who owns the N.E.¼ of the N.E.¼ of the same section? How many rods from Mr. B's west line to Mr. A's east line?

3. How many acres has Mr. C, who owns the N.½ of the S.E.¼ of the section? How many rods from Mr. B's north line to Mr. C's south line?

4. How many acres has Mr. D, who owns the W.½ of the S.W.¼ and the S.E.¼ of the S.W.¼ of the section?

5. What is the perimeter of Mr. A's farm? † Of Mr. B's? Of Mr. C's? Of Mr. D's?

(a) How many acres in a strip 17 rods wide on the north side of Mr. A's land?

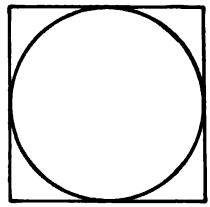
(b) How many acres in the E.½ of the S.E.¼ of the S.E.¼ of the section? Find this piece in the diagram.

* To be read, *north-west fourth*, or *north-west quarter*.
† Answer first in miles, then in rods.

Ratio and Proportion.

Circles and squares compared.

Teach the meaning and use of the terms circumference and diameter.



The circle is that which is *included within the circumference.*

1. If from a square piece of paper you cut the largest possible circle, what part of the paper will you cut away? *

2. *A circle is a little more than* $\frac{3}{4}$ † *of the smallest square from which it could have been cut.*

Observe that the diameter of a circle is equal to the side of the smallest square from which it could have been cut.

3. What is the area of a 1-inch circle? ‡

4. What is the area of a 2-inch square? Of a 2-inch circle?

5. What is the area of a 3-inch square? Of a 3-inch circle?

6. What is the area of a 4-inch square? Of a 4-inch circle?

Regarding $\frac{3}{4}$ as the ratio of the area of a circle to the area of the smallest square from which it could have been cut, first find the approximate area of each of the following without a pencil; then use .78 as the ratio and solve with a pencil:

(a) A 5-inch circle. (b) A 6-inch circle.

(c) A 7-inch circle. (d) An 8-inch circle.

(e) A 9-inch circle. (f) A 10-inch circle.

* Nearly one fourth of it. † .78 +

‡ A little more than $\frac{3}{4}$ of a square inch; more accurately, a little more than .78 of a square inch.

Percentage—Interest.

(a) Find the interest on \$500 at 6 %, from Feb. 10, 1895, to June 22, 1896.

From Feb. 10, 1895, to Feb. 10, 1896, it is ——— yr.
From Feb. 10, 1896, to June 10, 1896, it is ——— mo.
From June 10, 1896, to June 22, 1896, it is ——— da.
What per cent of the principal equals the interest?

(b) Find the interest on \$475 at 6 %, from March 25, 1894, to October 4, 1896.

From March 25, 1894, to Mch. 25, 1896, it is ——— yr.
From Mch. 25, 1896, to Sept. 25, 1896, it is ——— mo.
From Sept. 25, 1896, to Oct. 4, 1896, it is ——— da.
What per cent of the principal equals the interest?

(c) Find the interest on \$342 at 6 %, from Jan. 12, 1894, to Aug. 22, 1897.

From Jan. 12, 1894, to Jan. 12, 1897, it is ——— yr.
From Jan. 12, 1897, to Aug. 12, 1897, it is ——— mo.
From Aug. 12, 1897, to Aug. 22, 1897, it is ——— da.
What per cent of the principal equals the interest? †

(d) Find the interest on \$650 at 6 %, from July 24, 1895, to Nov. 5, 1897.

From July 24, 1895, to July 24, 1897, it is ——— yr.
From July 24, 1897, to Oct. 24, 1897, it is ——— mo.
From Oct. 24, 1897, to Nov. 5, 1897, it is ——— da.
What per cent of the principal equals the interest?

(e) Find the interest on \$830 at 6 %, from Sept. 15, 1894, to Mch. 21, 1897.

From Sept, 15, 1894, to Sept. 15, 1896, it is ——— yr.
From Sept. 15, 1896, to Mch. 15, 1897, it is ——— mo.
From Mch. 15, 1897, to Mch. 21, 1897, it is ——— da.
What per cent of the principal equals the interest?

Find the interest at 6 %.

(f) On \$740, from Jan. 1, 1894, to Apr. 13, 1896.

(g) On \$680, from Oct. 1, 1895, to March 1, 1897.

(h) On \$350, from May 20, 1894, to Oct. 4, 1897.

† The interest for 3 yr. equals 18 % of the principal; for 7 mo. $3\frac{1}{2}$ %; for 10 da. ($\frac{10}{30}$), $\frac{1}{6}$ %. 18 % + $3\frac{1}{2}$ % + $\frac{1}{6}$ % = $21\frac{2}{3}$ %.

Percentage—Interest.

Teach the meaning and use of the term face as applied to commercial paper.

On the first day of June, 1896, Mr. Hardy borrowed of Mr. Randall $375. For this Mr. Hardy gave to Mr. Randall the following promissory note:

$375. AURORA, Ill., June 1, 1896.

For value received, one year after date, I promise to pay to Louis Randall, or order, Three Hundred Seventy-five Dollars, with interest at six per cent.

H. F. HARDY.

(a) How much money must Mr. Hardy pay to Mr. Randall on June 1, 1897?

The following note was given in settlement of an account:

$346.50. SUGAR GROVE, Ill., July 12, 1895.

On or before January 1, 1897, I promise to pay to the order of J. S. Miller, Three Hundred Forty-six and $\frac{50}{100}$ Dollars, with interest at six per cent until paid.

HIRAM HENDERSON.

(b) How much money will be required to pay this note in full on the 24th day of Sept., 1896?

(c) Face of note, $430; rate, 6%; time, from Jan. 1, 1895, to July 1, 1896. Find the amount.

(d) Face of note, $364; rate, 7%; time, from March 1, 1896, to Sept. 1, 1896. Find the amount.

(e) Face of note, $560; rate, 6%; time, from Jan. 1, 1895, to May 21, 1897. Find the amount.

Review.

1. The square of 6 is ——. 8^2 =

2. If there are 6 linear units in the side of a square, there are —— surface units in the area of the square.

3. The square of $6\frac{1}{2}$ is ——. $(5\frac{1}{2})^2$ =

4. If there are $6\frac{1}{2}$ linear units in the side of a square, there are —— surface units in the area of the square.

5. The square of 2.5 is ——. (a) $(2.7)^2$ =

6. If there are 2.5 linear units in the side of a square there are —— surface units in the area of the square.

(b) If the side of a square is 2.8 feet, what is the area of the square? *

7. Twenty-five fathoms equal —— feet.

(c) One mile is how many fathoms?

8. The tallest horse I ever saw was 18 hands high. How many inches high was the horse? How many feet high?

9. The smallest pony I ever saw was a Shetland 27 inches high. How many hands high was the pony?

10. How many acres in a quarter section of land?

(d) Mr. Holland's farm is the south half of the N. W. $\frac{1}{4}$, and the west half of the S. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of section No. 20. Make a diagram showing the place of the farm in the section. How many acres in the farm? (e) How long is the farm? (f) How wide is it?

11. What is the area of a 20-foot square? Of a 20-foot circle?

(g) What is the area of a circle that is 45 feet in diameter? †

* You cannot multiply 2.8 feet by 2.8 feet; but the square of 2.8 is 7.84, and there are 7.84 square feet in the area of the square.

† Use .78 as the ratio of a circle to its circumscribed square.

Miscellaneous.

1. What number must be added to 53 that the sum may contain 12 exactly 5 times?

(a) What number must be added to 856 that the sum may contain 36 exactly 24 times?

(b) Mary's mother gives her 2 cents each time she washes the dishes, but requires Mary to pay for all the dishes she breaks. The first day of June was Wednesday, and during the month she washed them twice every Sunday and three times a day on other days of the week. She broke 25¢ worth of dishes. How much must her mother pay her for her work in June?

(c) Find the sum of all the integral numbers that can be expressed by the figures 2, 3, and 4, using all the figures each time.

(d) How many strokes does the hammer of a clock strike in a week?

(e) George Washington was born February 22, 1732. If he were living, how old would he be to-day?

2. Arthur and Harry together have 21 cents, and Arthur has 3 cents more than Harry. How many cents has each?*

(f) A and B together have $16.25, and A has $57 more than B. How many dollars has each?

3. William and Charles together have 84¢, and Charles has 6 times as much as William. How many cents has each?†

(g) C and D together have $595, and D has 6 times as much as C. How many dollars has each?

* Take 3 cents away from Arthur and the two boys together would then have 18 cents, and each would have one half that amount.

† 84¢ is how many times William's money?

Simple Numbers — Square Root.

Teach orally the meaning and use of the expressions, **square root** and **perfect square**, and of the *sign of square root.*

1. *The square root of a number is one of its two equal factors.*

2. The square root of 4 (2 × 2) is ——. $\sqrt{9}$ =

3. The square root of 16 (4 × 4) is ——. $\sqrt{36}$ =

4. The square root of 49 (7 × 7) is ——. $\sqrt{25}$ =

5. $\sqrt{81}$ = $\sqrt{144}$ = $\sqrt{100}$ = $\sqrt{64}$ = $\sqrt{121}$ = *

6. $\sqrt{225}$ (5 × 5 × 3 × 3) is (5 × 3) = ——.

7. $\sqrt{900}$ (5 × 5 × 3 × 3 × 2 × 2) is (5 × 3 × 2) = ——.

8. $\sqrt{256}$ (2 × 2 × 2 × 2 × 2 × 2 × 2 × 2) is (2 × 2 × 2 × 2) = ——.

(a) $\sqrt{196}$ = (b) $\sqrt{484}$ = (c) $\sqrt{324}$ =

(d) $\sqrt{1225}$ = (e) $\sqrt{400}$ = (f) $\sqrt{1600}$ =

9. All the numbers given thus far in the problems on this page are perfect squares; that is, each can be resolved into two equal factors. 8, 10, ——, ——, and —— are not perfect squares. *No number can be found which, when multiplied by itself, will give as a product* 8 *or* 10 *or* 12.

10. The square root of 12 is more than 3 and less than 4. Is it more or less than $3\frac{1}{2}$? $(3\frac{1}{2})^2$ =

11. The square root of 20 is more than —— and less than ——. Is it more or less than $4\frac{1}{2}$? $(4\frac{1}{2})^2$ =

12. In a certain square floor there are 144 square feet. How many feet across one side of it?

(g) In a certain square floor there are 576 square feet. How many feet across one side of it?

* The answers to problems 1 to 5 inclusive should be memorized.

Common Fractions.

1. *If the numerator and the denominator of a fraction are perfect squares, the fraction itself is a perfect square.* $\frac{4}{9}$ is a perfect square; its square root is ———. $(\frac{2}{3})^2 =$

2. Is $\frac{4}{5}$ a perfect square? $\frac{9}{16}$? $\frac{7}{16}$? $\frac{4}{25}$? *

3. $\sqrt{\frac{9}{25}} =$ $\sqrt{\frac{16}{25}} =$ $\sqrt{\frac{25}{36}} =$ $\sqrt{\frac{16}{49}} =$

4. $\sqrt{\frac{1}{4}} =$ † $\sqrt{\frac{1}{25}} =$ $\sqrt{\frac{1}{9}} =$ $\sqrt{\frac{1}{100}} =$

5. $\sqrt{\frac{25}{4}} =$ $\sqrt{\frac{144}{25}} =$ $\sqrt{\frac{121}{100}} =$ $\sqrt{\frac{81}{4}} =$

6. A square piece of tinfoil contains $\frac{1}{4}$ of a square foot. Its side is ——— ——— of a foot. $\sqrt{\frac{1}{4}} =$

$\frac{1}{2} \times \frac{1}{2} =$ $(\frac{1}{2})^2 =$

7. A square piece of oil-cloth contains $\frac{4}{9}$ of a square yard. Its side is ——— ——— of a yard. $\sqrt{\frac{4}{9}} =$

$\frac{2}{3} \times \frac{2}{3} =$ $(\frac{2}{3})^2 =$

8. A square piece of zinc contains $6\frac{1}{4}$ ($\frac{25}{4}$) square feet. Its side is ———. $\sqrt{\frac{25}{4}} =$ $\frac{5}{2} \times \frac{5}{2} =$

$2\frac{1}{2} \times 2\frac{1}{2} =$ $(2\frac{1}{2})^2 =$ $(\frac{5}{2})^2 =$

9. A square rug contains $12\frac{1}{4}$ ($\frac{49}{4}$) square yards. Its side is ———. $\sqrt{\frac{49}{4}} =$ $\frac{7}{2} \times \frac{7}{2} =$ $3\frac{1}{2} \times 3\frac{1}{2} =$

$(3\frac{1}{2})^2 =$ $(\frac{7}{2})^2 =$

10. $\sqrt{20\frac{1}{4}} =$ ‡ $\sqrt{30\frac{1}{4}} =$ $\sqrt{42\frac{1}{4}} =$

* Before deciding that a fraction is not a perfect square, be sure that it is in its lowest terms. Is $\frac{4}{16}$ a perfect square?

† The square root of 1 is 1.

‡ Change to 4ths.

Decimal Fractions.

1. The numerator of .25 is 25; the denominator is 100.

2. The numerator of .09 is ——; the denominator is ——.

3. The numerator of .7 is ——; the denominator is ——.

4. The numerator of .046 is ——; the denominator is ——.

5. *If the numerator of any number of hundredths is a perfect square, the fraction is a perfect square.**

6. Is .25 a perfect square? .17? .09?

7. $\sqrt{\frac{16}{100}}$ = $\sqrt{.16}$ = $\sqrt{\frac{25}{100}}$ = $\sqrt{.25}$ =

8. $\sqrt{\frac{1}{100}}$ = $\sqrt{.01}$ = $\sqrt{\frac{81}{100}}$ = $\sqrt{.81}$ =

9. $\sqrt{\frac{144}{100}}$ = $\sqrt{1.44}$ = $\sqrt{\frac{225}{100}}$ = $\sqrt{2.25}$ =

10. A square piece of tinfoil contains .25 of a square foot.† Its side is —— of a foot. $\sqrt{.25}$ = $.5 \times .5$ = $.5^2$ =

11. A square piece of zinc contains 6.25 ($\frac{625}{100}$) square feet. ‡ Its side is ——. $\sqrt{\frac{625}{100}}$ = 2.5×2.5 =

12. A square rug contains 12.25 ($\frac{1225}{100}$) square yards.§ Its side is ——. $\sqrt{\frac{1225}{100}}$ = $\frac{35}{10} \times \frac{35}{10}$ = 3.5×3.5 = $(3.5)^2$ = $(\frac{35}{10})^2$ =

13. $\sqrt{.64}$ = $\sqrt{1.21}$ = $\sqrt{.49}$ =

(a) $\sqrt{4.84}$ = (b) $\sqrt{3.24}$ = (c) $\sqrt{2.25}$ =

* Is this true of every number of tenths? thousandths?

† Compare this problem with problem 6, page 262. ‡ Compare this problem with problem 8, page 262.

§ Compare this problem with problem 9, page 262.

Denominate Numbers.

1 cental = 100 pounds.

1. One cental of oats equals —— and —— bushels.

2. One cental of wheat equals —— and —— —— bushels.

3. Five bushels of oats are —— and —— —— centals.

4. Five bushels of wheat are —— centals.

1 chain (ch.) = 4 rods.*

5. A square chain equals —— square rods.

6. A square chain is —— —— of an acre.

7. A piece of land 2 chains by 3 chains is what part of an acre? 4 ch. by 2 ch.?

Find the number of acres in each of the following:

(a) 6 ch. by 4 ch. (b) 6.5 ch. by 4 ch.

(c) 9 ch. by 5 ch. (d) 8 ch. by 20 rods.

(e) 6 ch. by 3.2 ch. (f) 12 ch. by 21 rods.

(g) 4 ch. by 4.5 ch. (h) 18.5 ch. by 6.2 ch.

(i) One mile is how many chains?

(j) The perimeter of a section of land is how many chains? (k) The perimeter of a quarter section?

(l) One hundred ninety eight feet equals how many chains?

8. Sixty-six feet = —— rd.

9. Sixty-six feet = —— ch.

10. Sixty-six feet = —— fth.

11. Sixty-six feet = —— yd.

* The chain to which reference is here made is known as the land surveyor's *chain*. *Civil engineers* use a chain 100 feet long.

Measurements.

A township = 36 sections.*

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

1. How far is it from the northeast corner of section 1 to the northwest corner of section 6?

2. Going on section lines, how far must one travel to go from the northwest corner of section 6 to the southeast corner of section 36?

(a) If there is a road on every section line, how many miles of road in and around one township?

(b) If the roads are 4 rods wide, how much land in the road extending from the northwest corner of section 1 to the northwest corner of section 36?

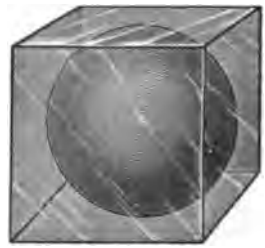
(c) Mr. A owns the northeast quarter of section 19; Mr. B owns the southwest quarter of section 2. How many miles by section lines from Mr. A's land to Mr. B's land?

(d) How many acres of land, including the roads, in a full township?

* Review page 255.

Ratio and Proportion.

Spheres and cubes compared.



1. If from a cube of dried clay you cut the largest possible sphere, what part of the clay will you cut away? *

2. *A sphere is a little more than* ½ † *of the smallest cube from which it could have been cut.*

Observe that the diameter of the sphere equals the side of the smallest cube from which it could have been cut.

3. What is the solid content of a 1-inch sphere? ‡

4. What is the solid content of a 2-inch cube? Of a 2-inch sphere?

5. What is the solid content of a 3-inch cube? Of a 3-inch sphere?

6. What is the solid content of a 4-inch cube? Of a 4-inch sphere?

Regarding ½ as the ratio of the solid content of a sphere to the solid content of the smallest cube from which it could have been cut, first find the approximate solid content of each of the following without a pencil; then use .52 as the ratio and solve with a pencil.

(a) A 5-inch sphere. (b) A 6-inch sphere.
(c) A 7-inch sphere. (d) An 8-inch sphere.
(e) A 9-inch sphere. (f) A 10-inch sphere.

* Nearly one half of it. † .52 +

‡ A little more than ½ a cubic inch; more accurately, a little more than .52 of a *cubic inch.*

Percentage — Bank Discount.

Teach the meaning and use of the terms **maturity** and **proceeds** as applied to commercial paper.

On June 10, 1896, Mr. E. R. Allen wished to borrow money from the Security Savings Bank. He therefore drew the following note and asked Mr. S. U. Spencer to sign the note with him. This, for good and sufficient reasons, Mr. Spencer consented to do.

$1250. No. 246.

WAUKEGAN, Ill., June 10, 1896.

On the 10th day of July, 1896, for value received, I, or we, or either of us, promise to pay to the order of the

SECURITY SAVINGS BANK,

One Thousand Two Hundred Fifty and $\frac{NO}{100}$ Dollars, payable at Security Savings Bank, at Waukegan, Ill., with interest at seven per cent per annum after July 10, 1896, until paid.

E. R. ALLEN.
S. U. SPENCER.

The Bank officials accepted this note, June 10, 1896, and paid to Mr. Allen $1250 less the interest at 6 % on this sum from the date of discounting the note to the date of maturity. The amount paid to Mr. Allen is called the proceeds of the note.

1. Upon what day was the note discounted?
2. What was the date of maturity?

(a) What were the proceeds of this note? that is, how much did Mr. Allen receive for it at the Bank?

3. When ought Mr. Allen to pay this note?
4. How much must he pay if he pays it at maturity?

(b) Find the proceeds of the following note:

Face, $460; date of discounting, April 15, 1896; date of maturity, June 15, 1896; rate of discount, 6 per cent.

Percentage—Partial Payments.

The expression partial payment as applied to commercial paper, means *part payment* of a note or other written agreement to pay a sum of money.

$800. JACKSONVILLE, Ill., Jan. 1st, 1895.

For value received, on or before the first day of January, 1897, I promise to pay to the order of E. M. Harris, Eight Hundred dollars, with interest at the rate of six per cent per annum until paid.

JOHN RAYMOND.

Upon the back of this note appear the following:
Received July 1, 1895, Two Hundred Dollars.
Received July 1, 1896, Three Hundred Dollars.

(a) How much was due on the above note, Jan. 1, 1897?

1. Amount due July 1, 1895, $800 + $24 interest = $824.
   Subtract first payment, $824 − $200 = $624.
2. Amount due July 1, 1896, $624 + $37.44 int. = $661.44
   Subtract the second payment, $661.44 − $300 = $361.44
3. Amount due Jan. 1, 1897, $361.44 + $10.84 interest, = $372.28

(b) Face of note, $750; date, June 1, 1895; rate, 6 %.
Partial payments were made upon this note as follows:
Dec. 1, 1895, $250. Apr. 1, 1896, $340.
Find the amount due, June 1, 1896.

(c) Face of note, $500; date, Aug. 10, 1895; rate, 7 %.
Partial payments were made upon this note as follows:
June 10, 1896, $300. Nov. 10, 1896, $100.
Find the amount due, Aug. 10, 1897.

TO THE TEACHER.—Explain to the pupil that any partial payment that is not equal to the interest due at the time when the payment is made, must be joined to the next payment if another is made; otherwise the sum paid must be deducted *from the amount* of the principal at the time of settlement. See Art. 98, page 288.

Review.

1. The square of 13 is ——. (a) 27^2 =

2. The square root of 400 is ——. (b) $\sqrt{3025}$ =

3. The square of $\frac{3}{7}$ is ——. $(\frac{3}{7})^2$ =

4. The square root of $\frac{9}{64}$ is ——. $\sqrt{\frac{16}{121}}$ =

5. The square of .6 is ——. $(1.5)^2$ =

6. The square root of .81 is ——. $\sqrt{.64}$ =

7. The square root of 40 is more than —— and less than ——. Is it more or less than $6\frac{1}{2}$?

8. In a certain square field there are 225 square rods. How many rods across one side of it?

9. What is the perimeter of the field described in problem 8? (c) How many acres in the field?

10. Tell which of the following are perfect squares and which are not: .9, 24, 1.6, .016, $\frac{9}{49}$.

11. Four bushels of oats are how many pounds more than 1 cental?

(d) Seventy-five bushels of oats are how many centals?

12. Four bushels of wheat are how many pounds less than 3 centals?

(e) Seventy-five bushels of wheat are how many centals?

13. A piece of land 7 chains by 3 chains is how much more than 2 acres?

(f) What is the value of a rectangular piece of land 15 chains by 25 chains at $50 per acre?

14. A 12-inch sphere is what part of a 12-inch cube?

(g) If a 12-inch cube of oak weighs 50 lb., how much does a 12-inch sphere of the same material weigh?

(h) How many pounds of water will fill a spherical tank 12 inches in diameter?

Miscellaneous.

(a) If sheep average $8\frac{1}{2}$ lb. of wool each per year, how much is the wool from 136 sheep worth for 5 years at 23¢ a pound?

(b) An electric light plant consumes 2500 lb. of coal per day. At $3.75 a ton, how much will the coal cost from 12 o'clock, noon, Nov. 10th, to 12 o'clock, noon, Dec. 25th?

(c) If $\frac{4}{5}$ of the value of a farm is $3462, what is the value of $\frac{1}{5}$ of the farm?

(d) If $\frac{3}{4}$ of an acre of land cost $56, how much will $36\frac{1}{2}$ acres cost at the same rate?

1. Divide $30 between two boys so that the ratio of their shares will equal the ratio of 2 to 3.

(e) Divide $8240 between two persons so that the ratio of their shares will equal the ratio of 3 to 5.

(f) A dealer bought lead pencils at $2.25 a gross and sold them at 5¢ each. How much was his profit on 5 gross?

(g) How many days of 8 hours each will it take a man to make 1 million figure 1's if he makes them at the rate of 50 a minute?

(h) If wheat costs $1.20 a cental, at what price per bushel must it be sold to gain 8¢ on each bushel?

(i) How many 160-acre farms in a township?

(j) Find the value at $45 per acre of the land in one township.

(k) How many feet of lumber in a piece of timber 36 feet long and 12 inches square?

DEFINITIONS, EXPLANATIONS, AND TABLES.

1. **Arithmetic** treats of numbers.

2. A **unit** is one.

3. A **number** is a unit or it is composed of units.

NOTE.—A unit, though regarded as a single thing, is itself made up of minor units; thus, in the expression 1 *dozen* the major unit, *one*, includes twelve minor units, but each of the minor units becomes a major unit when considered with reference to the halves, fourths, tenths, hundredths, thousandths, etc., of which it is composed. So we may have units of hundreds, as 5 hundred; units of thousands, as 5 thousand; units of hundredths, as 5 hundredths. No unit is so small that it is not made up of minor units, and no number is so great that it may not become a minor unit of some greater number.

4. **Notation** is the art of expressing numbers by figures or other characters.

ARABIC NOTATION.

5. In the **Arabic Notation** figures are employed. The figures are: **1, 2, 3, 4, 5, 6, 7, 8, 9, 0.**

In the number 7235.468, the figure 5 is said to stand in the first place and represents units of the first order; the figure 3 stands in the second place and represents units of the second order; the figure 2 stands in the third place and represents units of the third order; the figure 7 stands in the fourth place and represents units of the fourth order; the figure 4 stands in the first decimal place and represents units of the first decimal order; the figure 6 stands in the second decimal place and represents units of the second decimal order; the figure 8 stands in the third decimal place and represents units of the third decimal order.

A unit of any order equals ten units of the next lower order.

6. Three integral orders form a period.

The following shows the names of periods and orders:

hundreds.	tens.	units.	hundreds.	tens.	units.	hundreds.	tens.	units.	hundreds.	tens.	units.	hundreds.	tens.	units.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.
2	7	4,	3	5	8,	6	0	4,	3	6	0,	2	7	6.	1	4	5	6	3	8
trillion			billion			million			thous'd			units.								

(1) 1 tenth and 4 hundredths = 14 hundredths.

(2) 1 tenth, 4 hundredths, and 5 thousandths = ——— thousandths.

(3) 1 tenth, 4 hundredths, 5 thousandths, and 6 ten-thousandths = ——— ten-thousandths.

(4) 1 tenth, 4 hundredths, 5 thousandths, 6 ten-thousandths, and three hundred-thousandths = ——— hundred-thousandths.

(5) 1 tenth, 4 hundredths, 5 thousandths, 6 ten-thousandths, 3 hundred-thousandths, and 8 millionths = ——— millionths.

(6) In reading decimals the name of the lowest order is usually given to the entire decimal; thus, .345 is usually read 345 thousandths; not, 3 tenths, 4 hundredths, and 5 thousandths.

(7) A mixed decimal *may* be read by giving the name of the lowest order to the entire number; thus, 6.425 may be read 6425 thousandths; 27.4 may be read 274 tenths.

NOTE.—Observe that the name of the 1st period is units; of the second, thousand; of the 3d, million; of the 4th, billion, and of the 5th, trillion. The names of the periods above trillion are as follows: 6th, quadrillion; 7th, quintillion; 8th, sextillion; 9th, septillion; 10th, octillion; 11th, nonillion; 12th, decillion; 13th, undecillion; 14th, duodecillion; 15th, tredecillion; 16th, quatuordecillion; 17th, quindecillion; 18th, sexdecillion; 19th, septendecillion; 20th, octodecillion; 21st, novendecillion; 22nd, vigintillion.

ROMAN NOTATION.

7. In the **Roman Notation,** seven capital letters are employed. The letters are—I, V, X, L, C, D, M. I. = 1, V. = 5, X. = 10, L. = 50, C. = 100, D. = 500, M. = 1000.

PRINCIPLES.

(1) When a letter is repeated its value is repeated. XX. = 20. CC. = 200. III. = 3.

(2) When a letter is placed before one of greater value, the difference of their values must be taken. IX. = (10-1) = 9. IV. = (5-1) = 4.

(3) When a letter is placed between two letters, each of greater value, its value is taken from the last letter. XIX. = 10 + (10 — 1) = 19.

(4) Placing a short horizontal line over a letter, multiplies its value by 1000. $\bar{X}$. = 10,000.

NOTE.—Sometimes small letters are employed instead of capitals; thus, ii. = 2, iv. = 4, etc.

ADDITION.

8. Addition is the process of uniting two or more numbers into one number.

The **sum** is the number obtained by adding.

The **sign**, +, which is read *plus*, indicates that the numbers between which it is placed are to be added; thus, 6 + 4, means that 4 is to be added to 6.

The **sign**, =, which is read *equal* or *equals*, indicates that that which is on the left of the sign is equal to that which is on the right of the sign; thus, 3 + 4 = 7. 5 + 4 + 2 = 6 + 5.

NOTE.—There are 45 primary facts of addition which should be perfectly memorized. They are as follows:

1	2	2	3	3	4	3	4	5	4	5	6	4	5	6
1	1	2	1	2	1	3	2	1	3	2	1	4	3	2
2	3	4	4	5	5	6	6	6	7	7	7	8	8	8
7	5	6	7	8	5	6	7	8	9	6	7	8	9	6
1	4	3	2	1	5	4	3	2	1	5	4	3	2	6
8	9	9	9	9	10	10	10	10	10	11	11	11	11	12
7	8	9	7	8	9	7	8	9	8	9	8	9	9	9
5	4	3	6	5	4	7	6	5	7	6	8	7	8	9
12	12	12	13	13	13	14	14	14	15	15	16	16	17	18

The nine facts given in full-faced type should receive special attention. Pupils seldom fail to memorize the other thirty-six facts.

9. Subtraction is the process of taking one number from (out of) another.

The **minuend** is the number from which another number is taken.

The **subtrahend** is the number taken from another number.

The **difference** is the number obtained by subtracting.

The sign, —, which is read *minus*, indicates that the number that follows the sign is to be taken from (out of) the number that precedes it; thus, 8 — 3, means that 3 is to be taken from (out of) 8.

To "prove" a problem in subtraction: Add the subtrahend and the difference. The sum thus obtained should equal the minuend.

NOTE.—There are 81 primary facts of subtraction; that is, 2 for every primary fact of addition except the 1st, 3rd, 7th, 13th, 21st, 30th, 37th, 42nd, 45th. The facts of subtraction should be learned while learning the facts of addition. If a pupil really knows that 8 and 9 equal 17, he knows also that 17 less 8 = 9, and 17 less 9 = 8.

10. **Multiplication** is the process of taking one number as many times as there are units in another number.

The **multiplicand** is the number taken or repeated.

The **multiplier** is the number that shows how many times the multiplicand is taken or repeated.

The **product** is the number obtained by multiplying.

The **sign,** ×, which is read *multiplied by*, indicates that the number preceding the sign is to be repeated as many times as there are units in the number following the sign.

NOTE 1.—When this sign is followed by a fraction, it indicates that *a certain part* of the number preceding the sign is to be repeated as many times as there are units in the numerator of the fraction following the sign; thus, 12 × ¼, means that 1 fourth of 12 is to be repeated 3 times; 50 × .5, means that that 1 tenth of 50 is to be repeated 5 times.

NOTE 2.—This sign, ×, is sometimes so used that it means *times;* thus, 3 × 86, must be read 3 *times* 86; 3 × 6 may be read 3 *multiplied* by 6 or 3 *times* 6. As employed in this book, *the sign never means times.* Instead of 3 × 86, the author prefers 86 × 3. It is believed that the restriction of this sign to one use and to one meaning, at least in the first years of arithmetical study, will promote clearness of thought and accuracy in expression.

NOTE 3.—Without danger of ambiguity, the sign, ×, is sometimes used in this book and elsewhere, in place of the word *by;* thus, 1 pc. of 2 × 4, 12 (to be read, 1 *pc. of* 2 *by* 4, 12), means 1 piece of lumber 2 inches thick, 4 inches wide, and 12 feet long.

"To prove" a problem in multiplication: Divide the product by the multiplier. The quotient thus obtained should equal the multiplicand.

NOTE.—Besides those problems in which either the multiplicand or the multiplier is 1, and which require no effort on the part of the pupil beyond learning to count, there are 64 primary facts of multiplication that must be perfectly memorized before the pupil can acquire facility in the process. They are as follows;

2 times 2 = 4	3 times 2 = 6	4 times 2 = 8	5 times 2 = 10
2 " 3 = 6	3 " 3 = 9	4 " 3 = 12	5 " 3 = 15
2 " 4 = 8	3 " 4 = 12	4 " 4 = 16	5 " 4 = 20
2 " 5 = 10	3 " 5 = 15	4 " 5 = 20	5 " 5 = 25
2 " 6 = 12	3 " 6 = 18	4 " 6 = 24	5 " 6 = 30
2 " 7 = 14	3 " 7 = 21	4 " 7 = 28	5 " 7 = 35
2 " 8 = 16	3 " 8 = 24	4 " 8 = 32	5 " 8 = 40
2 " 9 = 18	3 " 9 = 27	4 " 9 = 36	5 " 9 = 45
6 times 2 = 12	7 times 2 = 14	8 times 2 = 16	9 times 2 = 18
6 " 3 = 18	7 " 3 = 21	8 " 3 = 24	9 " 3 = 27
6 " 4 = 24	7 " 4 = 28	8 " 4 = 32	9 " 4 = 36
6 " 5 = 30	7 " 5 = 35	8 " 5 = 40	9 " 5 = 45
6 " 6 = 36	7 " 6 = 42	8 " 6 = 48	9 " 6 = 54
6 " 7 = 42	7 " 7 = 49	8 " 7 = 56	9 " 7 = 63
6 " 8 = 48	7 " 8 = 56	8 " 8 = 64	9 " 8 = 72
6 " 9 = 54	7 " 9 = 63	8 " 9 = 72	9 " 9 = 81

Although a knowledge of the "elevens" and "twelves" of the table as it is usually given is convenient and helpful, it will be observed that it is not a necessity in the process of multiplication. The pupil who masters the table as given above will have in his mental possession the fundamental facts of simple multiplication.

11. Division is one of two processes;—

(1) It is finding how many times one number is contained in another number. $18 ÷ $2 = 9.

(2) It is finding one of the equal parts of a number. $18 ÷ 2 = $9.

The **dividend** is the number divided.

The **divisor** is the number by which we divide.

The **quotient** is the number obtained by dividing.

The **sign**, ÷, which is read *divided by*, has two meanings in concrete problems, which correspond to the two cases given above. In Case I., it means, *find how many times the divisor is contained in the dividend.* In Case II. it means, *separate the dividend into as many equal parts as there are units in the divisor and note the size of the parts.*

$18 ÷ $2, means, find how many times $2 are contained in $18.

$18 ÷ 2, means, find 1 half of $18.

NOTE.—The two cases of division are ultimately resolvable into one. But the teacher will find great difficulty, who attempts to make clear to young students the fact that we find one fifth of 15 apples, for instance, by finding how many times 5 apples are contained in 15 apples. "One fifth of 15 apples is as many apples as 5 apples are contained times in 15 apples. 5 apples are contained in 15 apples 3 times; therefore one fifth of 15 apples is 3 apples." A formal analysis of this kind

thrust upon fifth or sixth grade students, and its reproduction demanded, might be fatal to clear thinking on the part of the student. The author experimented along this line many years ago to his serious discomfiture.

To "prove" a problem in division, multiply the quotient by the divisor. The product thus obtained should equal the dividend.

NOTE 1.—There can be no "remainder" in a complete division. Therefore, the product of the quotient and divisor is *always* equal to the dividend. We may have a remainder when the process of dividing is incomplete. In such a case, the quotient, too, is incomplete, and the remainder is the undivided part of the dividend. To the product of the *incomplete quotient* and divisor, add the remainder, and the sum thus obtained will be equal to the dividend.

NOTE 2.—Besides the division of a number by a unit, there are 64 primary facts of division to be memorized by the pupil. These are the converse of the 64 facts of multiplication given on page 275. Little effort is required for their mastery beyond what must be expended in gaining a working familiarity with the facts of multiplication. The facts of division, however, must receive some attention and must be presented in their double aspect. The pupil must know that because 8 times 5 are 15 (11111 11111 11111), 5 is contained in 15 three times and one third of 15 is 5.

12. A **fractional number** is a number of the equal parts of some quantity considered as a unit; as, $\frac{2}{4}$, .7, 2 fifths.

13. An **integral number,** or an **integer,** is a number which is not, either wholly or in part, a fractional number; as, 27, 13, 246.

14. A **mixed number** is a number one part of which is integral and the other part fractional; as, $3\frac{1}{2}$, 7.3, $275\frac{1}{2}$.

15. An **exact divisor** of a number is a number that is contained in the number an integral number of times.

3 is an exact divisor of 12.
3 is not an exact divisor of 1.2.
$12\frac{1}{2}$ is an exact divisor of 100.

16. An **even number** is a number of which two is an exact divisor; as, 8, 36, 94.

An integral number is exactly divisible by two if its right-hand figure is 0, 2, 4, 6, or 8.

17. An **odd number** is an integral number of which two is not an exact divisor; as, 11, 15, 75.

An integral number is not exactly divisible by two if its right-*hand figure* is 1, 3, 5, 7, or 9.

18. A **prime number** is an integral number that has no exact integral divisors except itself and 1; as, 1, 2, 3, 5, 7, 11, 13, 17, 19, etc.

19. A **composite number** is an integral number that is not prime. It has one or more exact integral divisors besides itself and 1; as, 4, 6, 8, 9, 10, 12, 75, etc.

20. Any divisor of a number may be regarded as a **factor** of the number. An exact integral divisor of a number is an **integral factor** of the number.

3 is an integral factor of 12.
5 is an integral factor of 20.

21. An integral factor that is itself a prime number is called a **prime factor.**

3 is a prime factor of 12.
7 is a prime factor of 35.

22. An integral factor that is itself a composite number is called a **composite factor.**

6 is a composite factor of 18.
12 is a composite factor of 36.

23. *Every composite number is equal to the product of its prime factors.*

$18 = 2 \times 3 \times 3.$ $24 = 2 \times 2 \times 2 \times 3.$

NOTE.—A factor of a number may be integral, fractional, or mixed. a multiplied by $b = ab$. a and b are factors of ab. If $a = 2\frac{1}{2}$, and $b = 2$, then $ab = 5$, and $2\frac{1}{2}$ and 2 are factors of 5. All *prime* factors are integral since all prime numbers are integral.

24. A **multiple** of a number is a number that is exactly divisible by that number.

28 is a multiple of 7. It is exactly divisible by 7.
28 is a multiple of 4 and of 2.

25. *A multiple of an integral number contains all the prime factors of that number and some other integral factor or factors.*

60 ($2 \times 2 \times 3 \times 5$) is a multiple of 12 ($2 \times 2 \times 3$). It contains all the prime factors of 12, and, besides these, the factor 5. 12 is contained in 60, 5 times.

26. A **common multiple** of two or more numbers is a number that is exactly divisible by each of them.

24 and 48 and 72 are common multiples of 8 and 12. Each of these is exactly divisible by 8 and by 12.

A common multiple of two or more integral numbers contains all the prime factors of each of them and *may contain other prime factors.*

$$420 = 2 \times 2 \times 3 \times 5 \times 7.$$
$$12 = 2 \times 2 \times 3.$$
$$10 = 2 \times 5.$$

420 is a common multiple of 10 and 12. It contains all the prime factors of each of them, and besides these, the prime factor, 7.

27. The **least common multiple** (l. c. m.) of two or more numbers is the least number that is exactly divisible by each of them.

40 and 80 and 120 are common multiplies of 8 and 10. 40 is the least common multiple of 8 and 10.

The least common multiple of two or more integral numbers contains all the prime factors of each of them *and no other prime factors.*

$$60 = 2 \times 2 \times 3 \times 5.$$
$$20 = 2 \times 2 \times 5.$$
$$15 = 3 \times 5.$$

60 is the l. c. m. of 20 and 15. It contains as prime factors as many 2's, as many 3's, and as many 5's, as either of the numbers. *It contains no other prime factors.*

28. Two or more integral numbers are said to be **prime to each other** when they have no prime factor in common.

8 $(2 \times 2 \times 2)$, 9 (3×3), and 25 (5×5), are prime to each other.

When two or more numbers are prime to each other, their least common multiple is their product. The l. c. m. of 8 and 9 is 72. The l. c. m. of 5, 7, and 4 is $(5 \times 7 \times 4) = 140$.

29. The **square of a number** is the product obtained by using it twice as a factor.

The square of 7 is (7×7) 49.
The square of $\frac{3}{4}$ is $(\frac{3}{4} \times \frac{3}{4})$ $\frac{9}{16}$.
The square of .4 is $(.4 \times .4)$ .16.

30. The **square root** of a number is one of its two equal factors.

The square root of 9 is 3.
The square root of $\frac{9}{16}$ is $\frac{3}{4}$.
The square root of .16 is .4.

31. A number that is made up of two equal factors is said to be a **perfect square.**

144 (12×12) is a perfect square.
$\frac{1}{16}$ ($\frac{1}{4} \times \frac{1}{4}$) is a perfect square.
6.25 (2.5×2.5) is a perfect square.

32. A **fraction** is one or more of the equal parts of a unit.

Seven eighths is a fraction.
Five twelfths is a fraction.

33. A fraction may be expressed by two numbers, one of them being written above a short horizontal line and the other below it; thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{12}$.

The number above the line is called the **numerator.**
The number below the line is called the **denominator.**
The denominator shows the number of parts into which the unit is divided.
The numerator shows the number of parts taken.

NOTE.—The numerator tells the number of things and the denominator gives their name. In the fraction $\frac{3}{5}$ there are 4 things called fifths.

34. A fraction whose numerator is less than its denominator is called a **proper fraction.**

$\frac{3}{4}$, $\frac{5}{11}$, $\frac{1}{8}$, are proper fractions.

35. A fraction whose numerator is equal to or greater than its denominator is called an **improper fraction.**

$\frac{8}{8}$, $\frac{8}{5}$, $\frac{11}{8}$, are improper fractions.

36. Two or more fractions whose denominators are alike are said to have a **common denominator.**

$\frac{7}{13}$, $\frac{5}{13}$, $\frac{8}{13}$, and $\frac{1}{13}$, have a common denominator.
$\frac{3}{11}$, $\frac{5}{14}$, $\frac{8}{16}$, and $\frac{1}{11}$, do not have a common denominator.

37. The **terms of a fraction** are its numerator and its denominator. A fraction is said to be in its lowest terms when its numerator and its denominator are integral numbers which are prime to each other.

The terms of $\frac{3}{5}$ are 3 and 5. Three and 5 are integral numbers which are prime to each other; therefore the fraction is in its lowest terms.

38. Such expressions as the following are sometimes called **compound fractions:**

$\frac{3}{5}$ of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{7}{8}$, $\frac{3}{4}$ of $2\frac{1}{2}$.

39. A fraction either of whose terms is itself a fraction or a mixed number is called a **complex fraction.**

$\frac{\frac{2}{3}}{4}$, $\frac{3\frac{1}{2}}{3}$, $\frac{\frac{3}{5}}{\frac{3}{4}}$, are complex fractions.

40. Any fraction that is neither compound nor complex is called a **simple fraction.**

$\frac{5}{7}$, $\frac{3}{8}$, and $\frac{27}{30}$, are simple fractions.

41. A fraction whose denominator is 1 with one or more zeros annexed to it is a **decimal fraction.**

$\frac{3}{10}$, $\frac{7}{100}$, $\frac{28}{1000}$, are decimal fractions.

42. When a decimal fraction is expressed by figures, the denominator is not usually written. It is indicated by a decimal point; thus, $\frac{7}{10}$ is written, .7; $\frac{8}{100}$ is written, .08, etc. When so written, the expression may be called a **decimal.**

43. The **denominator of a decimal** is 1 with as many zeros annexed to it as there are decimal places; thus, the denominator of .8 is 1 with one zero annexed to it—that is, 10; the denominator of .027 is 1 with three zeros annexed to it—that is, 1000.

44. A **complex decimal** is a decimal and a common fraction combined in one number.

$.7\frac{1}{3}$, $.25\frac{1}{7}$, $.324\frac{2}{3}$, are complex decimals.

45. An **abstract number** is a number considered apart from any object. A **concrete number** is a number applied to objects. Seven, 5, 12, are abstract numbers. The *seven* in the expression, 7 *apples*, the *five* in the expression, 5 *books*, the *two* in the expression, 2 *inches*, are concrete numbers.

46 The expression **denominate numbers,** is applied to those concrete numbers used in exact measurements.

The *six* of the expression 6 *quarts*, the *four* of the expression 4 *inches*, are denominate numbers.

47. A **simple denominate number** is a denominate number whose units are of one kind. A **compound denominate number** is a denominate number whose units are of two or more kinds.

The *twenty-seven* of 27 feet is a simple denominate number.

The *two* and the *six* of the expression 2 *ft.* 6 *in.* together constitute a compound denominate number.

DENOMINATE NUMBER TABLES.

48. LINEAR MEASURE.

12	inches (in.)	= 1 foot (ft.).
3	feet	= 1 yard (yd.).
5½	yards	= 1 rod (rd.).
320	rods	= 1 mile (mi.).

4	inches	= 1 hand.
6	feet	= 1 fathom (fth.).
16½	feet	= 1 rod.
40	rods	= 1 furlong (fur.).
5280	feet	= 1 mile.
1760	yards	= 1 mile.
1	geographical mile (6080 feet)	= 1 knot.
3	knots	= 1 league.

49. SQUARE MEASURE.

144	square inches (sq. in.)	= 1 square foot (sq. ft.).
9	square feet	= 1 square yard (sq. yd.).
30¼	square yards	= 1 square rod (sq. rd.).
160	square rods	= 1 acre (A.).
640	acres	= 1 square mile (sq. mi.).
1	square mile	= 1 section.
160	acres	= 1 quarter section.

50. CUBIC MEASURE.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).
128 cubic feet	= 1 cord (cd.).

One foot of lumber is a piece 1 foot square and 1 inch in thickness *or its equivalent.*

NOTE.—For exception to this, see page 185.

51. LIQUID MEASURE.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).
31½ gallons	= 1 barrel (bbl.).

A kerosene barrel contains 48 to 52 gallons.
A vinegar barrel contains 45 to 50 gallons.
One gallon = 231 cubic inches.
One cubic foot = nearly 7½ gallons.

52. APOTHECARIES' FLUID MEASURE.

60 minims, or drops (♏)	= 1 fluid dram (ƒ3).
8 fluid drams	= 1 fluid ounce (ƒ℥).
16 fluid ounces	= 1 pint (O).
8 pints	= 1 gallon (Cong.).

53. DRY MEASURE.

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

One bushel = 2150.4 cubic inches.
One bushel = nearly 1¼ cubic feet.
A cylindrical vessel 18½ inches in diameter and 8 inches deep contains 1 bushel.
A "heaped bushel" = about 1½ cubic feet.

54. AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	= 1 pound (lb.).
2000 pounds	= 1 ton (T.).
20 hundredweight (cwt.).	= 1 ton.

1 cubic foot of water weighs 62½ lb., or 1000 oz.

1 bu. wheat	= 60 lb.	1 bu. rye	= 56 lb.
1 bu. potatoes	= 60 lb.	1 bu. barley	= 48 lb.
1 bu. beans	= 60 lb.	1 bu. oats	= 32 lb.
1 bu. corn	= 56 lb.	1 cental	= 100 lb.

55. APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ).
8 drams	= 1 ounce (℥).
12 ounces	= 1 pound (℔).

56. Troy Weight.

24 grains (gr.)	= 1 pennyweight (dwt. or pwt.).
20 pennyweights	= 1 ounce (oz.).
12 ounces	= 1 pound (lb.).

One pound "Troy" contains 5760 grains.
One pound "Apothecary" contains 5760 grains.
One pound "Avoirdupois" weighs 7000 grains.

Troy weight is used in weighing gold and silver.

Apothecaries' weight is used by physicians and by druggists in weighing drugs in small quantities.

Avoirdupois weight is used in weighing all ordinary substances.

57. Measure of Time.

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (da.).
7 days	= 1 week (wk.).
28 to 31 days	= 1 month (mo.).
12 months	= 1 year (yr.).
365 days	= 1 common year.
366 days	= 1 leap year.
52 wk. 1 da.	= 1 common year.
52 wk. 2 da.	= 1 leap year.

58. Miscellaneous Table.

12 ones	= 1 dozen.	14 lb.	= 1 stone.
12 dozen	= 1 gross.	100 " of nails	= 1 keg.
12 gross	= 1 great gross.	196 " of flour	= 1 bbl.
20 ones	= 1 score.	200 " of beef	= 1 bbl.
24 sheets of paper	= 1 quire.	200 " of pork	= 1 bbl.
20 quires	= 1 ream.	280 " of salt	= 1 bbl.

THE METRIC SYSTEM.

59. Linear Measure.

10 millimeters (mm.)	= 1 centimeter (cm.).
10 centimeters	= 1 decimeter (dm.).
10 decimeters	= 1 meter (m.).
10 meters	= 1 decameter (Dm.).
10 decameters	= 1 hectometer (Hm.).
10 hectometers	= 1 kilometer (Km.).
10 kilometers	= 1 myriameter (Mm.).

60. Square Measure.

100 square millimeters (qmm).	= 1 qcm.
100 square centimeters (qcm).	= 1 qdm.
100 square decimeters (qdm).	= 1 qm.
100 square meters (qm.)	= 1 qDm.

61. CUBIC MEASURE.

1000 cubic millimeters (cmm.) = 1 ccm. or cc.
1000 cubic centimenters (ccm.) = 1 cdm.
1000 cubic decimeters (cdm.) = 1 cu. m.

62. LIQUID MEASURE.

1 cubic decimeter is called a liter.
1000 cubic centimeters = 1 liter.
1000 liters = 1 cubic meter.

63. MEASURE OF WEIGHT.

The weight of 1 cubic centimeter of water is called a gram.

10 grams (g.) = 1 decagram (Dg.).
10 decagrams = 1 hectogram (Hg.).
10 hectograms = 1 kilogram (Kg.).

Observe that in all the weights and measures—

mili means *thousandths*, *deca* means *ten*,
centi means *hundredths*, *hecto* means a *hundred*,
deci means *tenths*. *kilo* means a *thousand*.

A centigram is a hundredth of a gram.
A kiloliter is a thousand liters.
A decimeter is a tenth of a meter.
A decameter is ten meters, etc.

64. Comparison of metric and non-decimal units.

1 meter is 39.37 inches.
1 kilometer is nearly ⅝ of a mile.
1 liter is a little more than a "liquid quart."
1 liter is a little less than a "dry quart."
1 gram is 15.4+ grains.
1 kilogram is 2.2+ pounds.

1 inch is 2.54+ centimeters.
1 yard is .9+ of a meter.
1 mile is 1.6+ kilometers.
1 quart (liquid) is .94+ of a liter.
1 grain is 64.7+ milligrams.
1 pound is .45+ of a kilogram.

65. A **rectangle** is a figure having four sides and four right angles. If the sides are equal the figure is a *square*. If two of the sides are longer than the other two the figure is an *oblong*.

66. A **triangle** is a figure having three sides. If the *sides are* equal it is said to be an *equilateral triangle*.

67. A **circle** is a figure bounded by a curved line every point in which is equally distant from a point within called the *center*.

68. A **regular pentagon** is a figure having five equal sides and five equal angles.

69. The **area** of a figure is the amount of its surface.

Area may be expressed in square inches, square feet, square centimeters, or any other authorized surface unit.

70. The **perimeter** of a figure is the distance around it.

71. A **solid** has length, breadth, and thickness.

72. A **rectangular solid** is bounded by six rectangular faces. If all the faces are squares the solid is a **cube.**

73. The **solid content** of a solid is the amount of its volume.

Solid content may be expressed in cubic inches, cubic feet, cubic centimeters, or any other volume unit.

74. **Ratio** is the quotient of one number divided by another number of the same kind.

The ratio of 6 inches to 2 inches is 3.
The ratio of 4 quarts to 8 quarts is $\frac{1}{2}$.

75. The **terms of the ratio** are the two numbers compared. The first term is the antecedent and the second term is the consequent.

76. The **sign** of ratio is the colon; thus, 16 : 8, means, the ratio of 16 to 8. Ratio may also be expressed by the division sign, thus, 16 ÷ 8; and as a fraction, thus, $\frac{16}{8}$.

77. The two terms of a ratio constitute a **couplet.** A **simple proportion** is made up of two couplets; thus, 2 : 6 = 5 : 15. This may be read, *The ratio of* 2 to 6 *equals the ratio of* 5 *to* 15.

The first and fourth terms of a proportion are called the **extremes,** and the second and third terms the **means.**

In every true proportion, the product of the *means* equals the product of the *extremes.*

NOTE.—In finding the product of the means and the product of the extremes, the numbers must be regarded as abstract. In the proportion, 88 : 84 = 20 bushels : 10 bushels, it would be absurd to speak of the product of 10 bushels multiplied by 84

78. Per cent means *hundredth* or *hundredths*. 1 per cent is $\frac{1}{100}$; 9 per cent is $\frac{9}{100}$; 200 per cent is $\frac{200}{100}$; $\frac{1}{2}$ per cent is $\frac{\frac{1}{2}}{100}$.

79. The **base** in percentage is the number of which hundredths are taken.

80. Commission is compensation paid by one person to another called an agent, for transacting business.

The agent's compensation is usually a sum equal to a certain per cent of the amount collected or expended.

81. Interest is compensation for the use of money.

The money upon which interest is paid is called the principal.

The principal and interest together are called the amount.

The interest is a sum equal to a certain per cent of the principal.

82. Partial payments are *part* payments on a note or other written agreement to pay money.

RULES.

83. To resolve a composite number into its prime factors.

RULE.—*Divide the number by any prime number that will exactly divide it; divide the quotient thus obtained by a prime number, and so on until a number is obtained which is itself prime. The several divisors and the last quotient are the prime factors.*

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ \hline & 5 \end{array}$$

84. To find the least common multiple of two or more numbers.

RULE.—*Divide the numbers by any prime number that is an exact divisor of two or more of them. "Bring down" undivided numbers. Divide the quotients and undivided numbers as before, and so continue until the quotients and undivided numbers are prime to each other. The product of the divisors, quotients, and undivided numbers, is the least common multiple.*

5	60	40	30
2	12	8	6
2	6	4	3
3	3	2	3
	1	2	1

$5 \times 2 \times 2 \times 3 \times 2 = 120.$

85. To reduce a fraction to higher terms.

RULE.—*Multiply each term of the fraction by the same number.*

$$\frac{7 \times 5}{8 \times 5} = \frac{35}{40}.$$

86. To reduce a fraction to its lowest terms.

RULE.—*Divide the terms by any number that will exactly divide each of them; divide these quotients as before, and so continue until quotients are obtained that are prime to each other; or, divide the numerator and the denominator by the product of their common prime factors.*

$\frac{50}{75} = \frac{10}{15} = \frac{2}{3}$

87. To reduce an improper fraction to a whole or a mixed number.

RULE.—*Divide the numerator by the denominator.*

$\frac{75}{8} = 9\frac{3}{8}$

88. To reduce a mixed number to an improper fraction.

RULE.—*Multiply the integral part of the number by the denominator of the fraction. To the product thus obtained add the numerator. Write the sum over the denominator.*

$24\frac{3}{4}$
$24 \times 4 = 96$
$96 + 3 = 99$
$24\frac{3}{4} = \frac{99}{4}$

89. To reduce fractions to equivalent fractions having a common denominator.

RULE.—*Find the l. c. m. of the denominators. This becomes the l. c. d. Take each numerator as many times as its denominator is taken to make the common denominator.*

$\frac{5}{18}, \frac{7}{24}$ l. c. d. 72
$72 \div 18 = 4$
$72 \div 24 = 3$
$\frac{5 \times 4}{18 \times 4} = \frac{20}{72}$
$\frac{7 \times 3}{24 \times 3} = \frac{21}{72}$

90. To add common fractions.

RULE.—*Reduce the fractions, if necessary, to equivalent fractions having a common denominator, add their numerators and write the sum over the common denominator.*

$\frac{5}{7} + \frac{3}{4}$
$\frac{5}{7} = \frac{20}{28}$ $\quad \frac{3}{4} = \frac{21}{28}$
$\frac{20}{28} + \frac{21}{28} = \frac{41}{28}$

91. To subtract one common fraction from another.

RULE.—*Reduce the fractions, if necessary, to equivalent fractions having a common denominator, find the difference of their numerators, and write it over the common denominator.*

$\frac{6}{7} - \frac{3}{5}$
$\frac{6}{7} = \frac{30}{35}$ $\quad \frac{3}{5} = \frac{21}{35}$
$\frac{30}{35} - \frac{21}{35} = \frac{9}{35}$

92. To multiply one common fraction by another.

RULE.—*Multiply the numerators together for the numerator of the product and the denominators together for the denominator of the product.*

$\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$

93. To divide one common fraction by another.

RULE.—*Reduce the fractions to equivalent fractions having a common denominator and find how many times the numerator of the divisor is contained in the numerator of the dividend; or, "invert the divisor and proceed as in multiplication of fractions."*

$\frac{3}{5} \div \frac{1}{4}$

$\frac{3}{5} = \frac{12}{20}$. $\frac{1}{4} = \frac{5}{20}$

$\frac{12}{20} \div \frac{5}{20} = 2\frac{2}{5}$

$\frac{3}{5} \times \frac{4}{1} = \frac{12}{5} = 2\frac{2}{5}$.

94. To "point off" in multiplication of decimals.

RULE.—*Place a separatrix in the multiplicand to indicate the part of the multiplicand that is to be multiplied by the number denoted by the right-hand figure of the multiplier. When, in the process of multiplication, the separatrix is reached, write a decimal point in the product. If this is a partial product, all other decimal points will fall directly under the point first placed.*

$$\begin{array}{r} 7^{\vee}4.63 \\ 2.3 \\ \hline 22.389 \\ 149.26 \\ \hline 171.649 \end{array}$$

95. To "point off" in division of decimals.

RULE.—*Place a separatrix in the dividend to the right of that figure which corresponds in denomination to the right hand-figure of the divisor. When, in the process of division, this separatrix is reached, write the decimal point in the quotient.*

$$.05\overline{)74.86^{\vee}5}\quad 1497.3$$

96. To reduce a common fraction to a decimal.

RULE.—*Divide the numerator by the denominator.* $\frac{3}{4} = 4\overline{)3.^{\vee}00}$ $.75$

97. To reduce a decimal to a common fraction.

RULE.—*Write the denominator under the numerator and reduce the fraction to its lowest terms.*

$.26 = \frac{26}{100} = 1\frac{3}{50}$

98. To find the amount due on a note when partial payments have been made.

"UNITED STATES RULE."—*Find the amount of the principal to the time when a payment was made which either alone or with payments previously made, equals or exceeds the interest at that time due. Subtract the payment (or the sum of payments) from the amount and regard the remainder as a new principal. Proceed in the same manner with the new principal and each succeeding one to the time of settlement.*

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